Fringe Parameter Estimation and Fringe Tracking

Mark Colavita

7/8/2003

Outline

- Visibility
- Fringe parameter estimation via fringe scanning
  - Phase estimation & SNR
  - Visibility estimation & SNR
- Incoherent and coherent averaging
- Estimator biases
- Fringe tracking

Visibility

- Visibility is the fundamental observable for interferometric imaging
  - Visibility is related to the object irradiance distribution via the van Cittert–Zernike theorem
- Visibility is generally complex, viz. $\Gamma = Ve^{j\phi}$
  - In optical/IR interferometry
    "visibility" generally refers to the visibility amplitude: $V = |\Gamma|$
  - Phase is just arg(Γ)
- While object visibility can be estimated with a two-element interferometer through the atmosphere, to get true object phase requires either phase referencing (multi-beam) or closure phase (3 apertures)

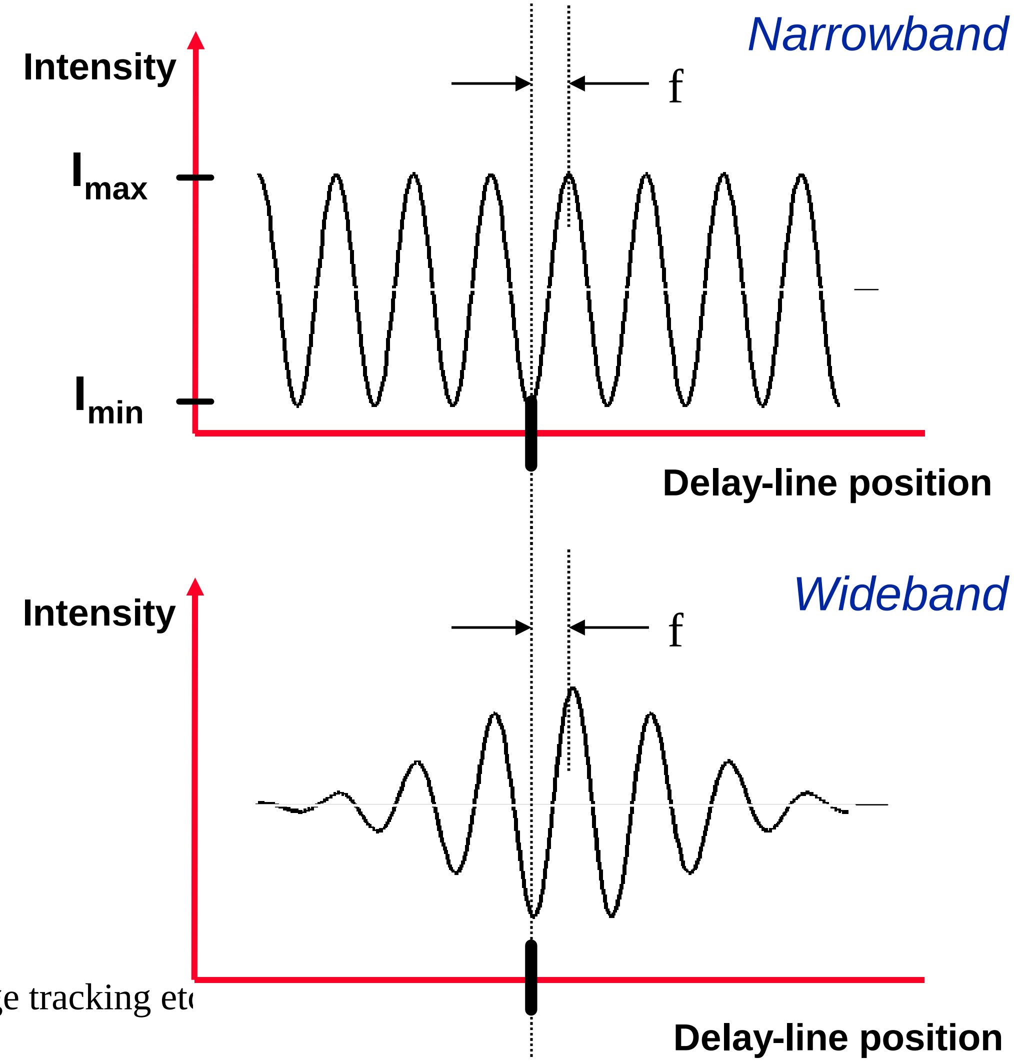
Measuring Visibility

•Visibility is just the contrast of the spatial fringe pattern

•Or using the traditional Michelson definition:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$0 < V < 1$$



Measuring Visibility

- Most measurement schemes involve converting the spatial pattern to a temporal pattern
  - We know how to measure the contrast of an electrical sinusoid
  - These are all variants of schemes used for phase shifting interferometry (PSI) for optical testing
    - » Options
      - Step or continuous scanning
      - 4, 6, or 8 bins
      - Triangle or sawtooth waveform
  - NB: all this discussion is in context of a fringe-tracking interferometer than scans over a single interference cycle

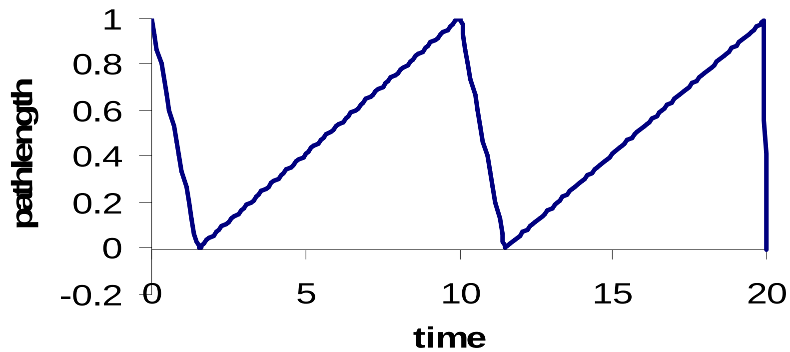
Fringe Measurements (PTI, Keck example)

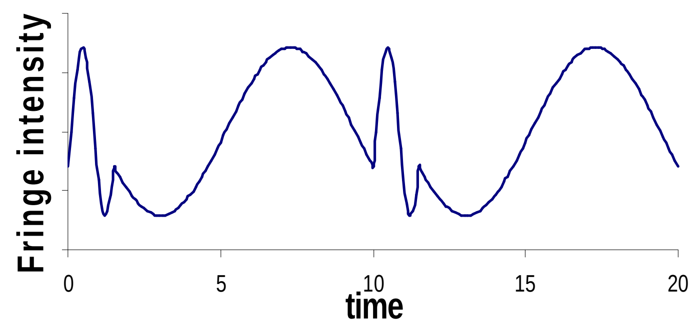
- Fringe-scanning modulation, implemented on delay line
- Sawtooth waveform to minimize number of reads per frame
- Retrace occurs during array settling time

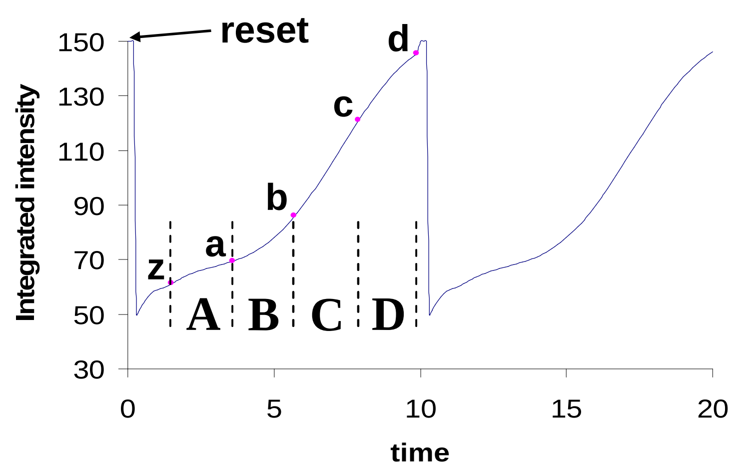
- A, B, C, D ¼-wave intensity bins computed as
  - A = a - z, B = b – a, etc.
- Let X = A - C, Y = B - D, N = A+B+C+D

$$f = \arctan\left(\frac{Y}{X}\right)$$

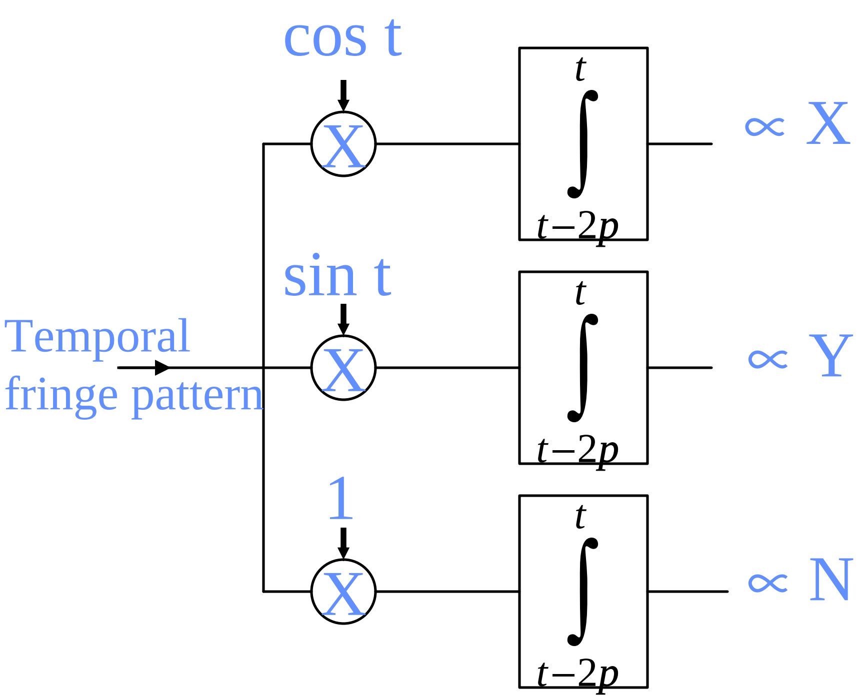
$$V^2 \propto \frac{X^2 + Y^2 - \text{bias}}{N^2}$$







