Interferometer Optical Design

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Goals of this talk:

1. Learn to think in terms of wavelets.
2. Learn how to calculate the interference of wavefronts for any optical system.
3. Learn how to separate astrophysical from instrumental effects.

Note: Direct combination of wavefronts (homodyne detection) is discussed here, i.e. $\lambda < 10\ \mu m$; see radio for heterodyne detection.

Michelson Summer School
Caltech, Pasadena   8 July 2003

Summation of wavelets

Born and Wolf (7th edition, p. 428) define the wavelet summation integral as the Fourier-transform relation between amplitude in the pupil $A_{in}(x,y)$ and amplitude in the focal plane $A_{out}(u,v)$.

Image amplitude = Sum of wavelet amplitudes

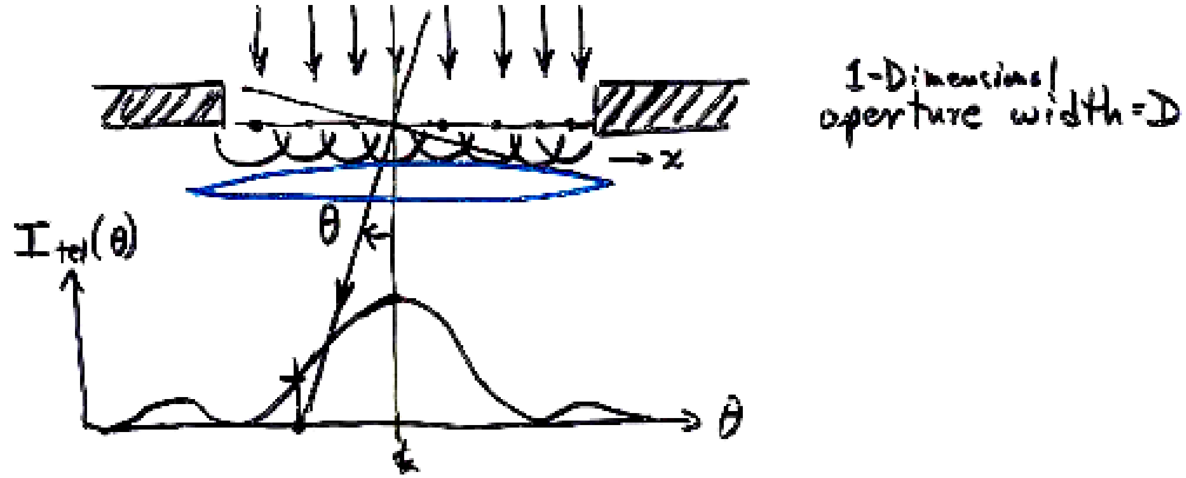
$$A_{out}(u,v) = \frac{1}{\lambda f} \int A_{in}(x,y) e^{-ik(xu+yv)/f} \, dxdy$$

where

$$|A(x,y)|^2 = energy/area = Intensity$$

Simplify: (1) 2D→1D; (2) coef.= 1; (3) u/f = θ = angle in focal plane.

Derive single-telescope response to point source



amplitude. $A_{tel}(\theta) = \sum_{wavelets} = \int_D e^{i(\text{phase at } x)}\,dx$

$$= \int_{-D/2}^{+D/2} e^{i\left(2\pi \frac{x\theta}{\lambda}\right)}\,dx \Big/ \int dx$$

$$= \frac{\lambda}{2\pi i\theta}\left[e^{+i\pi\theta D/\lambda} - e^{-i\pi\theta D/\lambda}\right] / D$$

$$= \frac{\sin(\pi\theta D/\lambda)}{(\pi\theta D/\lambda)}$$

intensity. $I_{tel}(\theta) = |A_{tel}|^2 = \left[\frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda}\right]^2$

1st zero. $I_{tel}(\theta_{tel}) = 0$ when $\theta_{tel} = \lambda/D$

circular aperture. $\int_{circle} \cdots \Rightarrow I_{tel}(\theta) = \left[\frac{2J_1(\pi\theta D/\lambda)}{\pi\theta D/\lambda}\right]^2$

$\theta_{tel} = 1.22\,\lambda/D$

Single telescope again

add constant phase ϕ.

$$A_{tel}(\theta) = \int_{-D/2}^{+D/2} e^{i\left(2\pi \frac{x\theta}{\lambda} + \phi\right)} dx \Big/ \int dx = \frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda} \cdot e^{i}$$

I_{tel} is unchanged.

