Outline

- Imaging – Worth a Thousand Words?
  - Image and Model: which is which?
- Deconstructing an image
  - Fourier nuts and bolts.
- Recovering Phase
  - Closure Phase and how to handle them
- Deconvolution and Regularization
  - Mapping, a priori information, examples

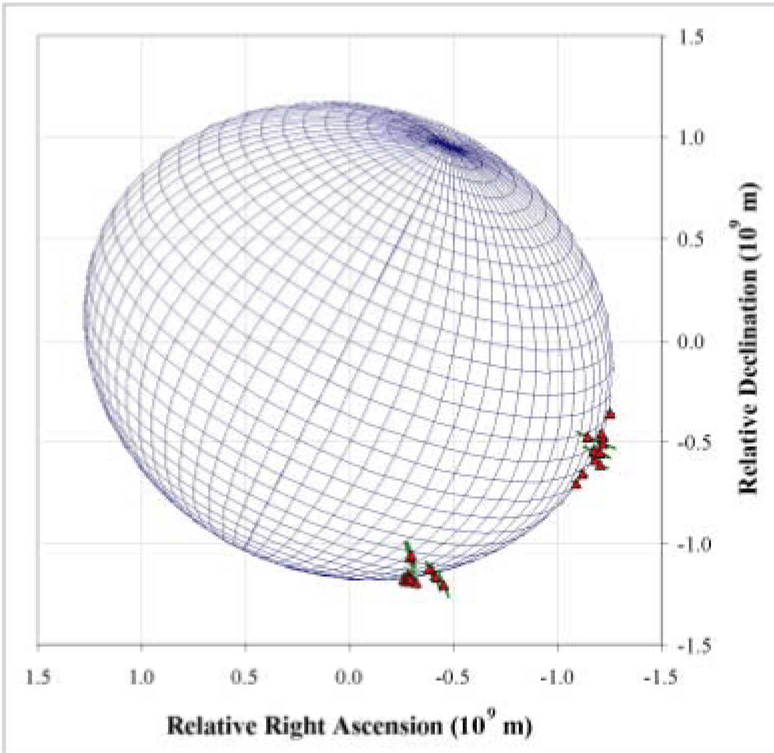
Do I really want to make an image?

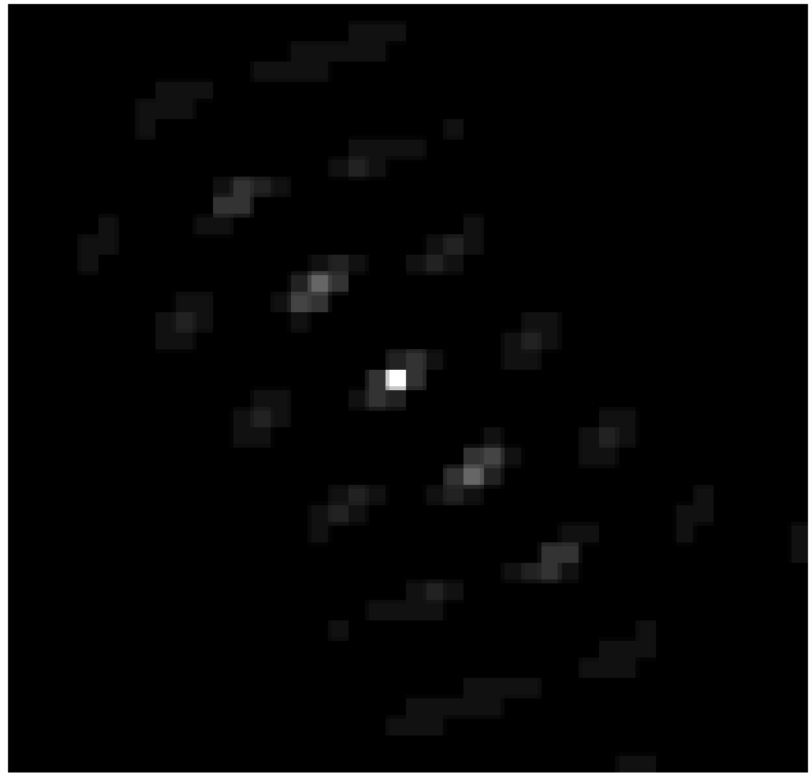
NATURE, instructions to Authors: “Letters are not to exceed 3 pages (180 word abstract plus 1500 word body)”. Converting this by the well-known picture-to-word law, we need 1.68 good images to make a Nature paper. Right?

“You gotta know when to walk away;
... know when to run” (Kenny Rogers)

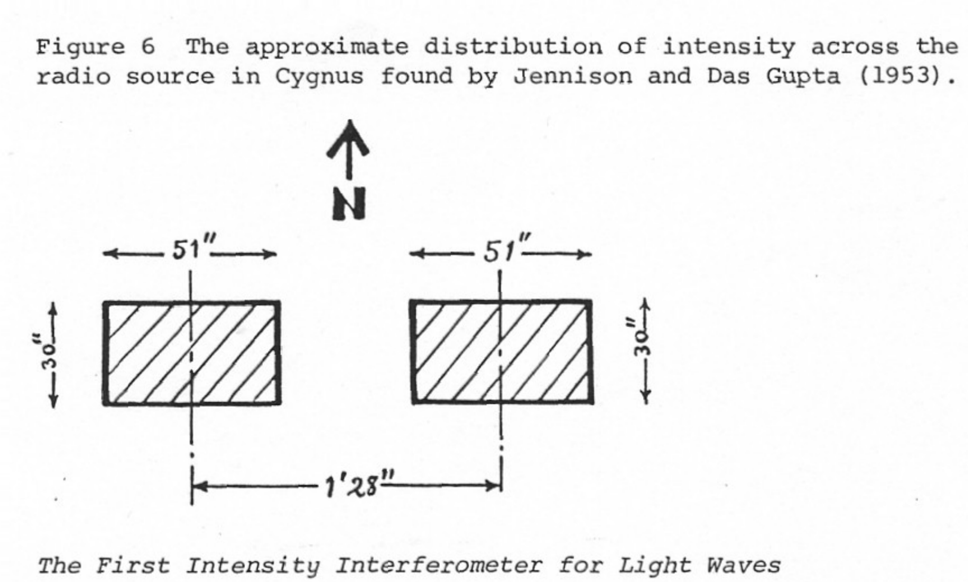
1. Is an image the best way to view my data?
2. Where is the Physics?
3. Imaging and Modelfitting: a blurry distinction.

Unchained fidelity





Fine Feathers make not a Fine Fowl...



Cyguns A: VLA 6cm continuum image

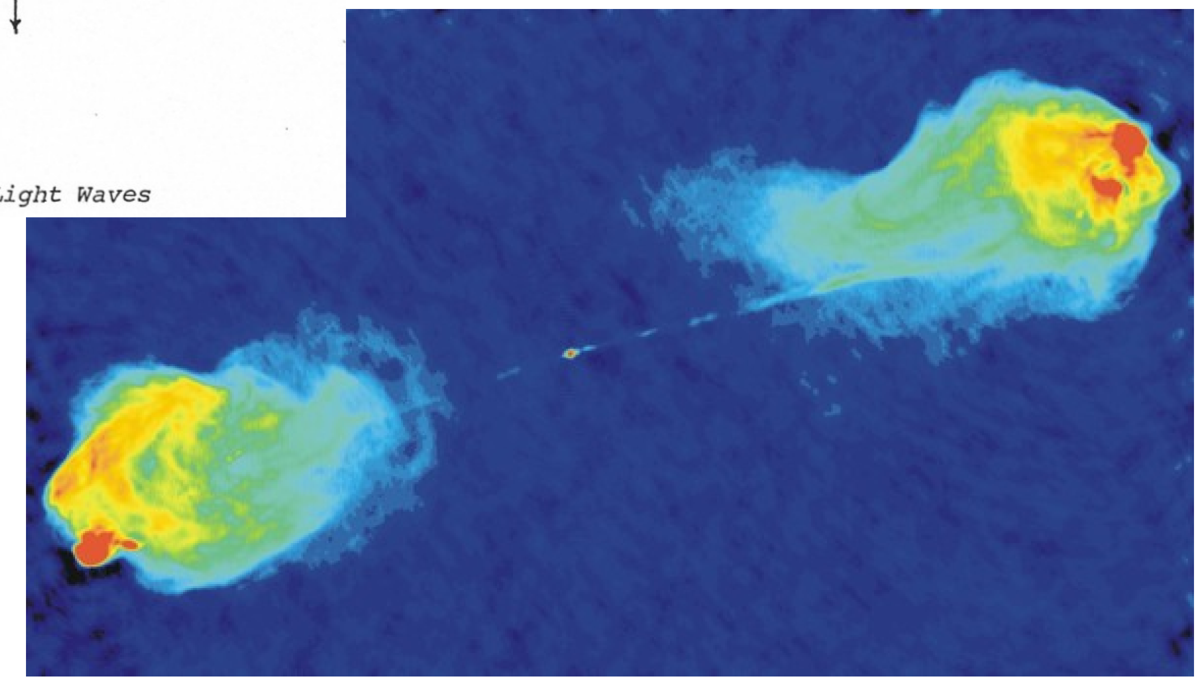
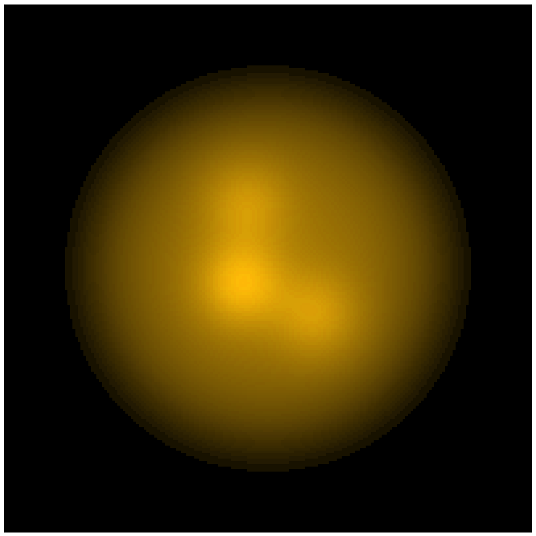
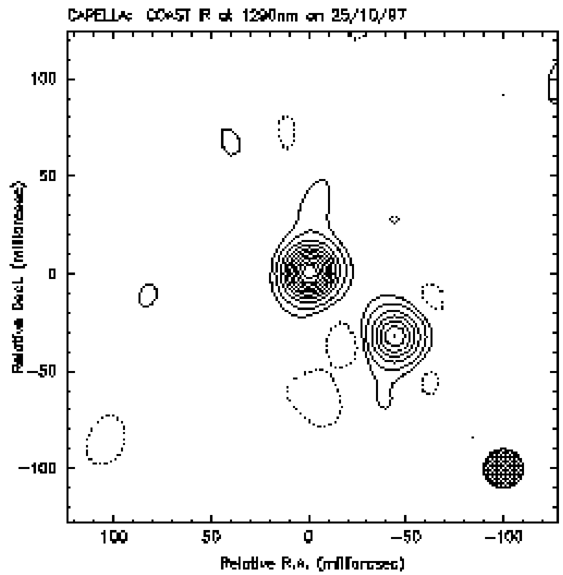
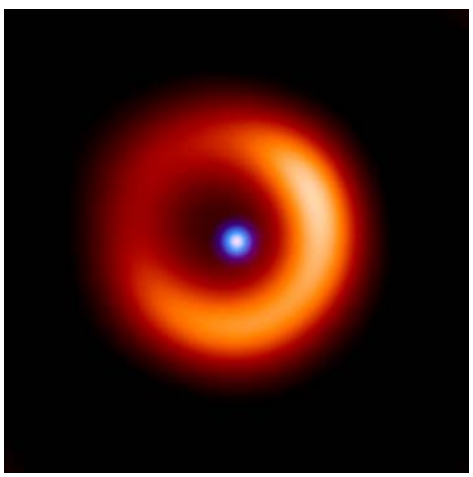


Image or Model?