- Visibility Estimation can also be understood as a standard communication problem, aka
  - Coherent demodulation
  - Quadrature demodulation
  - Matched filtering
- Use fringe scanning to convert spatial pattern to a temporal pattern



$$I = N(1 + V\cos(t + f)) = N + X\cos t + Y\sin t$$

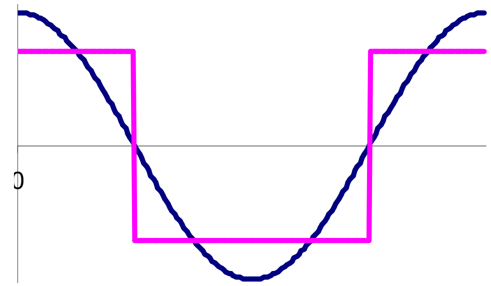
$$\hat{f} = \tan^{-1}\frac{\hat{Y}}{\hat{X}}$$

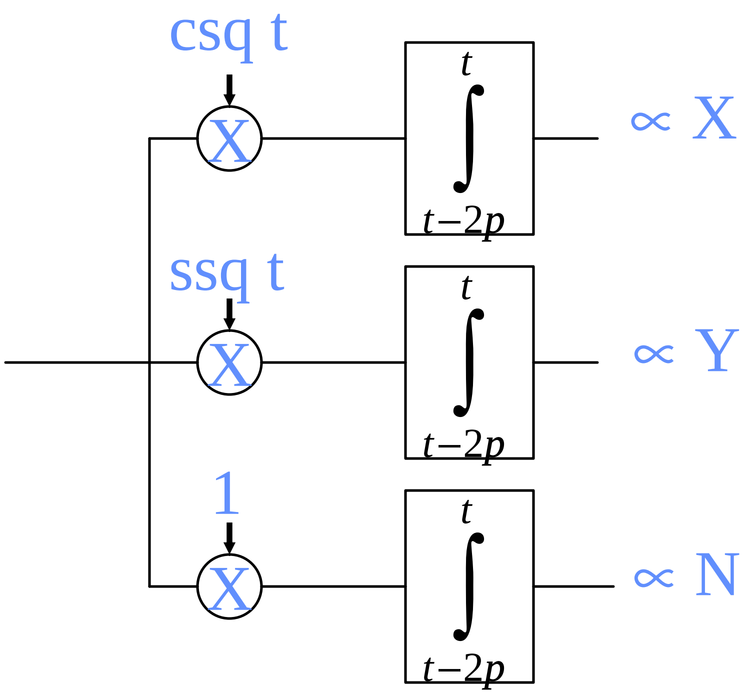
$$N\hat{V} \propto \sqrt{\hat{X}^2 + \hat{Y}^2}$$

$$\hat{V} \propto \frac{\sqrt{\hat{X}^2 + \hat{Y}^2}}{N}$$

4-Bin Algorithm

- Approximate sines, cosines with square waves





Slightly non-optimal, as it’s a mismatch to the proper waveform

10-20% more photons needed vs. ideal case

But minimizes number of reads

Estimating Phase

- Typically $$\hat{f} = \tan^{-1}\frac{Y}{X}$$

$$SNR \equiv \frac{1}{s_f} = \sqrt{g\,\frac{1}{2}\,\frac{N_{tot}^2 V^2}{N_{tot} + b s^2_{cds\_read\_noise}}}$$

$\propto \sqrt{N}\ \ V$, photon-noise limited

$\propto N\ \ V$, read-noise limited

4-bin: $\frac{4}{p^2}$ $\quad$ 4

It's a non-linear estimator; SNR >~ 3 for proper phase estimates

Example: To obtain SNR = 5 with $V^2 = 0.5$

125 phots, total, photon-noise limit
300 phots, total, with 10 electrons read noise

Improving SNR?

General don't average phase. Can average phasors if phase reference or closure phase – more later

Estimating Visibility

- Usually estimate V^2, rather than V, to avoid taking a square root on a noisy quantity (adds bias)

$$\hat{V^2} = \frac{p^2}{2} \frac{X^2 + Y^2 - Bias}{N^2}$$

- Typically, inadequate SNR to get a good estimate in one sample
- Average numerator and N separately

$$\left\langle \hat{V^2} \right\rangle = \frac{p^2}{2} \frac{\left\langle X^2 + Y^2 - Bias \right\rangle}{\left\langle N \right\rangle^2}$$

SNR for V^2

V^2 is a squared quantity of Gaussian & Poisson RVs; need 4th-order statistics to compute SNR

Typically assume all noise in numerator; N (in denominator) constant

Photon-noise only

$$SNR \equiv \frac{1}{s_{V^2}} \propto \begin{bmatrix} \sqrt{N}, & N >> 1 \\ N, & N << 1 \end{bmatrix}$$

Read-noise only

$$SNR \equiv \frac{1}{s_{V^2}} \propto N^2$$

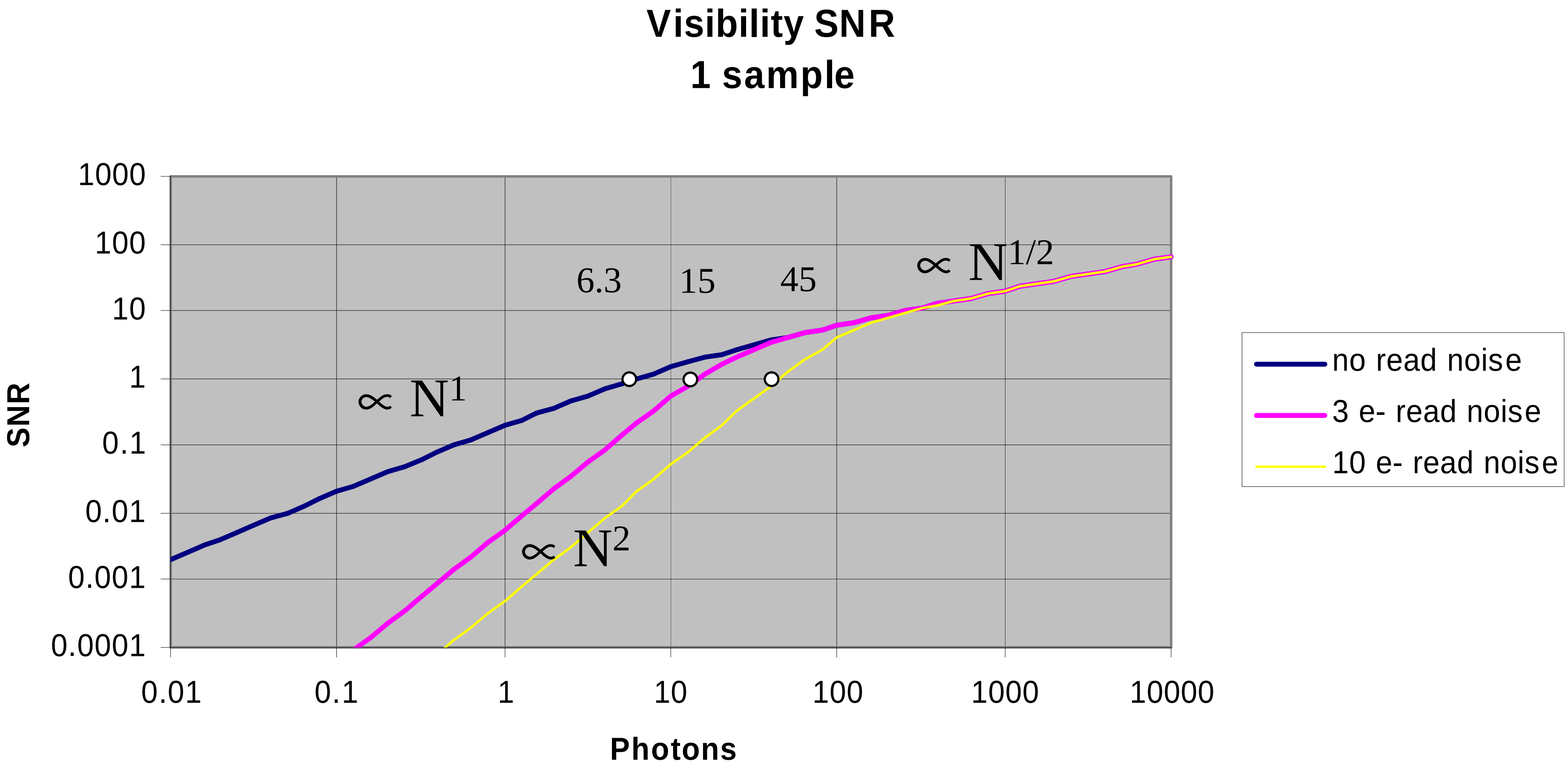
NB: when photon-starved, or read noise limited, SNR ≠ √N