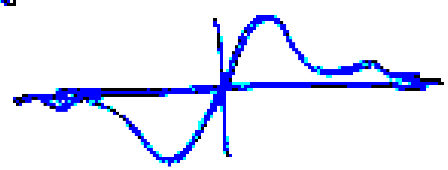
add off-axis angle θ_0.

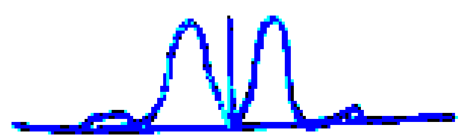
$$A_{tel}(\theta) = \int_{-D/2}^{+D/2} e^{i\left(2\pi x(\theta - \theta_0)/\lambda\right)} dx \Big/ \int dx = \frac{\sin\left(\pi(\theta-\theta_0)D/\right.}{\pi(\theta-\theta_0)D/\lambda}$$

I_{tel} is shifted to center at θ_0.

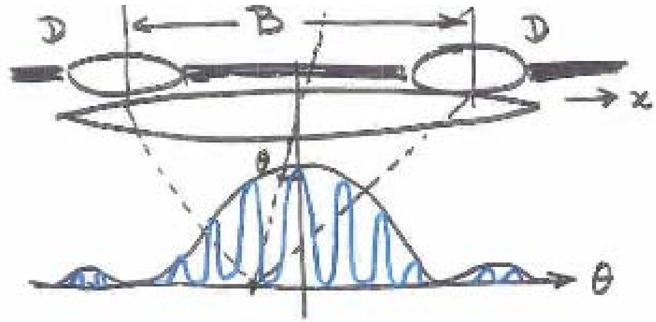
add phase step (across 1/2 aperture) of π.

$$A_{tel}(\theta) = \left[\int_{0}^{+D/2} e^{i(2\pi x\theta/\lambda + \pi/2)} dx + \int_{-D/2}^{0} e^{i(2\pi x\theta/\lambda - \pi/2)} dx\right] \Big/ \int$$

$=$ 

$I_{tel}(\theta) =$  i.e. 2 speckles.

Derive interferometer (2-tel.) response



<u>amplitude.</u> $A_{int}(\theta) = \sum_{wavelets} = \int e^{i(\text{phase at } x)}\,dx$

$D+D$

$$= \left[\int_{+\frac{1}{2}B-\frac{1}{2}D}^{+\frac{1}{2}B+\frac{1}{2}D} \sim \; + \; \int_{-\frac{1}{2}B-\frac{1}{2}D}^{-\frac{1}{2}B+\frac{1}{2}D} \sim \right] / 2D$$

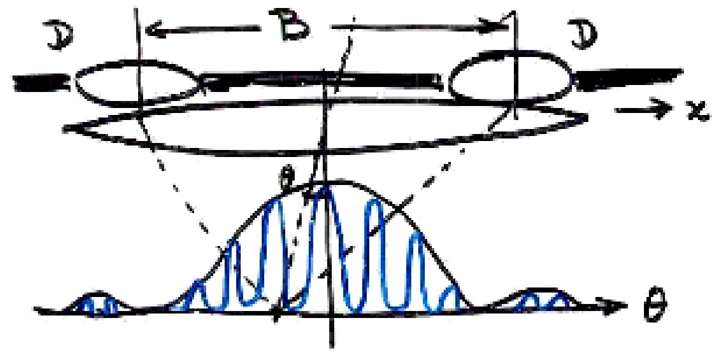
$$= \frac{\sin(\pi \theta D/\lambda)}{\pi \theta D/\lambda} \cdot \cos(\pi \theta B/\lambda)$$

<u>intensity.</u> $I_{int}(\theta) = I_{tel}(\theta) \cdot \frac{1}{2}\left[1 + \cos(2\pi\theta B/\lambda)\right]$

<u>1st zero.</u> $I_{int}(\theta_{int}) = 0$ when $\theta_{int} = \frac{\lambda}{2B}$ = width of fringe.