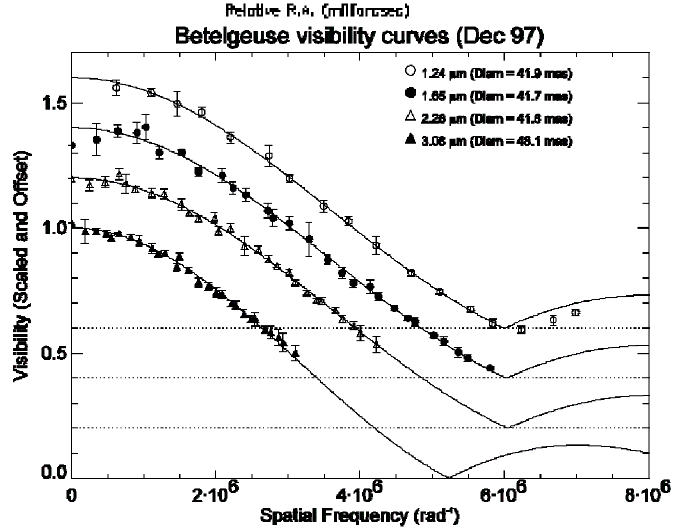


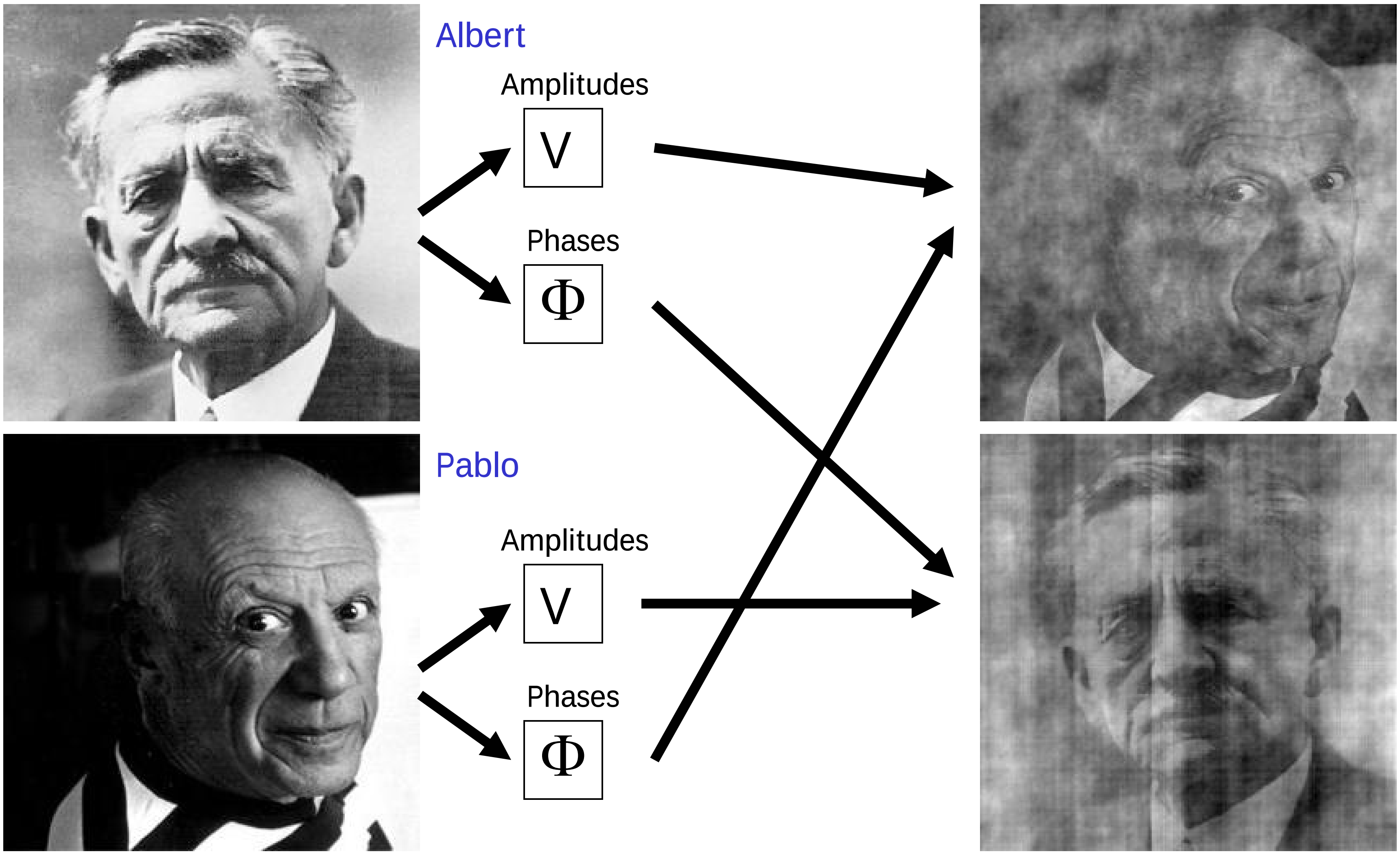
Image and Model:

Both
- representation of data
- contain a priori assumptions

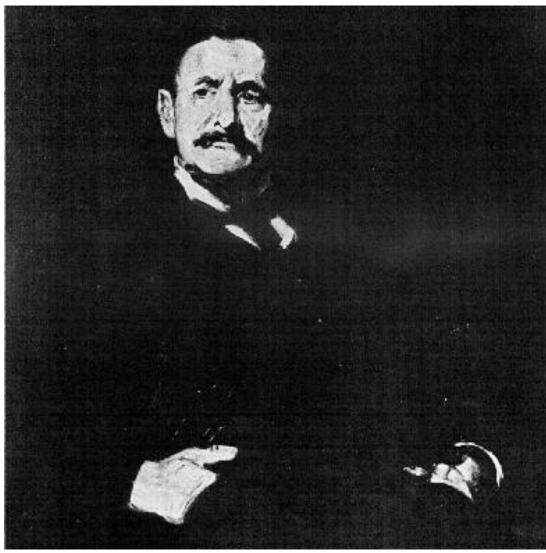
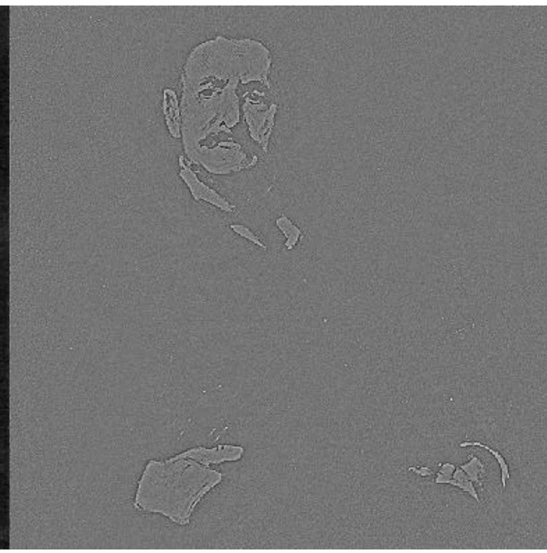
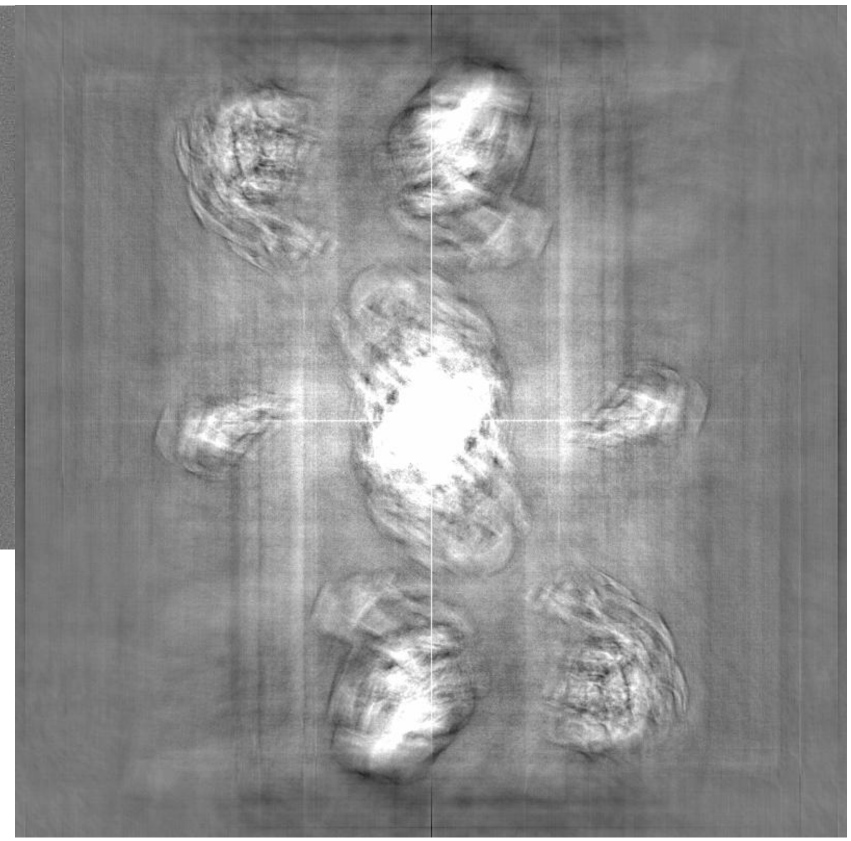
Semantic difference?

When extracting quantitative information from data, stay as close to the native data form as possible.

Amplitudes and Phases



Amplitudes and Phases II

Requirements for Imaging

- Complex Fourier Data (Vis + Phase)
  - Presently, require good SNR
  - Fourier Coverage is important
- Reliable errors
  - You might be held to them!
- Prior Knowledge
  - No negative flux; finite extent
  - Astrophysical constraints?