With 2nd or higher-order estimators like for V^2, can get SNR dependencies steeper than N^1

In general

$$SNR \equiv \frac{1}{s_{V^2}} \propto \left(\frac{N^4}{N^2 + aN^3V^2 + bs^4_{cds\_read\_noise}} \right)^{1/2}$$

Signal-to-Noise Ratio



Coherent vs. Incoherent Averaging

Incoherent averaging (sum the magnitude squared of the fringe phasor)

- Averaging V^2 (strictly the numerator term) doesn't require phase stability between samples
  - Can combine many independent estimates of V^2
  - At PTI, 5 spectral channels over 125 sec at 50-100 samples/sec are combined to produce a synthetic white-light V^2 estimate
    - » Increases final SNR by ~200
    - » Scatter on 25 sec points allow estimation of internal errors
  - SNR increases as $\sqrt{}$ #samples

Coherent vs. Incoherent Averaging

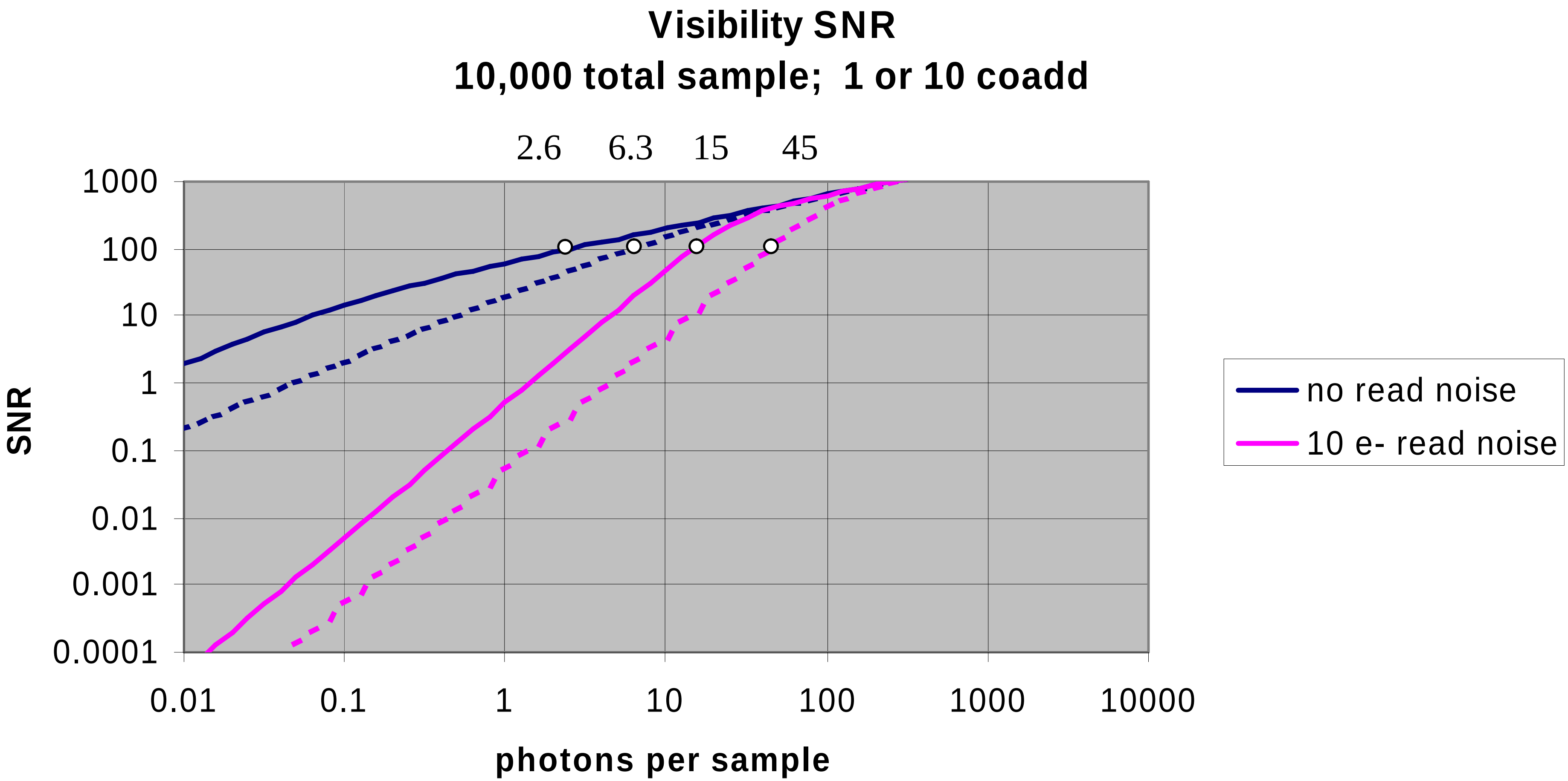
Coherent averaging (coadding: sum the visibility phasor $NVe^{j\phi}$)

- Use a phase reference to measure the phasor rotation
- Derotate the fringe phasor ($NVe^{j\phi} \times e^{-j\phi ref}$)
- Sum the fringe quadratures X+ jY

- Compared to incoherent average
  - No advantage when samples are shot noise limited (SNR $\propto \sqrt{N}$)
    - » Actually, some disadvantage due to extra biases
  - Advantage occurs when samples are photon starved
    - » SNR gains faster than $\sqrt{}$ #samples
- Can also be used to increase fringe SNR to get an estimator into a linear regime
  - E.g., increase SNR to compute the arctan phase estimate

- Using a phase estimate to rotate phasors to a common angle so they can be coherently averaged is *phase-referencing*, a powerful technique for increasing sensitivity

Signal-to-Noise Ratio with Averaging and Coadding



Estimating Detection Bias Terms, I

- Most detectors have imperfections which must be accommodated to get good measurement accuracy
- Offsets $B_?$

$N = N_{raw} - B_N$ (from dark sky)

This bias is just dark current + background

$X = X_{raw} - B_X$ (from dark sky)

$Y = Y_{raw} - B_Y$ (from dark sky)

With a perfectly linear detector, these biases are zero

Estimating Detection Bias Terms, II

Numerator biases

$$\text{NUM} \propto \langle X^2 + Y^2 - \mathit{bias} \rangle$$

Photon noise

$$\langle X^2 + Y^2 \rangle = k\,N$$

(can get k from slope of bias vs. N)

Sometimes, correction for imperfect photon counting needed, too.

- N: Counts (adc units)
- k: Electronic gain (adc units / e^-)

+ Detector noise

$$\langle X^2 + Y^2 \rangle = 4\,k^2\,\sigma^2_{\text{cds read noise}}$$

(from dark sky)

- σ^2: Read noise variance
- 4: 4 CDS reads for 4-bin algorithm

The detector-noise term dominates when read-noise limited. It also has the same noise statistics as V^2, so care must be taken in estimating it well.

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Other Biases

- Atmospheric biases
  - Spatial $$\langle V^2 \rangle \cong \exp(-2\sigma_f^2) = \exp\left(-2.06\left(\frac{d}{r_0}\right)^{5/3}\right)$$ *(slow guiding)*
    - » Single mode fibers can eliminate most of this
  - Temporal $$\langle V^2 \rangle \cong \exp\left(-\left(\frac{T}{T_{0,2}}\right)^{5/3}\right)$$
    - » Some post-processing calibration possible
- Instrumental
  - Mismatched stroke vs. wavelength
- Longitudinal coherence
  - Off peak of fringe envelope
    - » Narrow spectral channels for science help

NB:
The issue is not the visibility reduction, but its variability

Fringe Tracking

- What: following the interference phase - phase tracking - to stay on the central fringe to maintain coherence
  - Typically follow to ~radian
  - Maintains high duty cycle; necessary for cophasing
- [There's also envelope tracking, which maintains centration on the fringe envelope, not discussed here]
- Issues
  - Phase measurement - already discussed
  - Sampling time
  - Phase unwrapping
  - Fringe centering
  - Atmospheric residuals

Coherence Time and Sample Spacing

- Many different definitions
  - $T_{0,2}$ - integration time during which phase fluctuations are 1 rad rms
  - $\tau_{0,2}$ - sample spacing for which phase difference = 1 rad rms