<u>number of fringes in packet.</u> $(\text{no. fringes}) = \frac{2\,\theta_{tel}}{2\,\theta_{int}} = \frac{1.22\,\lambda/D}{0.50\,\lambda/B} = 2.44\,\frac{B}{D}$

Derive binary-star response



amplitude. $A_{int}(\theta) = \sum_{wavelets} = \int e^{i(\text{phase at } x)}\,dx$

$$= \left[\int_{+\frac{1}{2}B-\frac{1}{2}D}^{+\frac{1}{2}B+\frac{1}{2}D} \sim \; + \int_{-\frac{1}{2}B-\frac{1}{2}D}^{-\frac{1}{2}B+\frac{1}{2}D} \sim \right] / 2D$$

$$= \frac{\sin(\pi\theta D/\lambda)}{\pi\theta D/\lambda} \cdot \cos(\pi\theta B/\lambda)$$

intensity. $I_{int}(\theta) = I_{tel}(\theta) \cdot \frac{1}{2}\left[1 + \cos(2\pi\theta B/\lambda)\right]$

1st zero. $I_{int}(\theta_{int}) = 0$ when $\theta_{int} = \frac{\lambda}{2B}$ = width of fringe.

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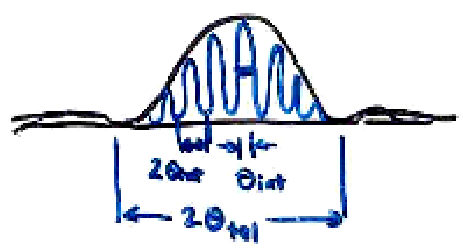
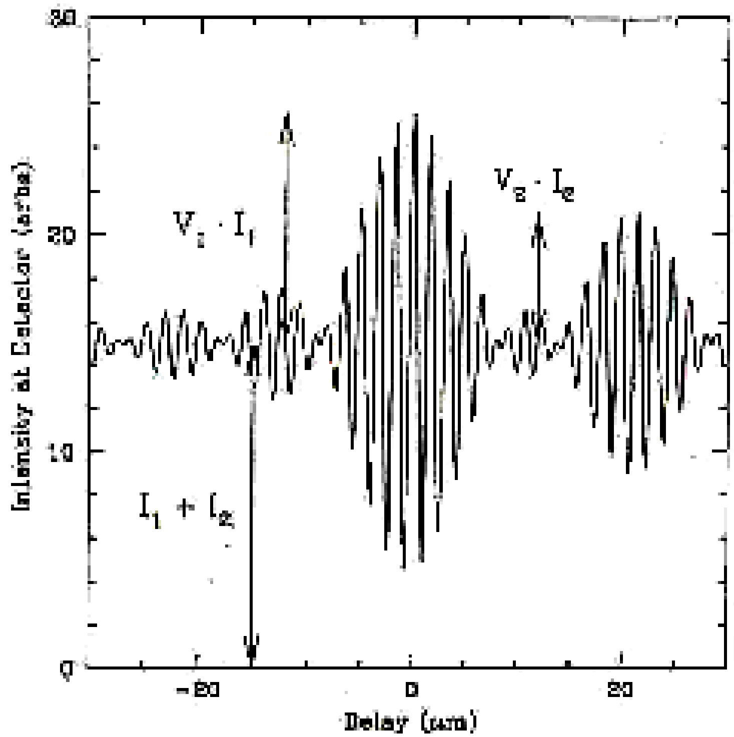
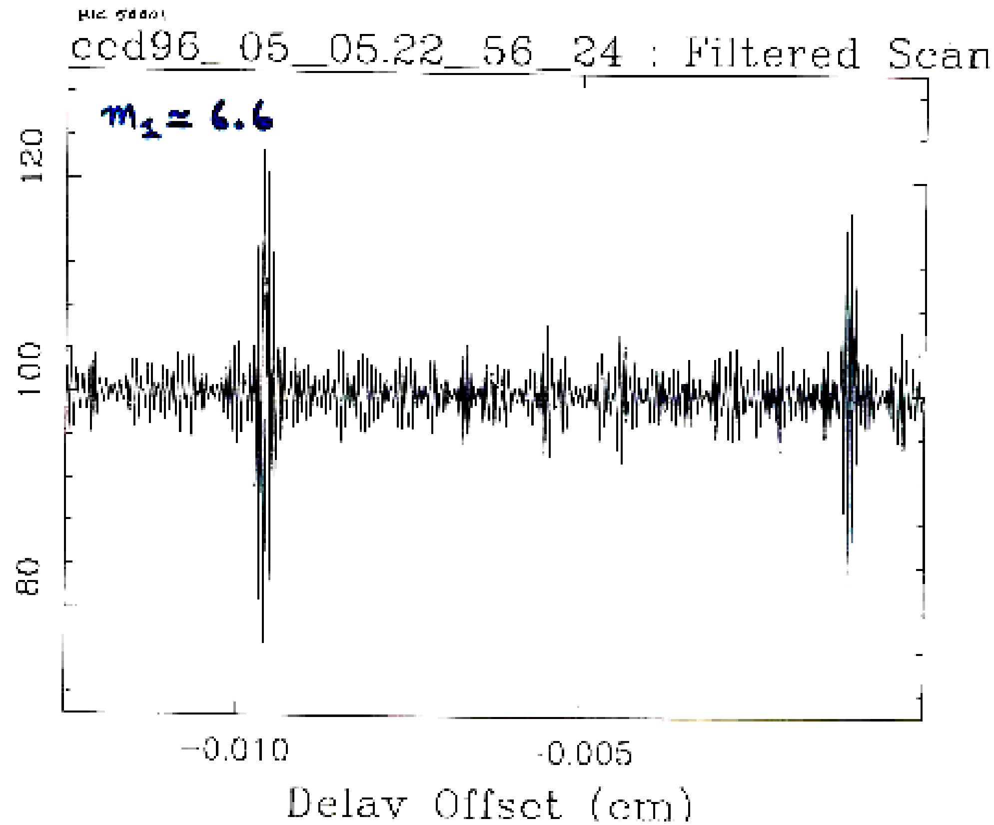