Complex Antenna Gain

$$\tilde{E}_i^{\rm measured} = \tilde{G}_i \tilde{E}_i^{\rm true}$$
$$= |G_i| e^{i\Phi_i^G} \tilde{E}_i^{\rm true}.$$

Telescope Gain
(e.g., coupling efficiency
into single-mode fiber)

Telescope Phase Shift
(e.g., atmospheric piston,
bad baseline, thermal drifts)

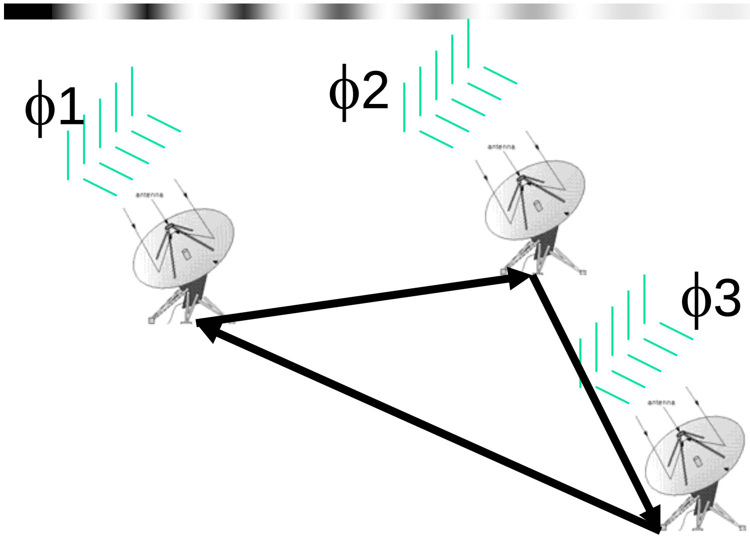
$$\text{Since} \quad \tilde{\mathcal{V}}_{ij} \propto \tilde{E}_i \cdot \tilde{E}_j^*,$$
$$\tilde{\mathcal{V}}_{ij}^{\rm measured} = \tilde{G}_i \tilde{G}_j^* \tilde{\mathcal{V}}_{ij}^{\rm true}$$
$$= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\rm true}$$

Phase shift of
detected Fringe

Phase Retrieval

- Phase Referencing
  - Nod to a phase cal (OK if long atm coherence time)
  - Phase calibrator within Isoplanatic Patch?
  - Self-Reference (e.g. reference from different wavelength)
- Absolute estimation of $\Delta\Phi$
  - Works in mm and sub-mm.
  - Perhaps possible in optical (at least to a degree)...
- Closure Phase
  - Recover most of the phase information
  - Good interferometric observable

Closure Phase



Observed		Intrinsic		Atmosphere
$\Phi(1\text{-}2)$	=	$\Phi_o(1\text{-}2)$	+	$[\phi(2)-\phi(1)]$
$\Phi(2\text{-}3)$	=	$\Phi_o(2\text{-}3)$	+	$[\phi(3)-\phi(2)]$
$\Phi(3\text{-}1)$	=	$\Phi_o(3\text{-}1)$	+	$[\phi(1)-\phi(3)]$

$$\boxed{\text{Closure Phase (1-2-3)} = \Phi_o(1\text{-}2)+\Phi_o(2\text{-}3)+\Phi_o(3\text{-}1)}$$

Related to the Bispectrum, B_{ijk}:

$$\begin{aligned}
\tilde{B}_{ijk} &= \tilde{\mathcal{V}}_{ij}^{\rm measured}\,\tilde{\mathcal{V}}_{jk}^{\rm measured}\,\tilde{\mathcal{V}}_{ki}^{\rm measured} \\
&= |G_i||G_j|\,e^{i(\Phi_i^G-\Phi_j^G)}\,\tilde{\mathcal{V}}_{ij}^{\rm true}\cdot|G_j||G_k|\,e^{i(\Phi_j^G-\Phi_k^G)}\,\tilde{\mathcal{V}}_{jk}^{\rm true}\cdot|G_k||G_i|\,e^{i(\Phi_k^G-\Phi_i^G)}\,\tilde{\mathcal{V}}_{ki}^{\rm true} \\
&= |G_i|^2\,|G_j|^2\,|G_k|^2\,\tilde{\mathcal{V}}_{ij}^{\rm true}\cdot\tilde{\mathcal{V}}_{jk}^{\rm true}\cdot\tilde{\mathcal{V}}_{ki}^{\rm true}.
\end{aligned}$$

How much phase information?

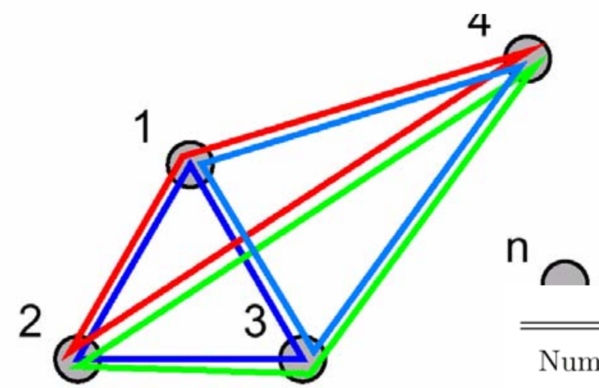
Closure Phases are not all independent from each other.

Number of Closure Phases

$$\binom{N}{3} = \frac{(N)(N-1)(N-2)}{(3)(2)},$$

Number of Fourier Phases

$$\binom{N}{2} = \frac{(N)(N-1)}{2}$$

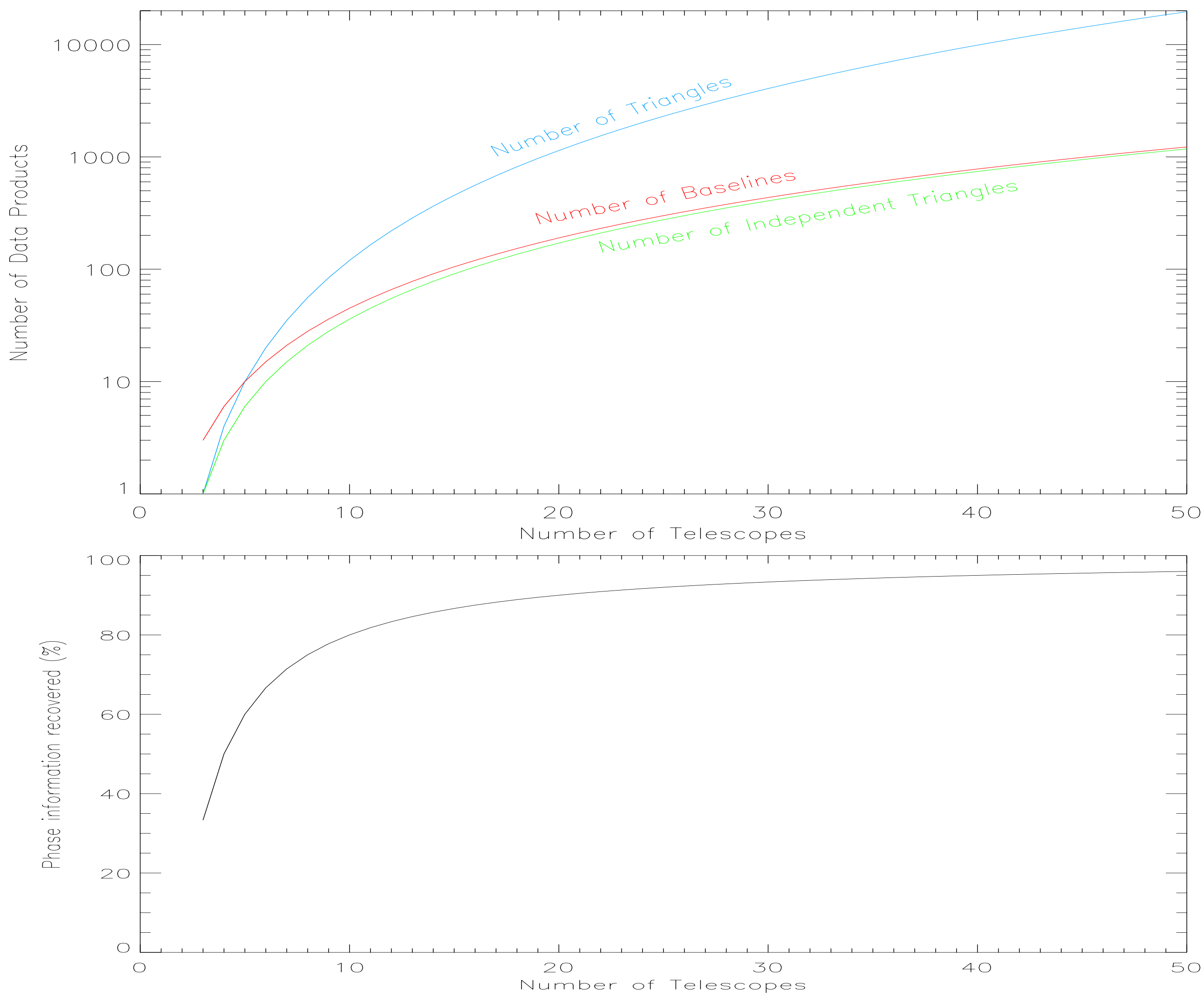


Number of Independent Closure Phases

$$\binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$$

Number of Telescopes	Number of Fourier Phases	Number of Closing Triangles	Number of Independent Closure Phases	Percentage of Phase Information
3	3	1	1	33%
7	21	35	15	71%
21	210	1330	190	90%
27	351	2925	325	93%
50	1225	19600	1176	96%