    $\tau_{0,2} \cong \frac{1}{4} T_{0,2}$
- Integration time $T < T_{0,2}$ to maintain coherence (high V^2)
  - rms phase fluctuations during interval = $(T/T_{0,2})^{5/6}$
- Sample spacing $t < \tau_{0,2}$ for phase continuity
  - Usually t=T, and this requirement dominates

$$1 = \int df\, A(f) W(f,t)$$

Atmospheric power spectrum: 1 or 2 aps

Window function for phase difference or variance

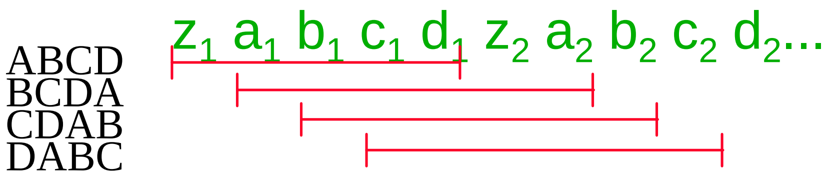
Phase Continuity

- Phase being measured is typically >> 2π rads
  - But arctan phase estimator $-\pi < \phi < +\pi$
- Phase unwrapping
  - Simple

    $\Phi_i = 2\pi M_i + \phi_i$

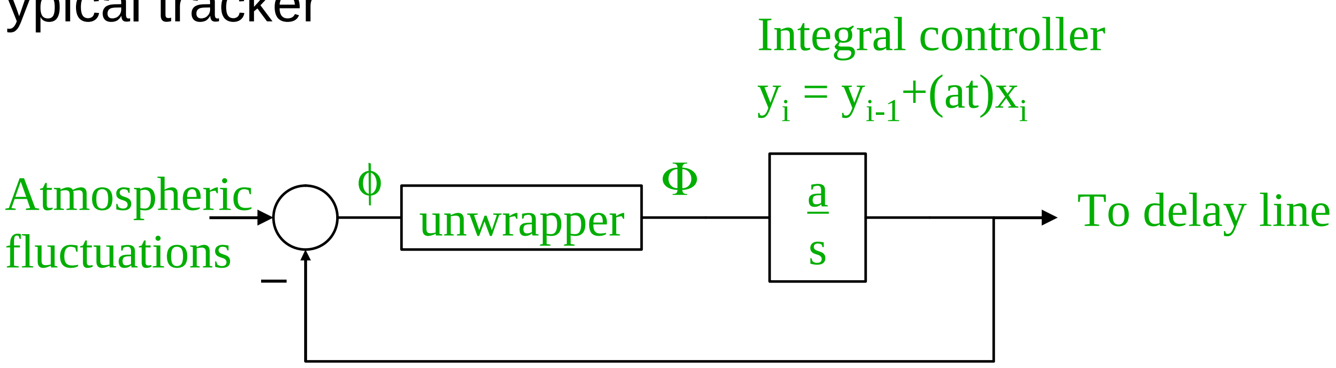
    » Chose M_i for each sample s.t. $| \Phi_i - \Phi_{i-1} | < \pi$
  - Better

    » Chose M_i for each sample s.t. $| \Phi_i - \Phi_{i \mid i-1} | < \pi$
    - Estimate with low pass filter or Kalman filter, matched to sample spacing, atmospheric parameters, etc.
- Sliding window can be used to improve continuity



Tracking Performance

- Typical tracker



» Closed loop bandwidth $f_c \approx a/(2\pi)$ for $f_c << 1/t$

- rms tracking error $\approx (f_c/f_{G,2})^{5/6}$
  – where $f_{G,2}$ is the two-aperture Greenwood frequency $\propto 1/T_{0,2}$

- Example
  – $T_{0,2}$ = 50 ms
  – $\tau_{0,2}$ = 13 ms
  – $f_{G,2}$ = 11 Hz
  – T = t = 10 ms
  – f_c = 5 Hz (1/20th sample rate)
  – tracking error = 1.9 rads

Required Bandwidth

- In standard servo design, you want to optimize parameters to minimize the tracking error
- For the interferometer, you can accurately measure the tracking error
  - Often, you need a small enough tracking error to stay well centered on the fringe
  - You can still co-phase even if the tracking error > 1 rad if you can feedforward to a separate delay line for the secondary channel

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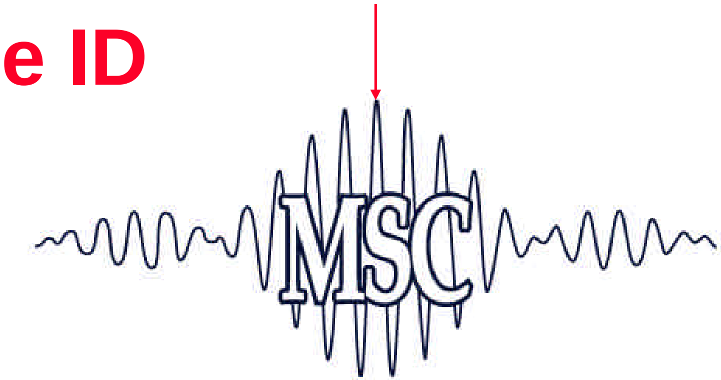
Central Fringe ID



- Want to stay on the central fringe
  - – Highest contrast - best SNR
  - – V^2 for science also refers to central fringe

    (Typically, also use spectrometer channels with their longer coherence lengths to reduce sensitivity to tracking errors)
- How?
  - – 1) Measure dependence of V^2 on phase, and move in direction of higher V^2
  - – Issues
    - » V^2 estimator typically noisier than phase estimator
    - » Need “wobble” -- natural or induced -- to resolve direction to move

Group Delay Estimation

- White-light fringe ≡ interference peak ≡ phases of all colors match up

E field as function of group delay **x**

$$E = A\exp(jkx), k = \frac{2p}{l}$$

$$x = \frac{\partial \Phi}{\partial k}$$

- Group delay estimate $\hat{x}$ gives absolute fringe position without unwrapping errors
- Why not use all the time?
  - In the infrared, SNR for group delay worse than for phase
    - » More read noise from reading additional channels
    - » Incoherent group-delay estimator includes a noise term proportional to fringe envelope width $\lambda^2/\Delta\lambda$

Group Delay Estimation, cont.

- Usual approach to group delay in the IR
  - » Use white-light phase tracking for high bandwidth control
  - » Use group-delay centering at a lower bandwidth
- Different in the visible (ex: NPOI)
  - » When photon count, no penalty to dispersing
  - » Wide optical bandwidth reduces GD noise
    - Allows use of a coherent delay estimator which has same SNR as WL phase estimator for moderate SNRs
- Other issues
  - » Atmospheric dispersion will introduce differences between the WL phase and the group delay

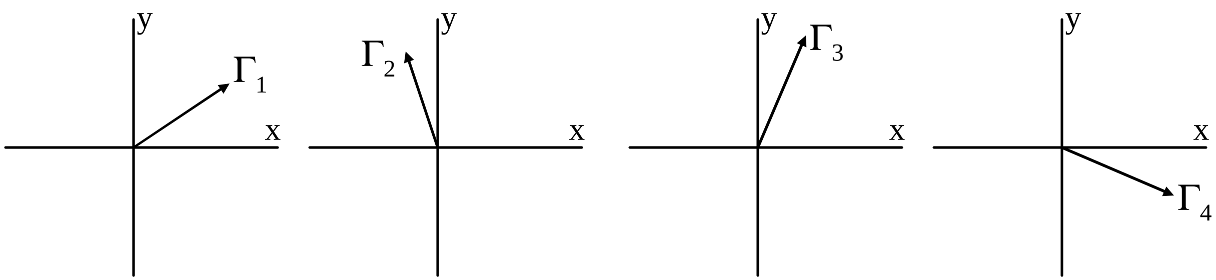
Conclusion

- You typically measure visibility phase and visibility amplitude by converting a spatial fringe pattern to a temporal one
  - Becomes a matched-filter problem
- You can derive SNR expressions: not everything goes at ÖN
  - Leads to differences between incoherent and coherent averaging
- Calibration is critical
  - Stability of biases is what frequently limits data accuracy
- Fringe tracking is implemented using the measured fringe phase

The End

Fringe derotation and stacking (coadding)