Binary star interferograms





Model interferogram for a binary star, with well-separated fringe packets.

Observed interferogram of a very wide-spaced binary. CCD detector, no filter, IOTA interferometer, 1996 data.

Derive uniform disk response

Add up (incoherent) fringe patterns from square disk:

intensity. $I_{USD}(\theta) = \sum_{sq.\,disk} (\text{intensities}) = \int_{sq.\,disk} I_{int}(\theta - \theta_x) \cdot d\theta_x \, d\theta_y \Big/ \int d\theta$

$$= \int_{-\theta_{disk}/2}^{+\theta_{disk}/2} I_{tel}(\theta) \cdot \tfrac{1}{2}\left[1 + \cos 2\pi(\theta - \theta_x) B/\lambda\right] \cdot \theta_{disk} \cdot d\theta_x \Big/ \theta_{disk}^2$$

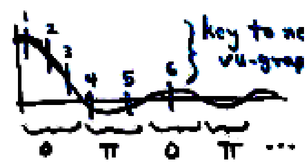
$$\approx I_{tel}(\theta) \cdot \tfrac{1}{2}\left[1 + \underbrace{\left(\frac{\sin \pi B \theta_{disk}/\lambda}{\pi B \theta_{disk}/\lambda}\right)}_{V_{USD}} \cdot \cos \frac{2\pi\theta B}{\lambda}\right]$$

visibility. $V_{USD} = \dfrac{\sin(\pi B \theta_{disk}/\lambda)}{\pi B \theta_{disk}/\lambda}$, square disk.