Most of the phase is recovered



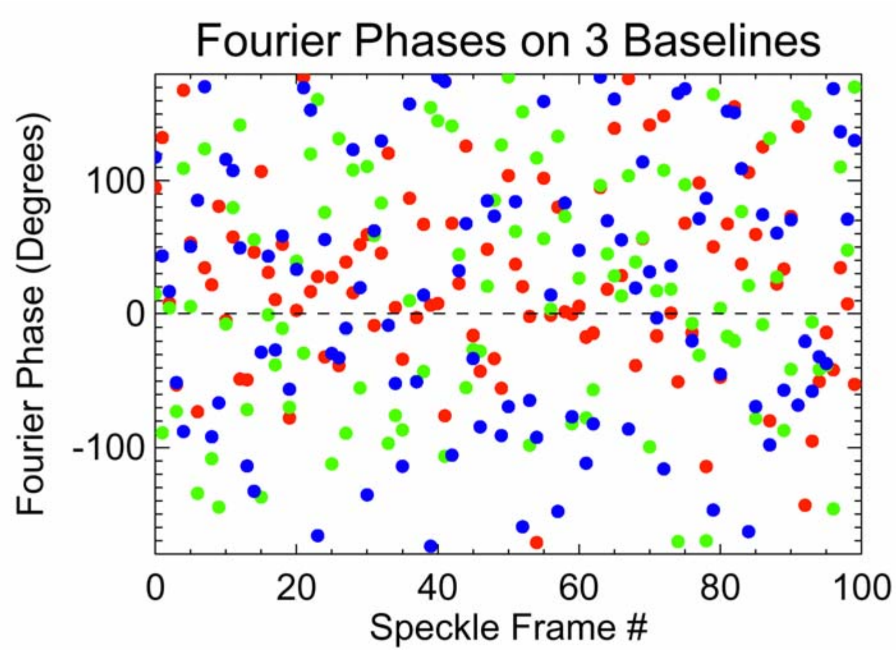
Closure Amplitudes

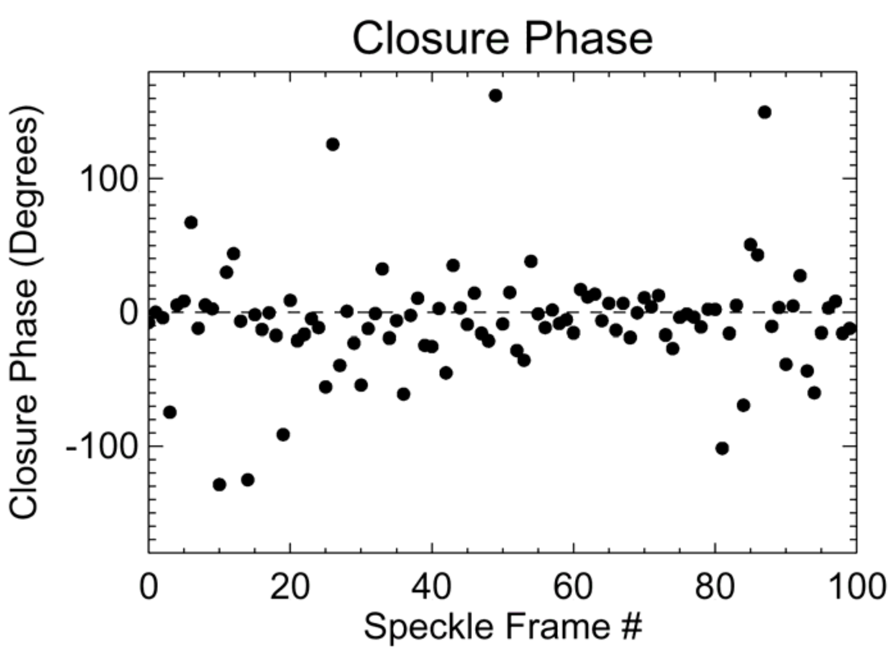
$$
\begin{aligned}
A_{ijkl} &= \frac{|\tilde{\mathcal{V}}_{ij}^{\text{measured}}||\tilde{\mathcal{V}}_{kl}^{\text{measured}}|}{|\tilde{\mathcal{V}}_{ik}^{\text{measured}}||\tilde{\mathcal{V}}_{jl}^{\text{measured}}|} \\
&= \frac{|\tilde{G}_i||\tilde{G}_j||\tilde{\mathcal{V}}_{ij}^{\text{true}}||\tilde{G}_k||\tilde{G}_l||\tilde{\mathcal{V}}_{kl}^{\text{true}}|}{|\tilde{G}_i||\tilde{G}_k||\tilde{\mathcal{V}}_{ik}^{\text{true}}||\tilde{G}_j||\tilde{G}_l||\tilde{\mathcal{V}}_{jl}^{\text{true}}|} \\
&= \frac{|\tilde{\mathcal{V}}_{ij}^{\text{true}}||\tilde{\mathcal{V}}_{kl}^{\text{true}}|}{|\tilde{\mathcal{V}}_{ik}^{\text{true}}||\tilde{\mathcal{V}}_{jl}^{\text{true}}|}.
\end{aligned}
$$

Closure amplitudes have not been used effectively in optical interferometry because fringe amplitude fluctuations are mostly caused by variable atmospheric coherence (and because there are few 4-telescope arrays).

However, closure amplitudes should be useful for interferometers using spatial filters such as single-mode fibers.

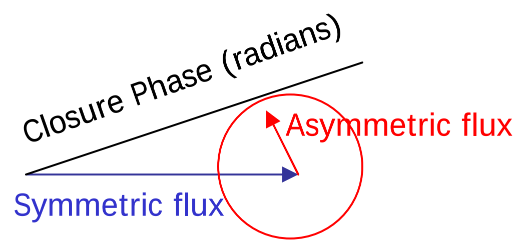
Example: Phase and Closure Phase from Keck data



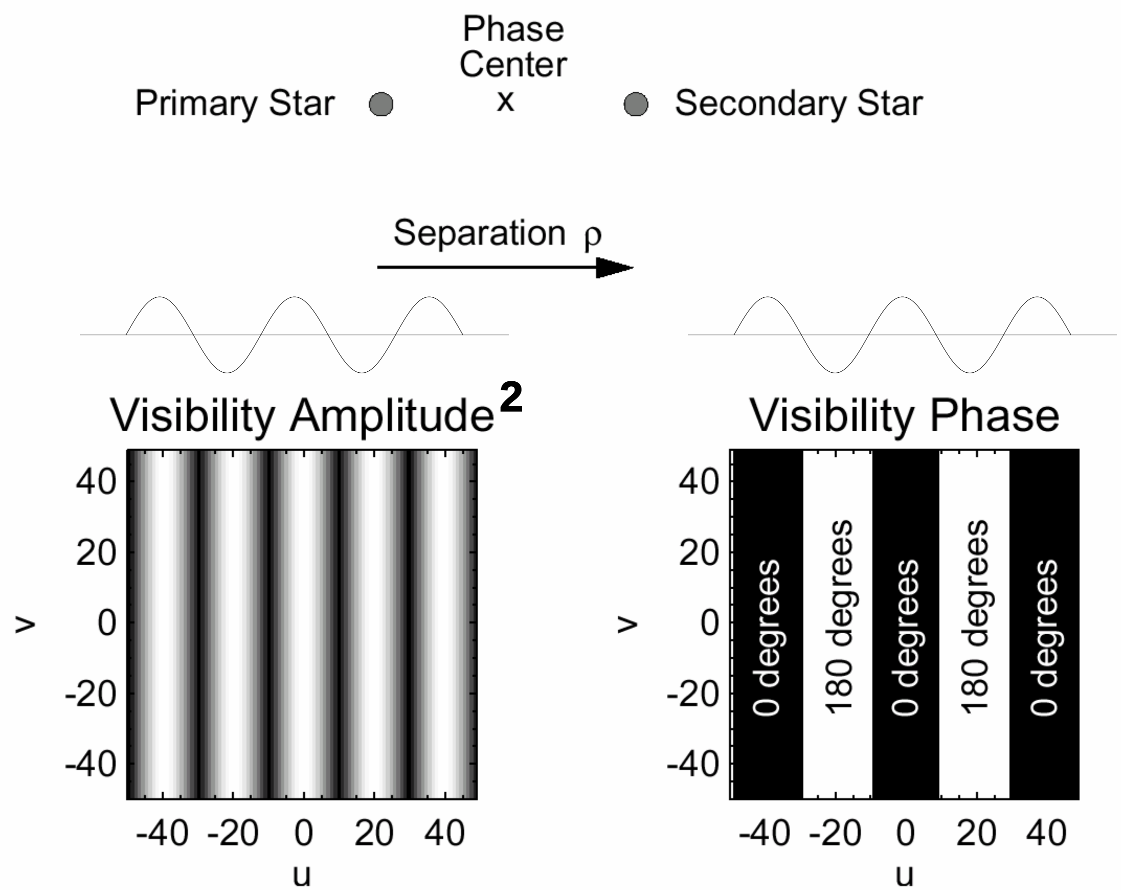


Properties of Closure Phase

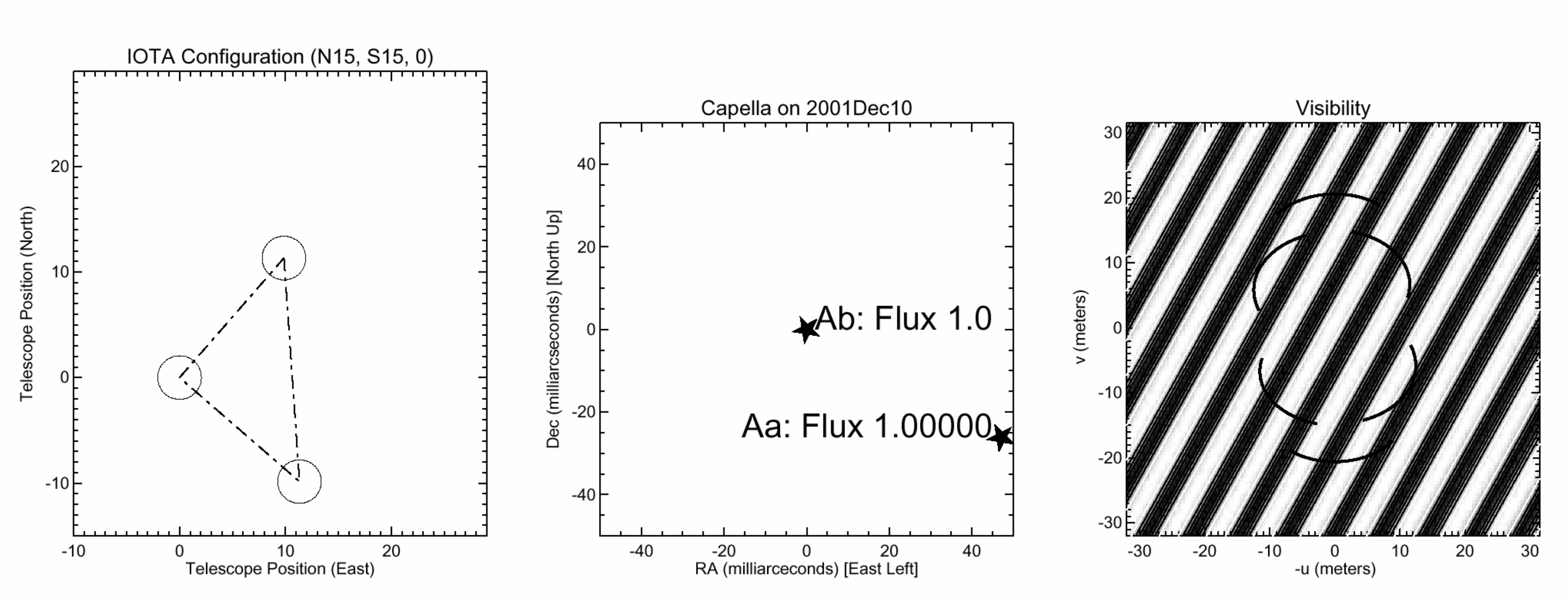
- Robust to *telescope-specific* calibration errors
  - Atmospheric turbulence generally does not bias phase measurements (unlike Visibility^2). But biases (e.g. photon noise) and errors (e.g. from pairwise beam combination) can exist.
  - Hope to beat down measurement error as $\sqrt{N}$
- Sensitive to asymmetries in brightness distribution
  - The Bispectrum is REAL for point-symmetry (Φ_{CP} = 0 or 180 degs)
  - Magnitude of CLP signal given by the ratio of symmetric to asymmetric flux at that resolution