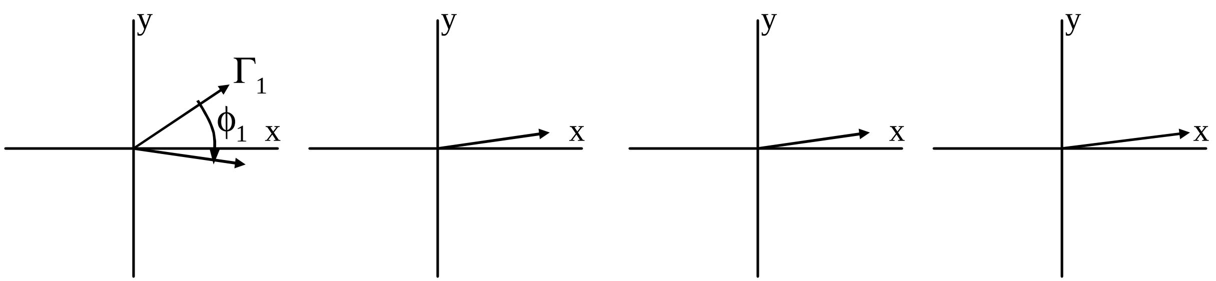
Raw phasors



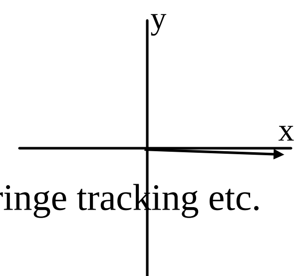
Phase reference ϕ_1 ϕ_2 ϕ_3 ϕ_4

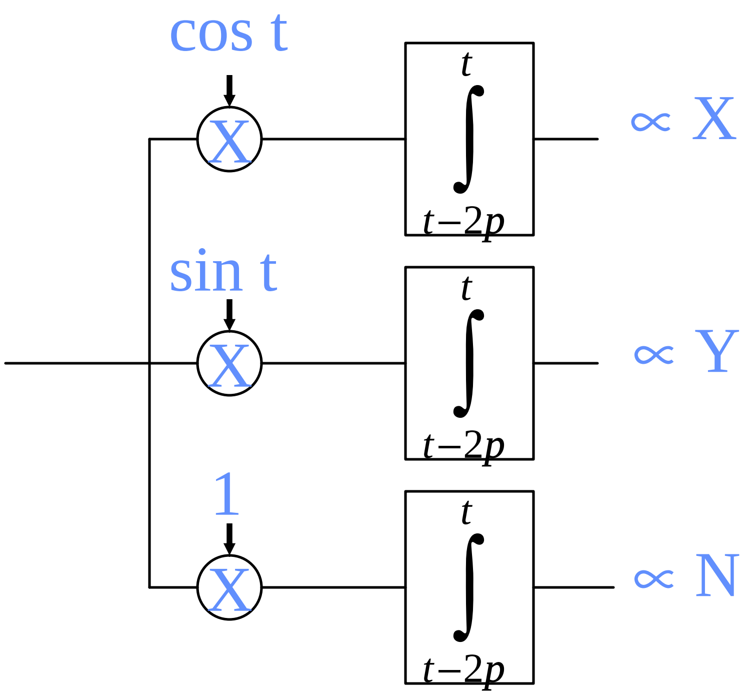
De-rotate

(transformation matrix)



Sum (average)




cos t
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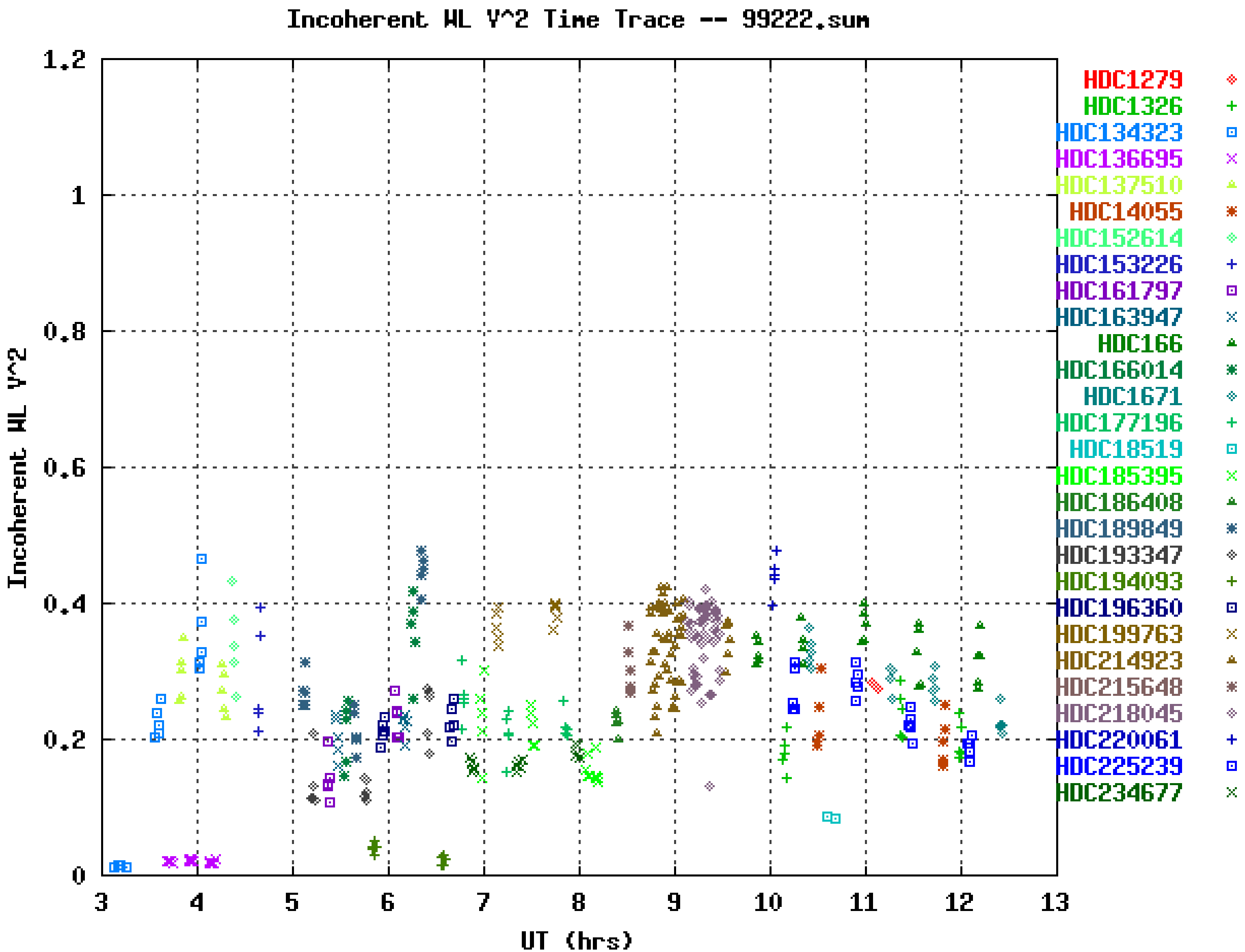

Incoherent WL V^2 Time Trace -- 99222.sum
Incoherent WL V^2
UT (hrs)
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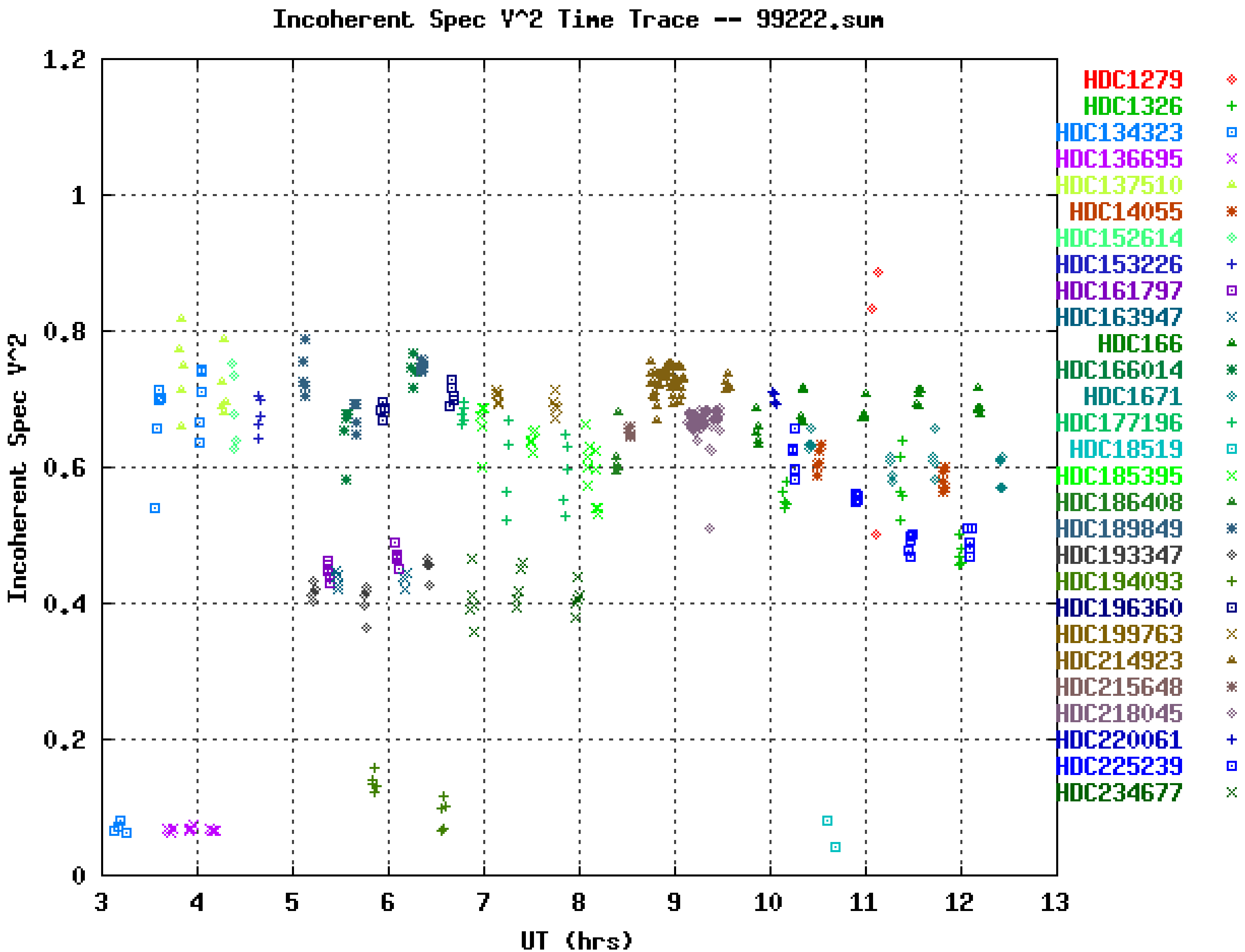