$V_{UD} = \dfrac{2 J_1(\pi B \theta_{disk}/\lambda)}{\pi B \theta_{disk}/\lambda}$, round disk.

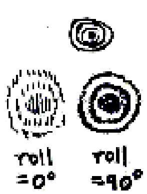
1st zero. $V_{UD} = 0$ when $\theta_{disk} = 1.22\, \lambda / B$, $B = 1.22\, \lambda/\theta_{disk}$

phase. phase $= \begin{cases} 0 & \text{inside odd lobes} \\ \pi & \text{inside even lobes} \end{cases}$



example 1. see next vu-graph from Born & Wolf.

example 2. SIM pocket demonstration card.



$D \simeq 0.07$ mm so $\theta_{tel} = 1.22\, \lambda/D \simeq 2000.''$ $\simeq$ sun, moon.

$B \simeq 0.25$ mm so $\theta_{int} = \frac{\lambda}{2B} \simeq 200.''$ $\simeq$ Mag-light at 6-18 inches.

(# fringes in packet) $= 2.44\, B/D \simeq 8$.

Uniform disk: interferograms

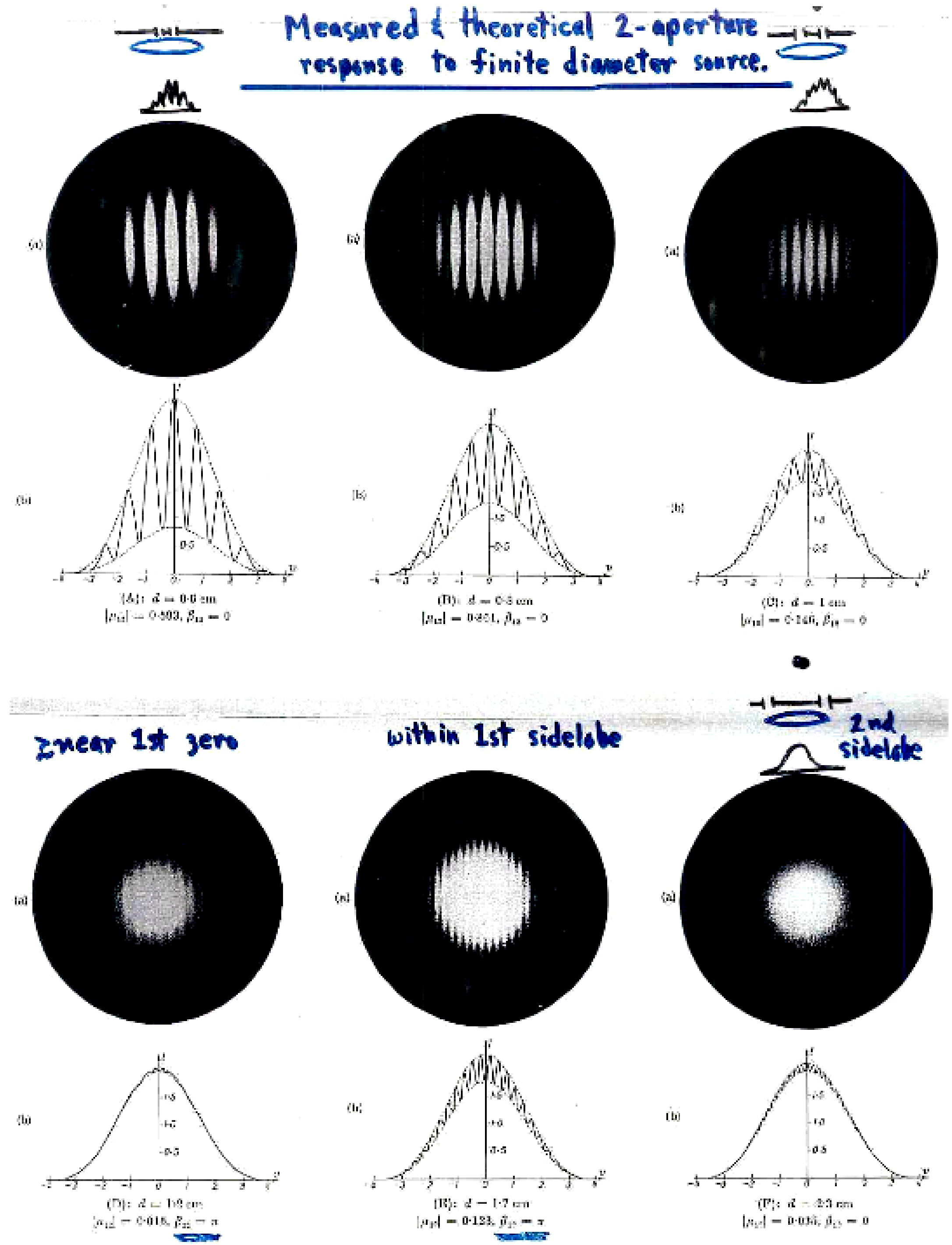


Fig. 10.6. Two-beam interference with partially coherent light.

Van Cittert-Zernike theorem

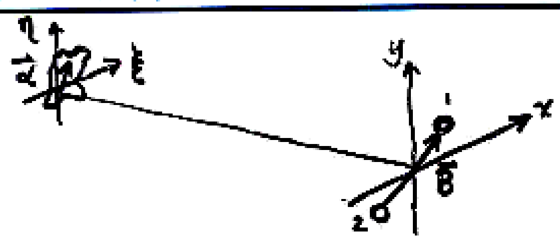
Beam pattern on sky.

Think like a radio astronomer.

The antenna pattern is considered to be projected out from the receiver horn & antenna & array onto the sky. As you move the antenna, or change the phase at an array element, the pattern sweeps across the sky. The received signal is the convolution of the moving pattern and the sources in the sky.

A sinusoidal pattern picks out the Fourier component at that spacing of fringes.