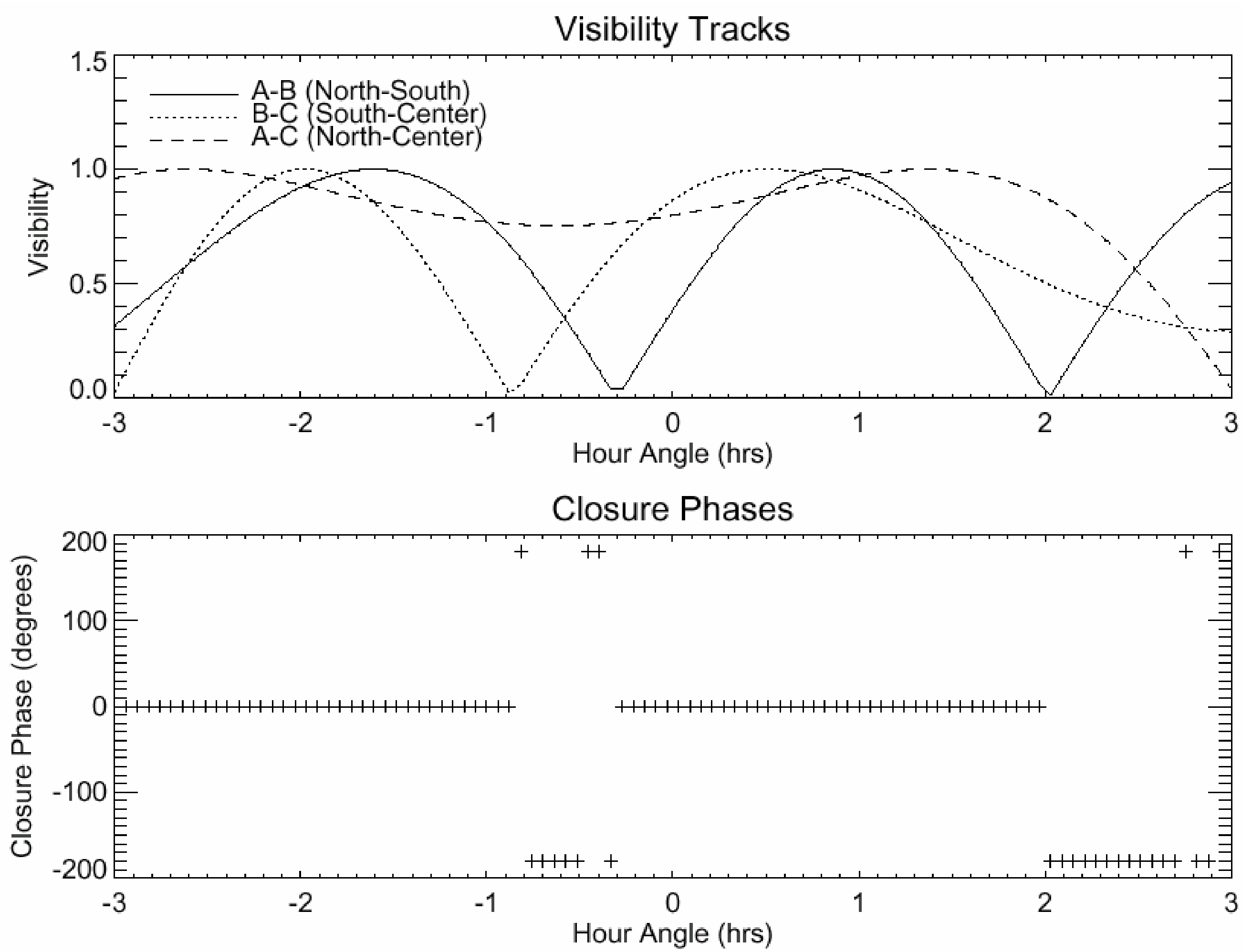
Simple Example: equal binary

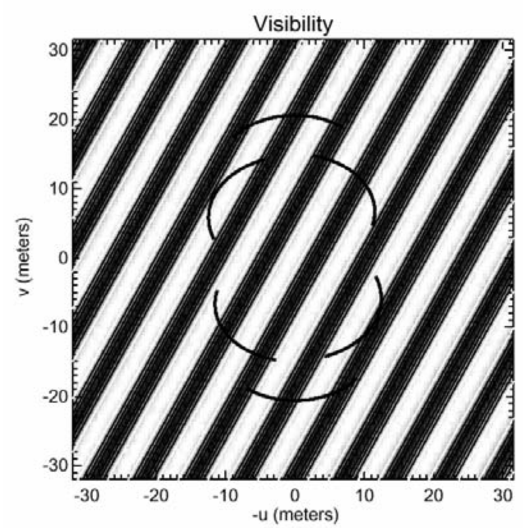


Equal Binary: Simulation with 3 telescope synthesis

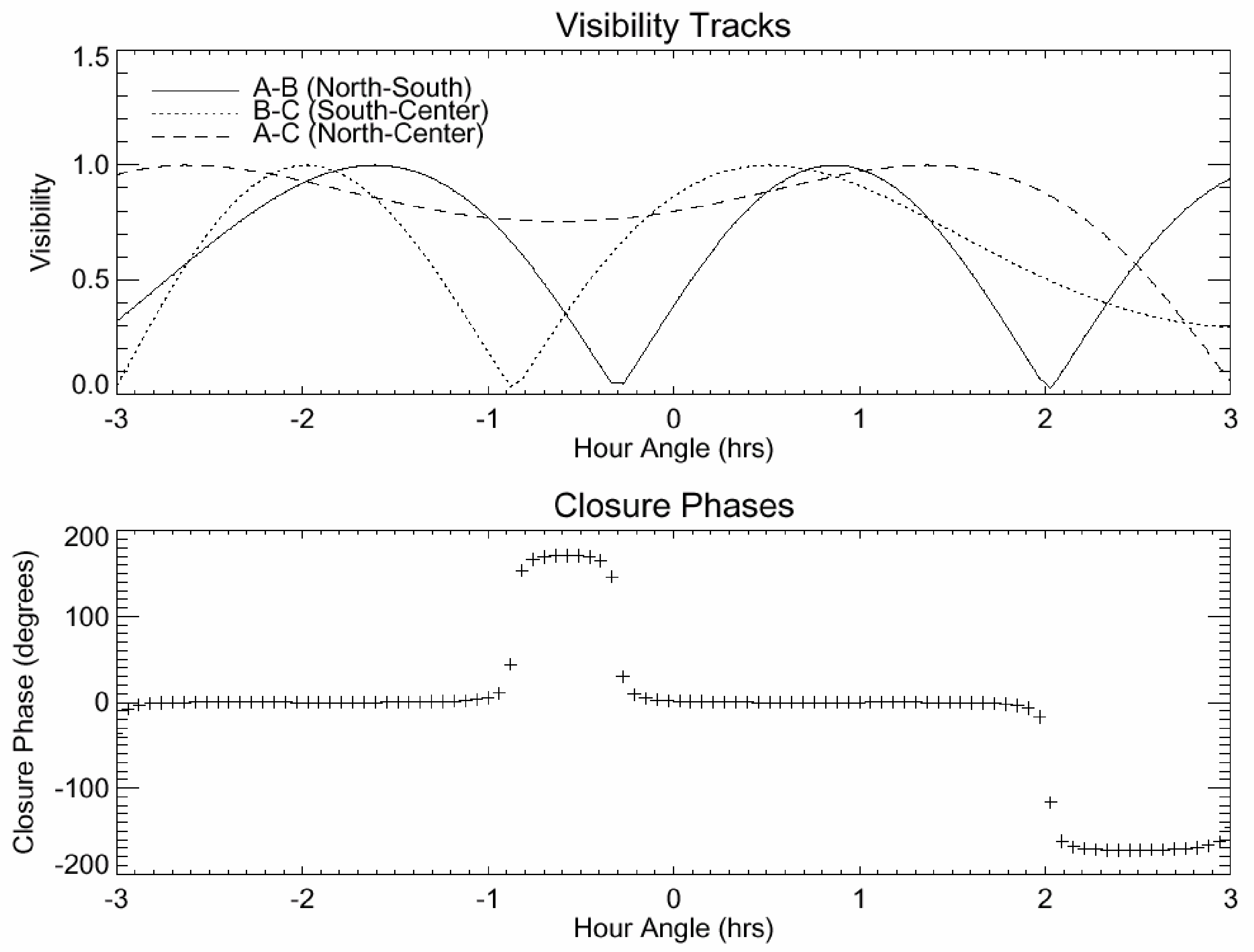


Equal Binary: Vis and CLP tracks





Almost-Equal binary 1.05 : 1



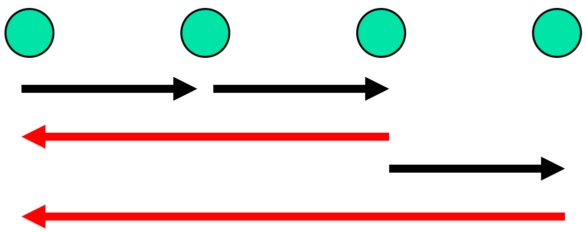
Getting from data to Image

Current status of Imaging for Optical Interferometry

- Fourier Data are Incomplete
- Fourier Data have Noise
- Phases need to be found from Closure Phases
- External assumptions are required