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Incoherent Spec V^2
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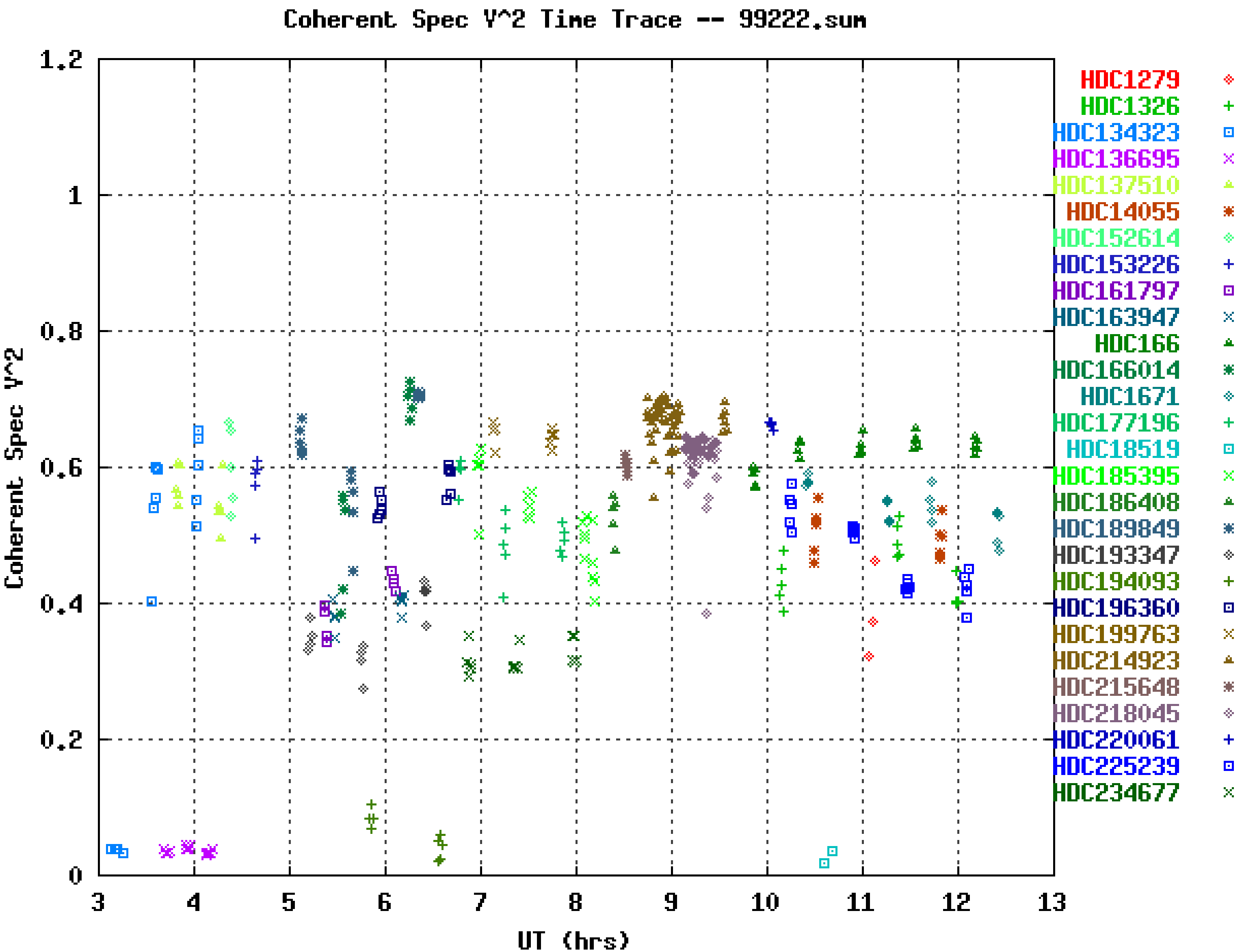
Coherent Spec V^2 Time Trace -- 99222.sum
Coherent Spec V^2
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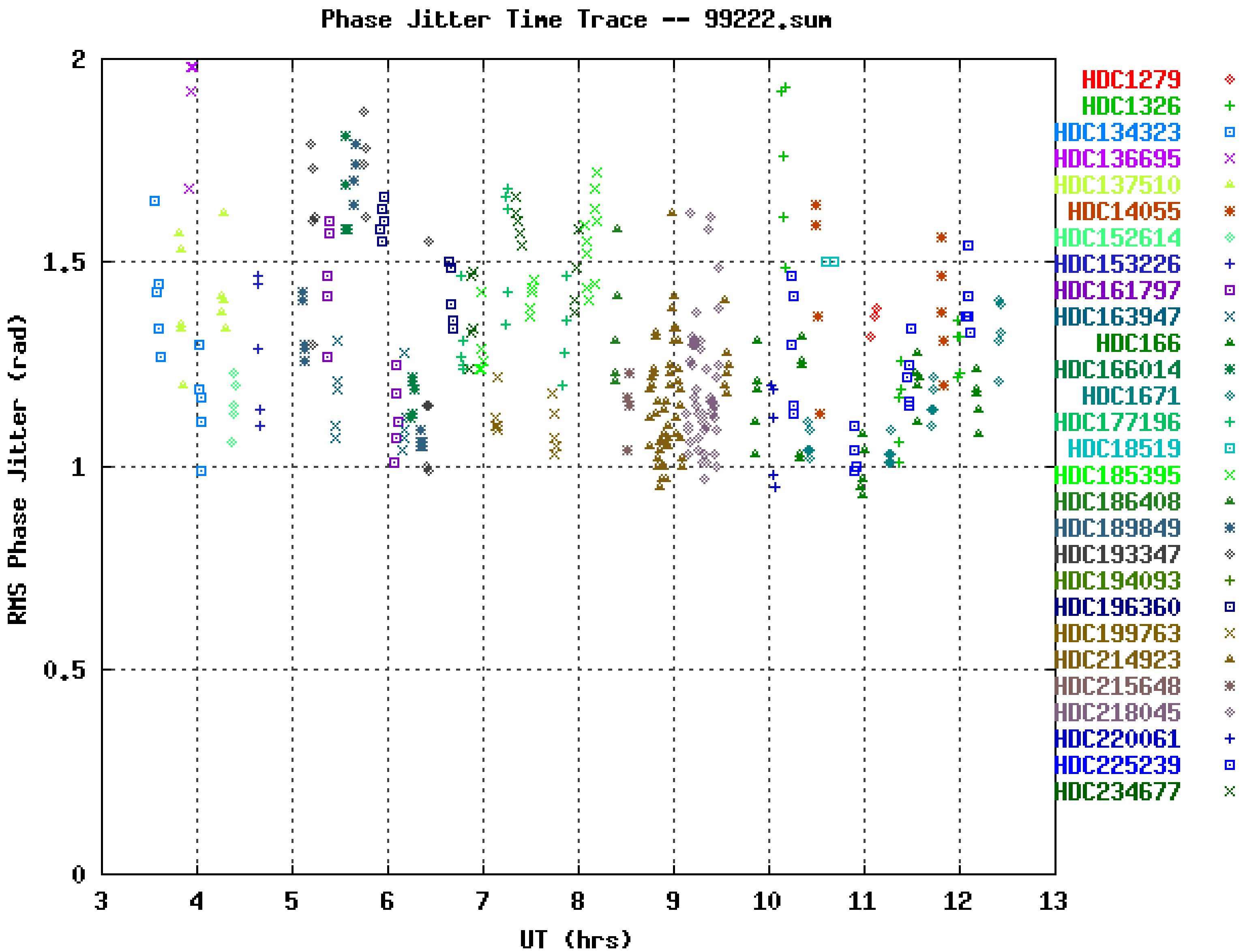