van Cittert – Zernike theorem (1934, 1938).



<u>Complex degree of coherence</u>.

$$\mu_{12}(\vec{B}) = \int_{FOV} I(\vec{\alpha}) \cdot e^{-ik\vec{B}\cdot\vec{\alpha}} \cdot d\vec{\alpha} \Big/ \int_{FOV} I(\vec{\alpha}) \cdot d\vec{\alpha}$$

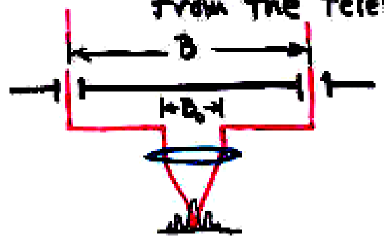
<u>degree of coherence</u>. $|\mu| \equiv V$ = visibility ; <u>phase</u> = $\arg(\mu)$.

<u>inverse relation.</u>

$$I(\vec{\alpha}) \Big/ \int_{FOV} I(\vec{\alpha})\, d\vec{\alpha} = \int_{\text{all } \vec{B}} \mu(\vec{B}) \cdot e^{+ik\vec{B}\cdot\vec{\alpha}} \cdot d\vec{B}$$

Michelson's stellar interferometer

Suppose we decouple the collecting apertures at B from the telescope feed apertures at B_0.



The coherence is measured by B.

The display pattern is set by B_0.

$$I_{int}(\theta) = \underbrace{I_{tel}(\theta)}_{\text{envelope vs } \theta} \cdot \frac{1}{2}\left[1 + \underbrace{\left(\frac{\sin \pi B \theta_{disk}/\lambda}{\pi B \theta_{disk}/\lambda}\right)}_{\text{degree of modulation indep. of } \theta} \underbrace{\cos\left(2\pi\theta B_{tel}/\lambda\right)}_{\text{modulation vs } \theta \text{ with period indep. of B and } \theta_{disk}.}\right]$$

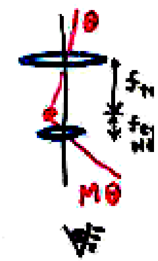
magnification. So can make B_0 any convenient value. Michelson used $B_0 = 1.14$

so the fringe width is $\theta_{int} = \frac{\lambda}{2B_0} = 0.045$ arcsec.