Baseline Bootstrapping

Redundant array:
baseline bootstrapping

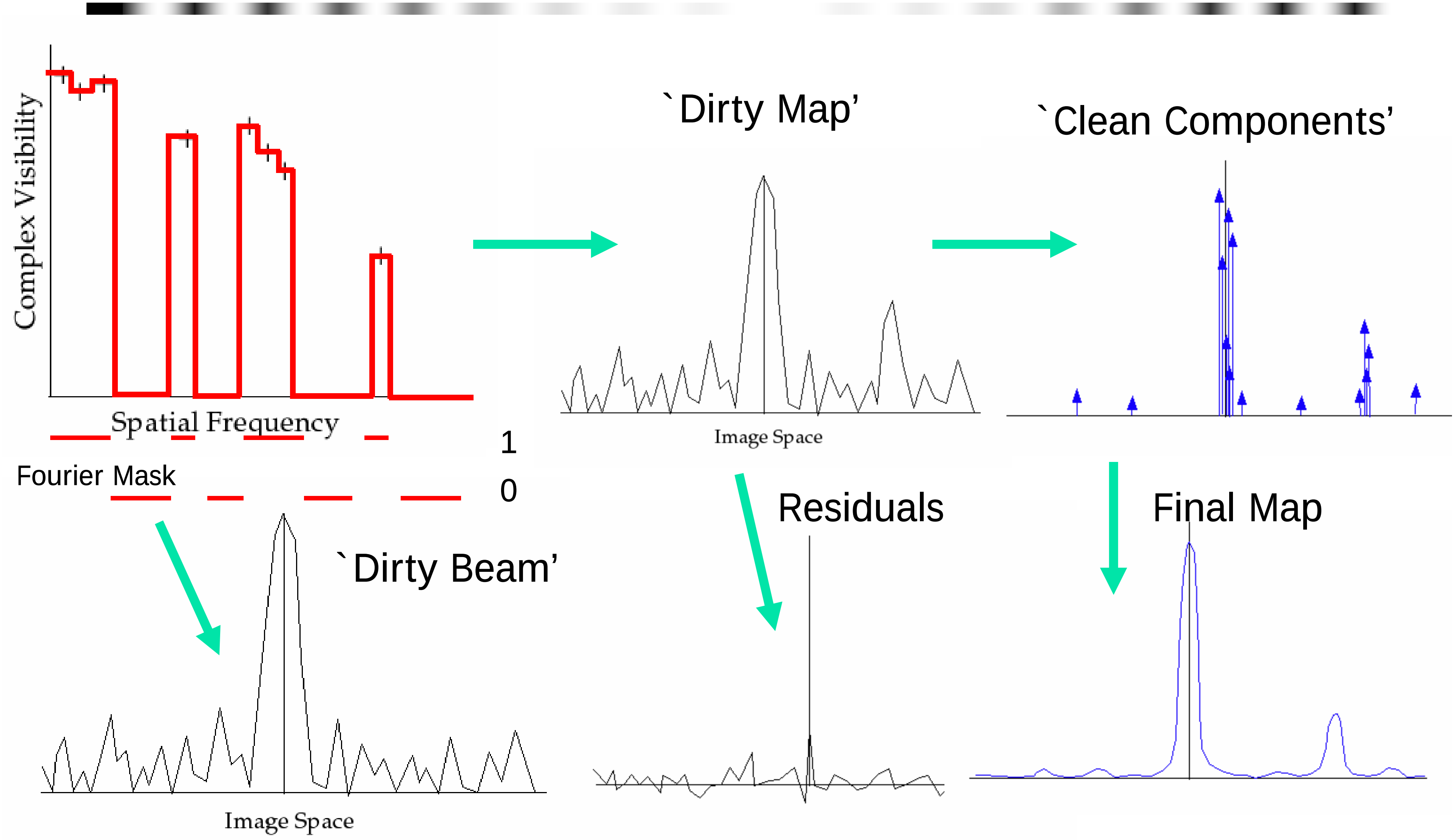


- **Pros**
  - Direct recovery of the phase
  - Minimimal initial assumptions
- **Cons**
  - Cumulative error propagation
  - Array must be redundant

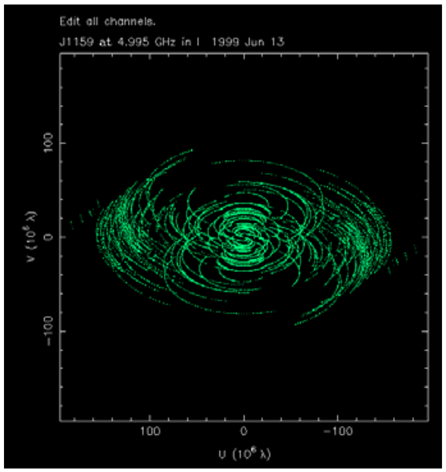
Phase Recovery: Image Plane Constraints

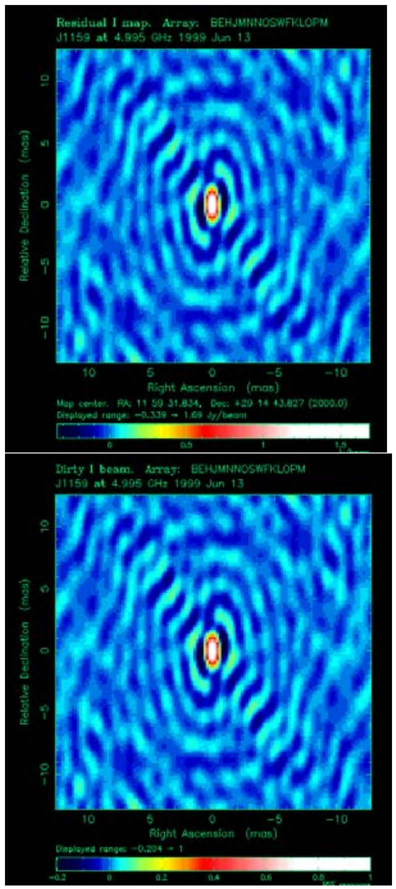
- Finite extent
  - Flux from a bounded region of image space
- Positive Definite
  - No negative photons
- Smoothness
  - Image as smooth as is consistent with the data
- Prior Information
  - From other observations or theoretical considerations

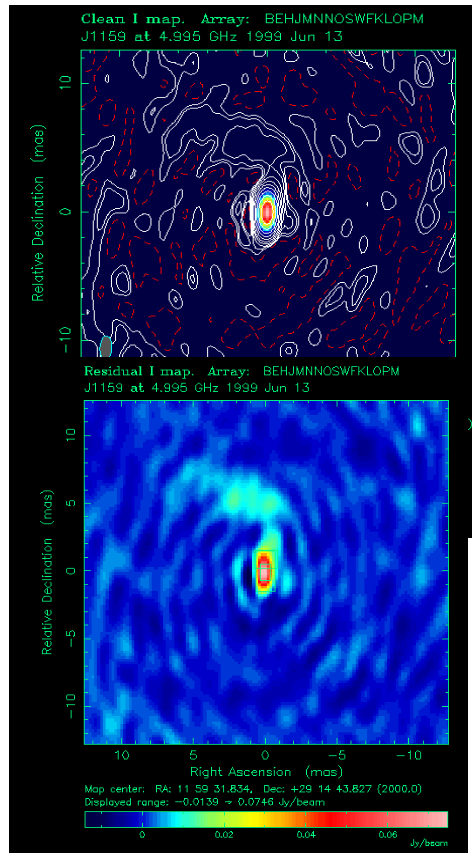
Deconvolution: Clean

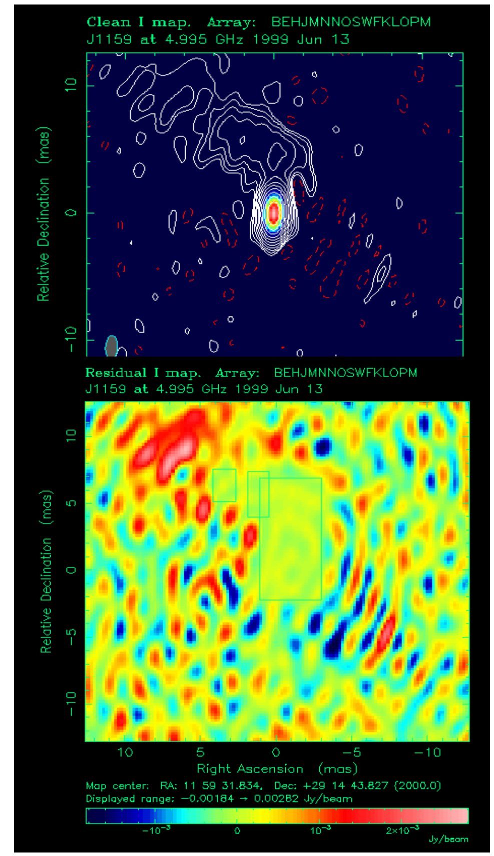


CLEAN: example VLA 5 GHz









From Mike Garrett, NRAO summer school

Clean 30

Clean 2300

CLEAN Comments

- Pros:
  - Intuitive, numerically quick, intensively developed in radio
- Cons:
  - Generates Negative Flux
  - After reconvolving, solution no longer fits data
  - Difficult to incorporate errors
  - Arbitrary parameters: gain, stopping point
  - Diverges if left to iterate.
  - Very difficult to put on rigorous statistical footing

SIRIUS CYBERNETICS CORPORATION PRODUCTS:
"It is very easy to be blinded to the essential uselessness of them by the sense of achievement you get from getting them to work at all … their fundamental design flaws are completely hidden by their superficial design flaws."

Douglas Adams, Hitch Hiker's Guide to the Galaxy.

A Bayesian Approach to Regularization: MEM

- Maximum Entropy Method
  - Philosophy: Of that set of images which all fit the data adequately (in a chi-squared sense), we should choose the one with the *least* information *(Occam's Razor)*
  - Information content of an image can be quantified by the ENTROPY:

Fraction of flux in pixel i

Skilling & Bryan (1984)

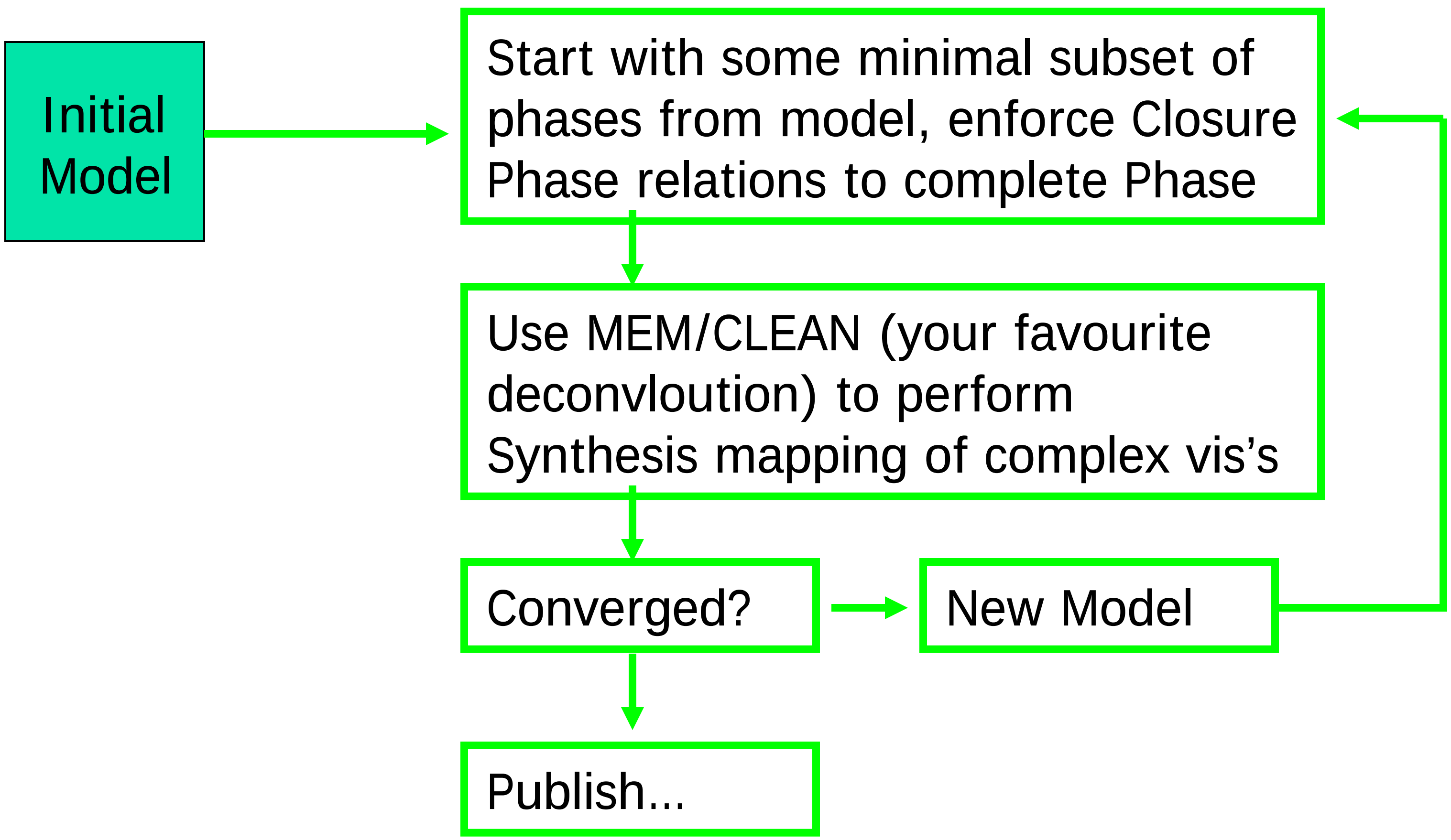
Entropy
$$S = -\sum_i f_i \ln \frac{f_i}{I_i}$$
Image prior