Phase Jitter Time Trace -- 99222.sum
RMS Phase Jitter (rad)
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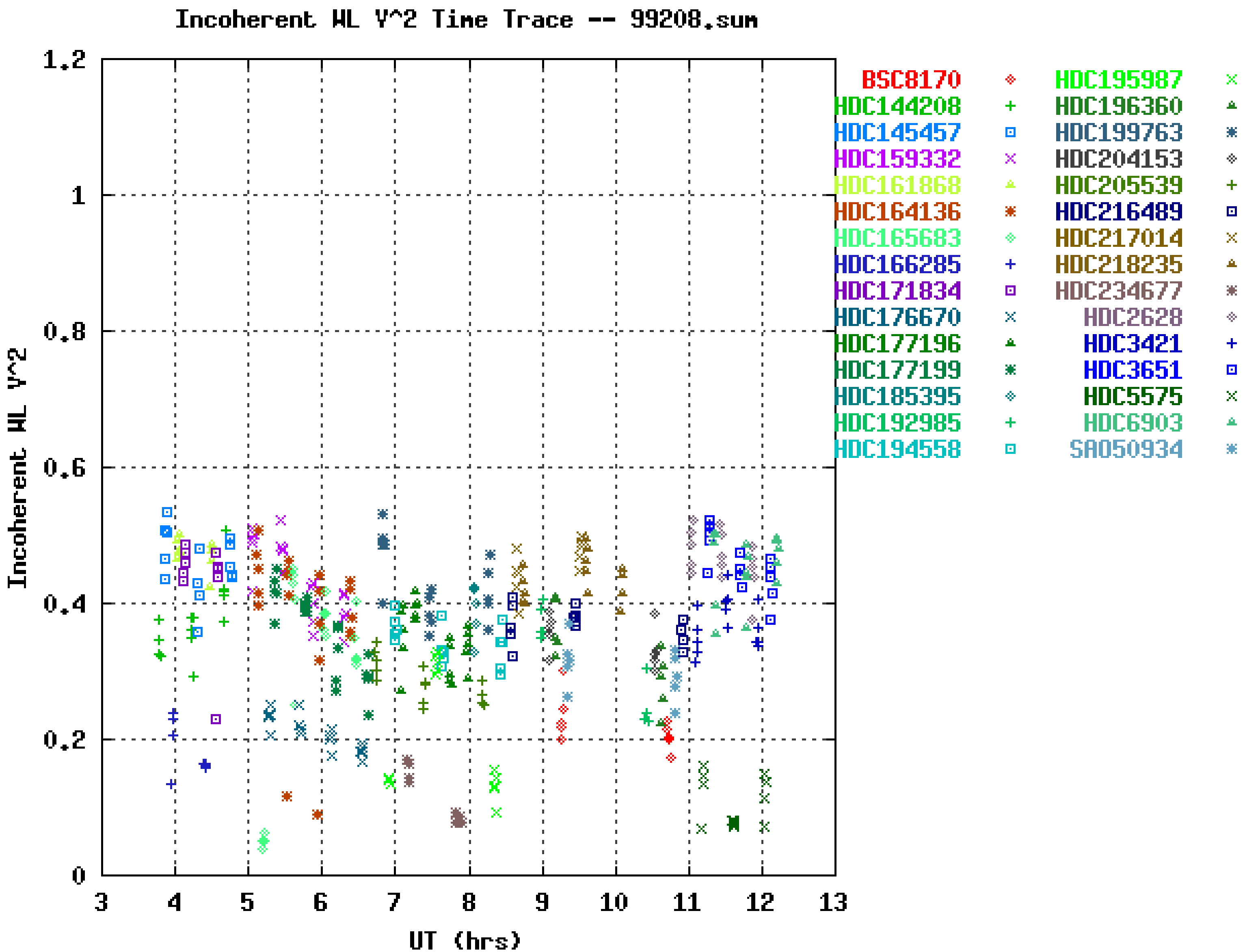

Incoherent WL V^2 Time Trace -- 99208.sum
Incoherent WL V^2
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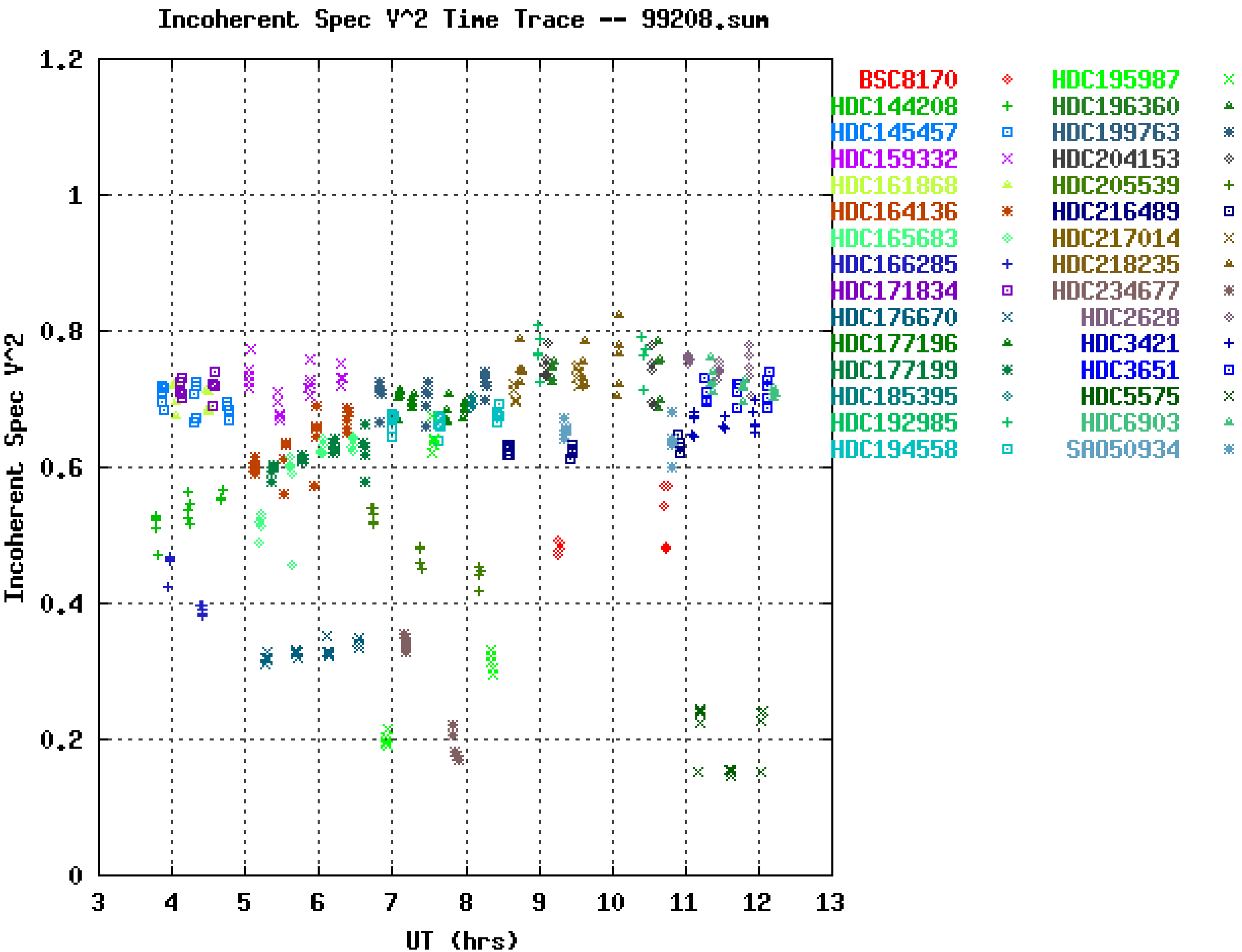
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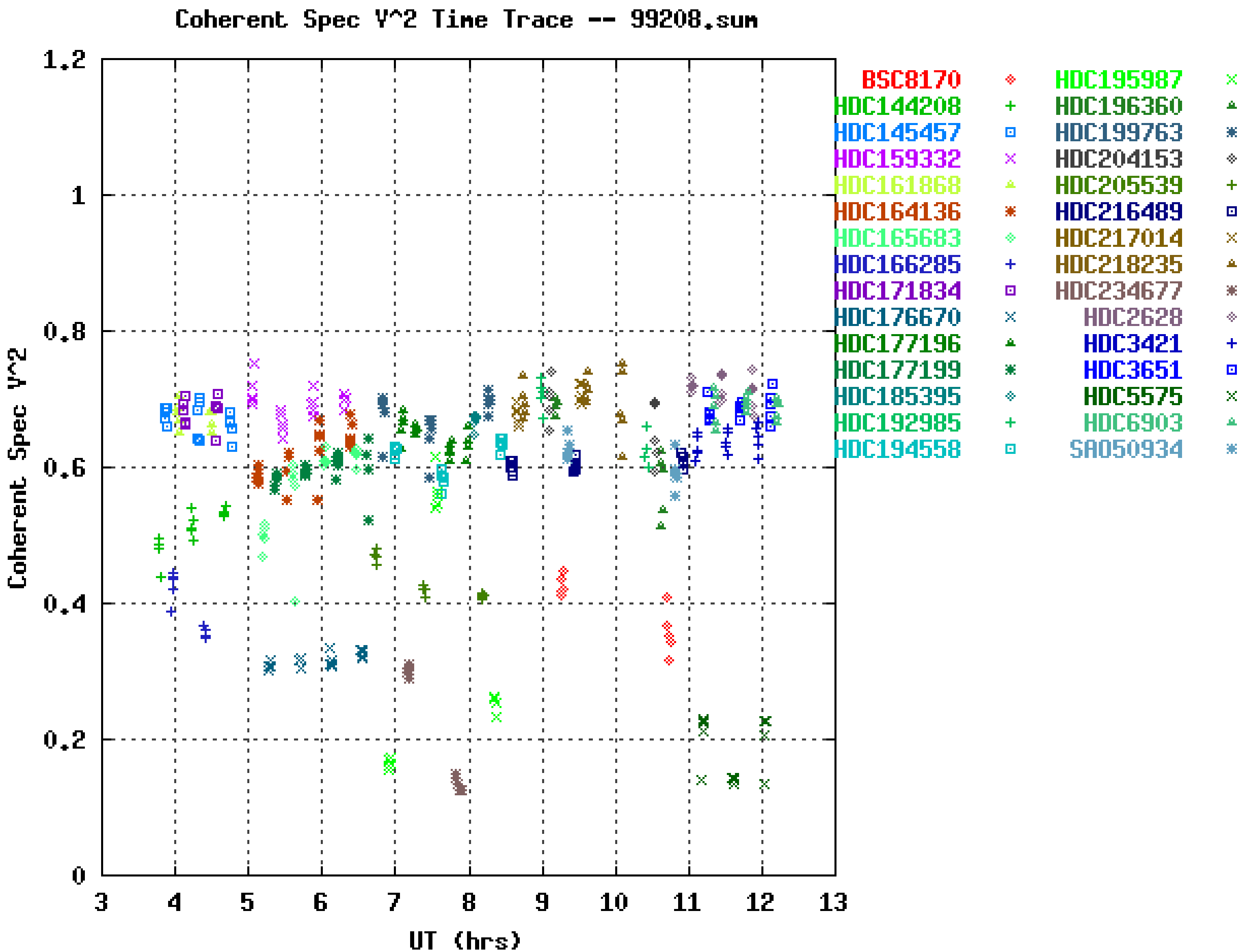
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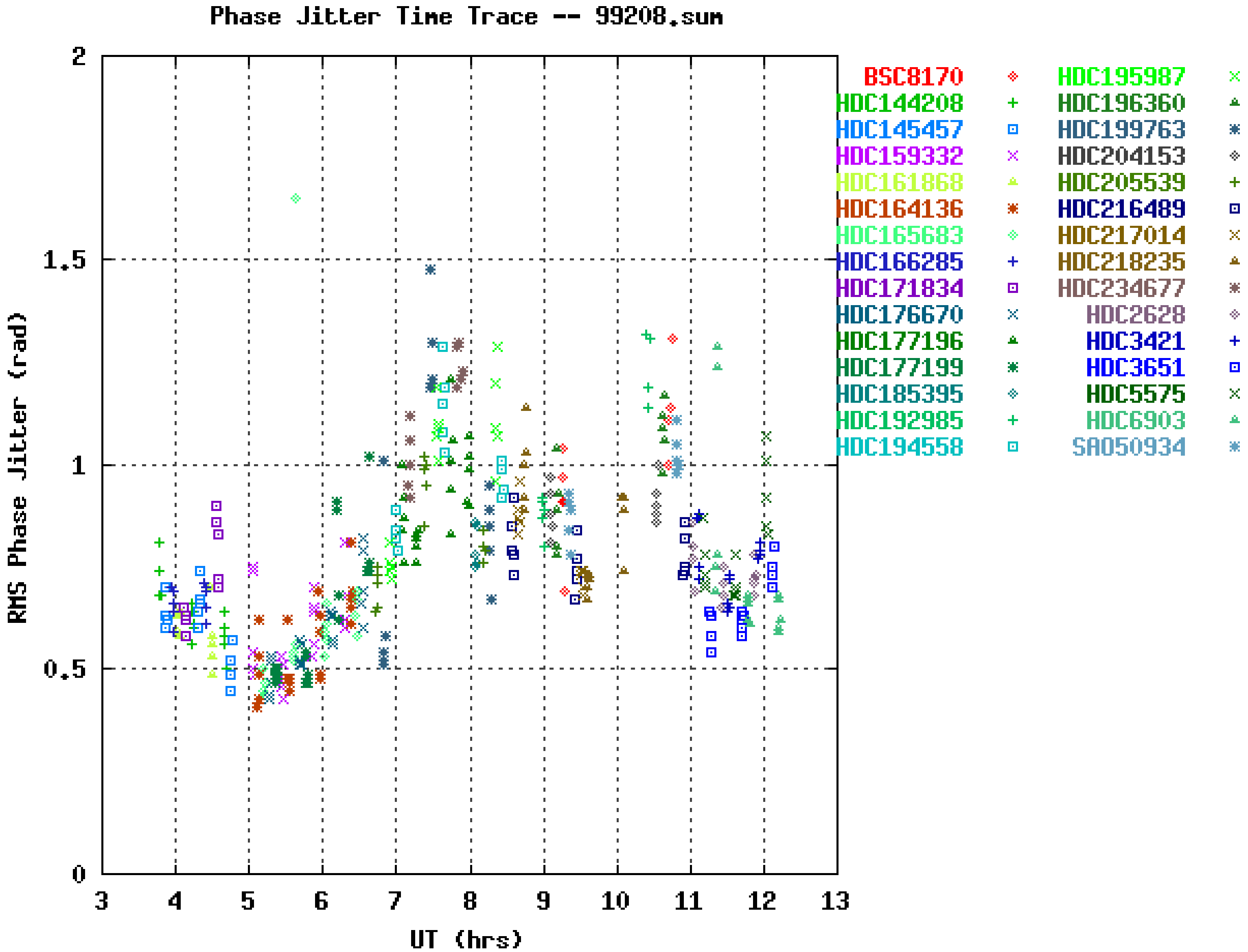
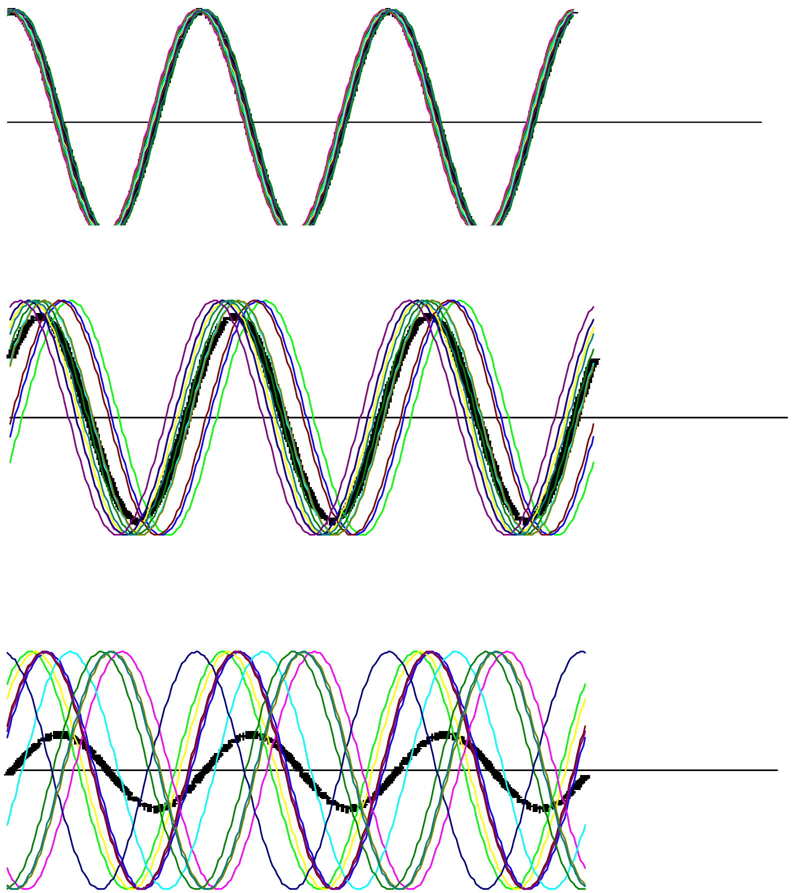

Phase Jitter Time Trace -- 99208.sum
RMS Phase Jitter (rad)
UT (hrs)
0
0.5
1
1.5
2
3
4
5
6
7
8
9
10
11
12
13
BSC8170
HDC144208
HDC145457
HDC159332
HDC161868
HDC164136
HDC165683
HDC166285
HDC171834
HDC176670
HDC177196
HDC177199
HDC185395
HDC192985
HDC194558
HDC195987
HDC196360
HDC199763
HDC204153
HDC205539
HDC216489
HDC217014
HDC218235
HDC234677
HDC2628
HDC3421
HDC3651
HDC5575
HDC6903
SAO50934

7/8/2003

Requirements on Fringe Stabilization

Vibrations blur out the fringe - reduce fringe visibility



10 nm rms

50 nm rms

200 nm rms

Need real-time control of pathlength to ~10 nm (λ/50) for high fringe visibility