Sum over all pixels

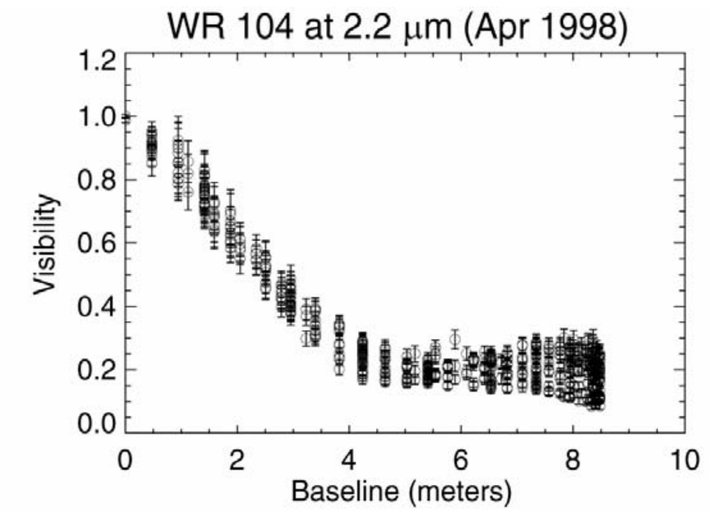
  - Naturally enforces positivity
  - Statistically tractable
  - Incorporates Prior information (no data: output=prior)
  - Smoothness criterion results in super-resolution

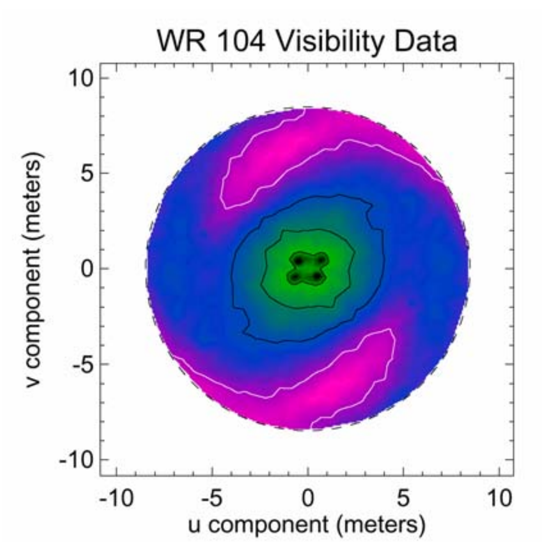
Self-Cal

Problem: We can’t start deconvolution without complex vis’s (not CLP’s)

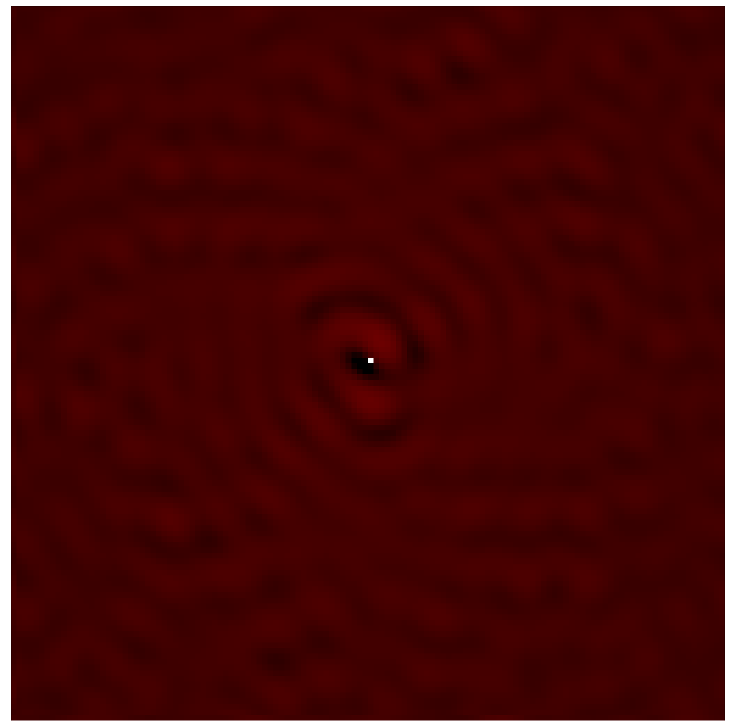


Example: WR 104 data



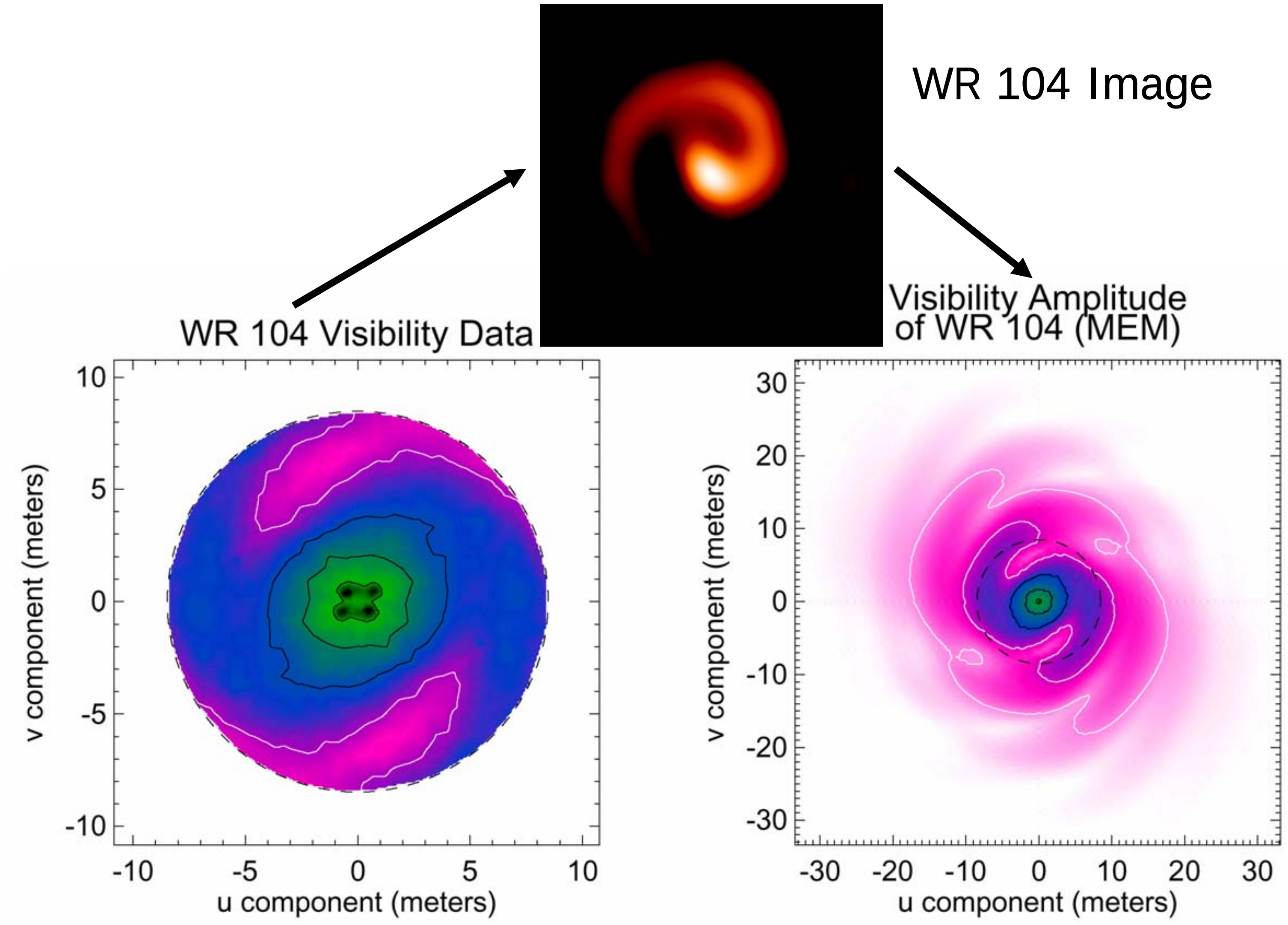


Iterations 1 to 30

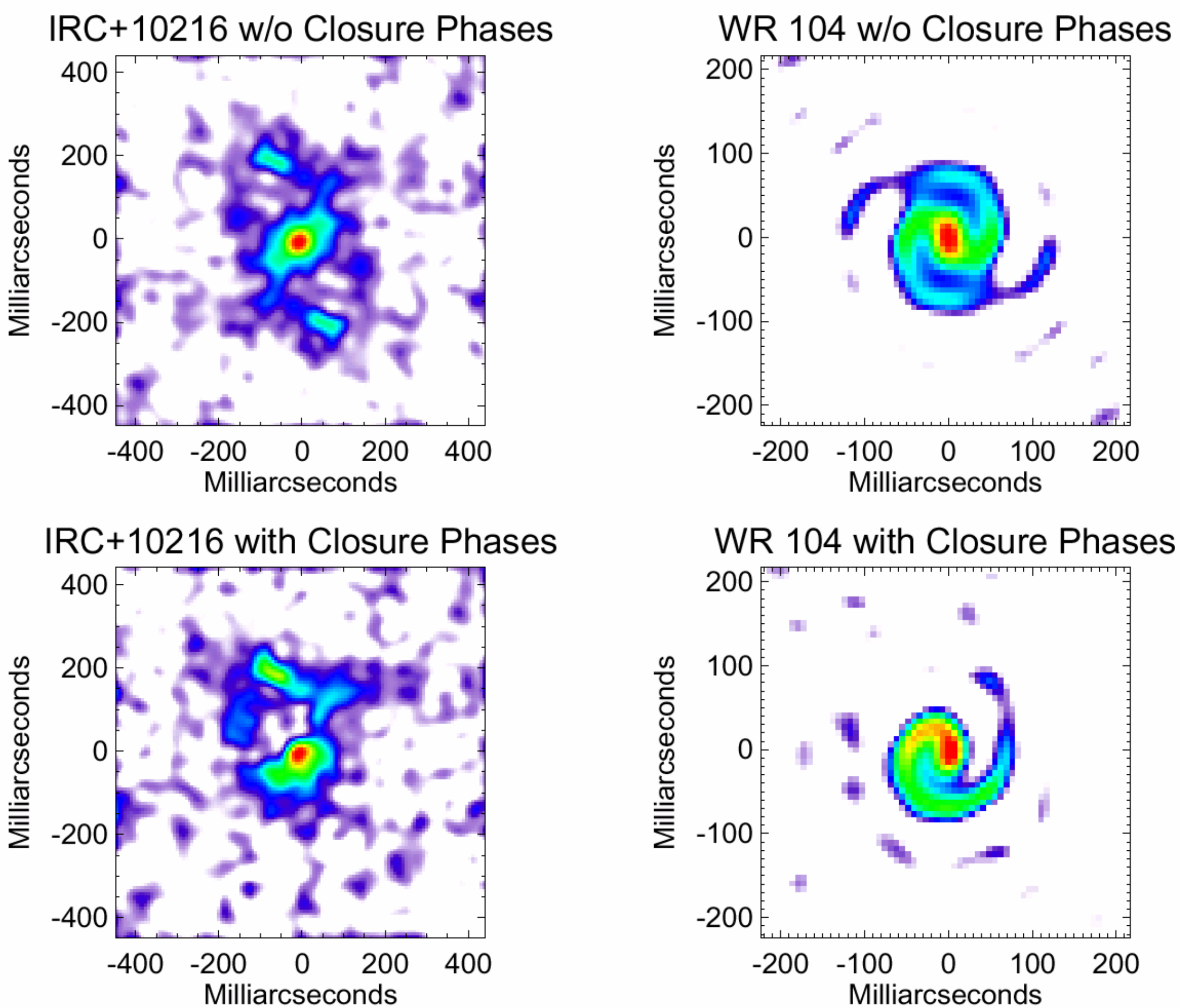


WR 104 (2.2 microns)

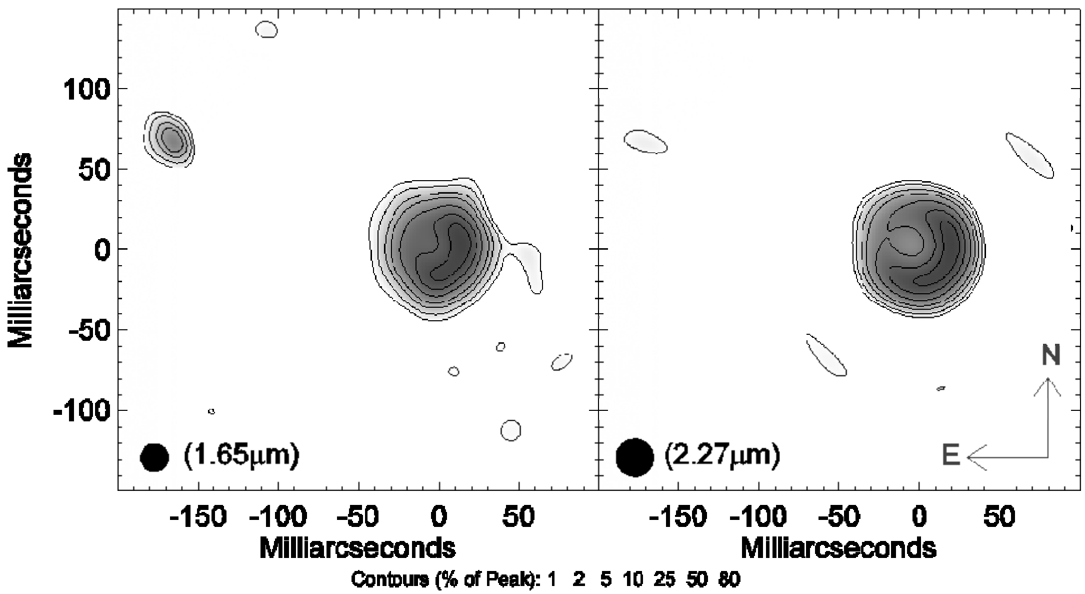
Super-Resolution (and snake oil)

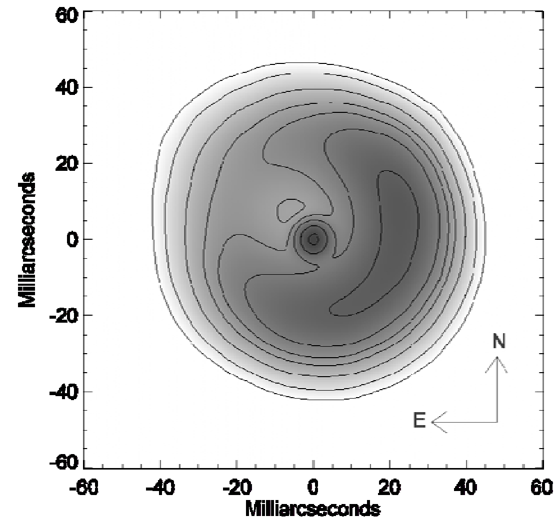


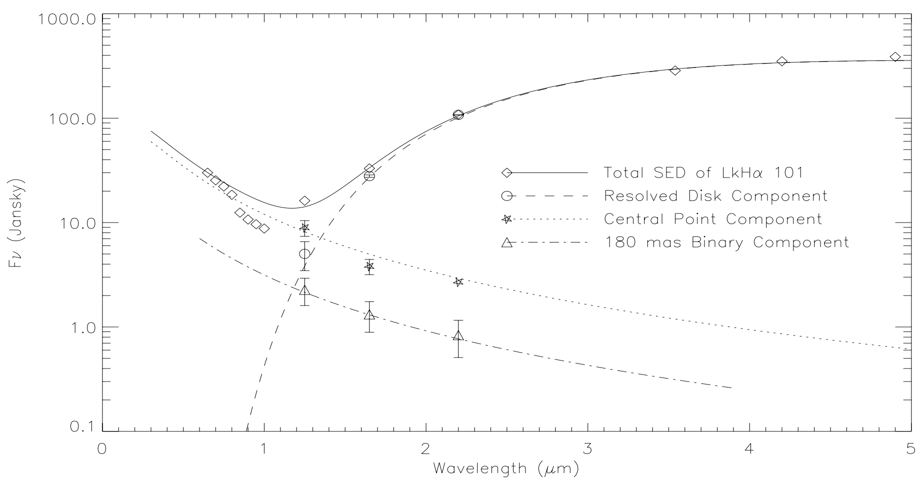
The importance of closure phase

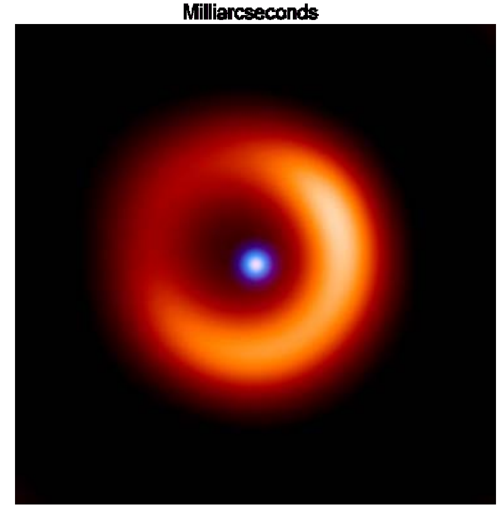


Using extra information in mapping: Case Study



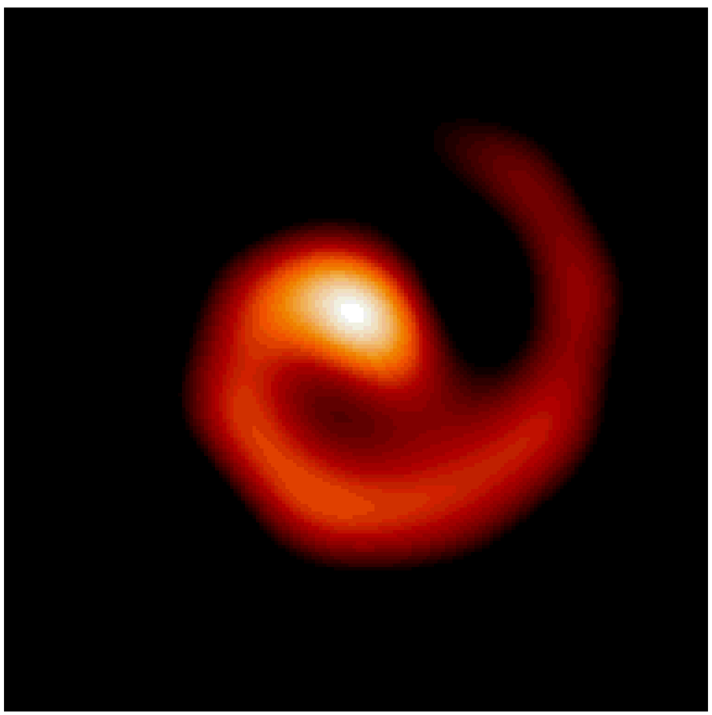


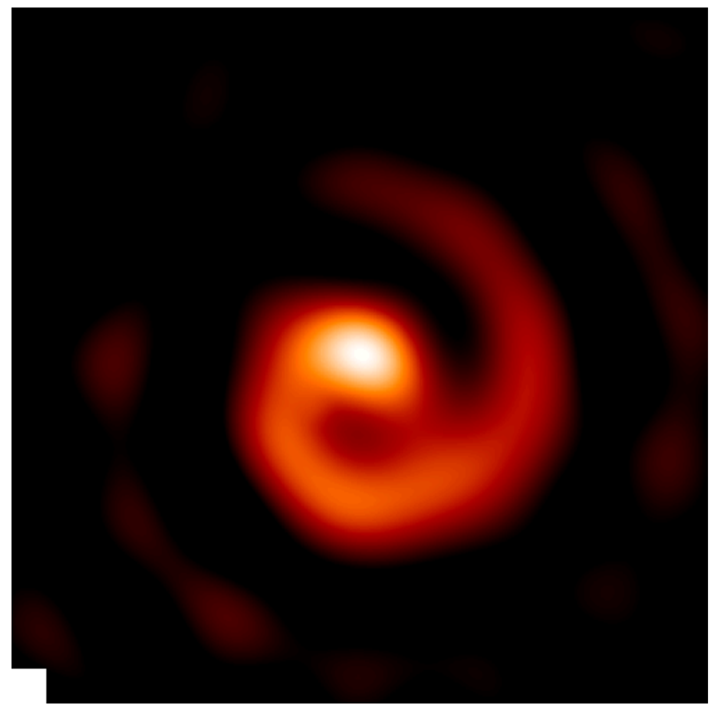




Colliding Winds in WR 104

75 AU





Actual Observations
Period 243.5 days

Tuthill, Monnier & Danchi 1999