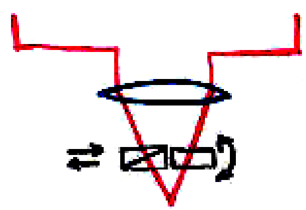
Assume his eye had $\theta_{eye} \simeq 1.22 \frac{\lambda}{5\,mm} \approx 25.$ arcsec.

Magnify θ_{int} to match θ_{eye} with eyepiece M,

$$M = \theta_{eye}/\theta_{int} \approx \frac{25}{.045} \approx 600.$$



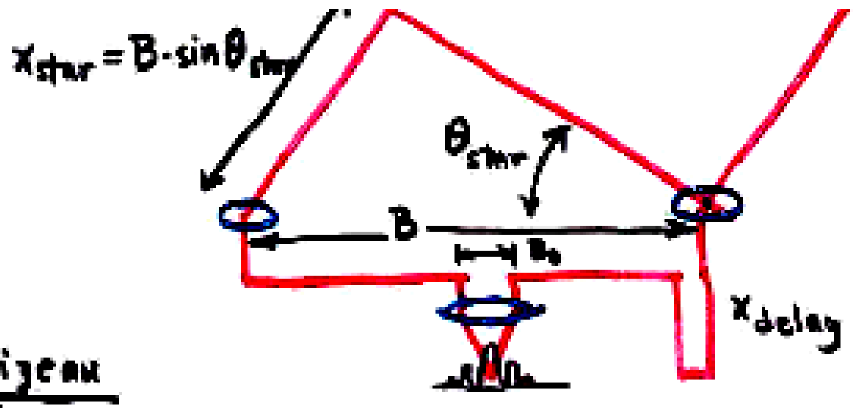
details.



Tilt plate gives angle motion,
+ superposes images,
ie., makes wavefronts parallel at entrance pupil.

Wedge plates give variable thickness,
to compensate for tilt plate's thickness,
ie., makes all color wavefronts arrive at same time
as in other beam.

Image-plane interferometer



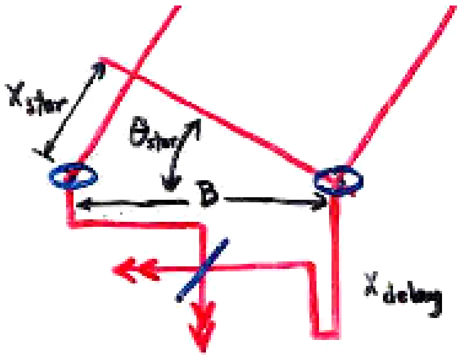
Phase difference between beams

$$\phi = \frac{2\pi}{\lambda}\left(x_{delay} - x_{star}\right)$$

Fizeau type.

$$I_{int}(\theta) = \underbrace{I_{tel}(\theta)}_{\text{envelope vs }\theta} \cdot \frac{1}{2}\left[1 + \underbrace{V}_{\text{visib. of star}} \cdot \cos\frac{2\pi}{\lambda}\left(\underbrace{\theta B_0}_{\text{fringe modulation}} + \underbrace{x_{delay} - x_{star}}_{\text{fringe phase or position.}}\right)\right]$$

Pupil-plane interferometer



Michelson type.

Phase difference between beams

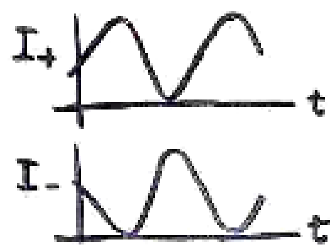
$$\phi = \frac{2\pi}{\lambda}\left(X_{delay} - X_{star}\right) + \frac{\pi}{2}$$

conservation of energy at lossless beamsplitter

$$I_{int}(t) = I_{tel}(\theta)\cdot\frac{1}{2}\left[1 \pm V\cdot\sin\frac{2\pi}{\lambda}\left(X_{delay} - X_{star}\right)\right]$$

Note: B_o here is zero; use time modulation $X_{delay}(t) - X_{star}(t) = vt$

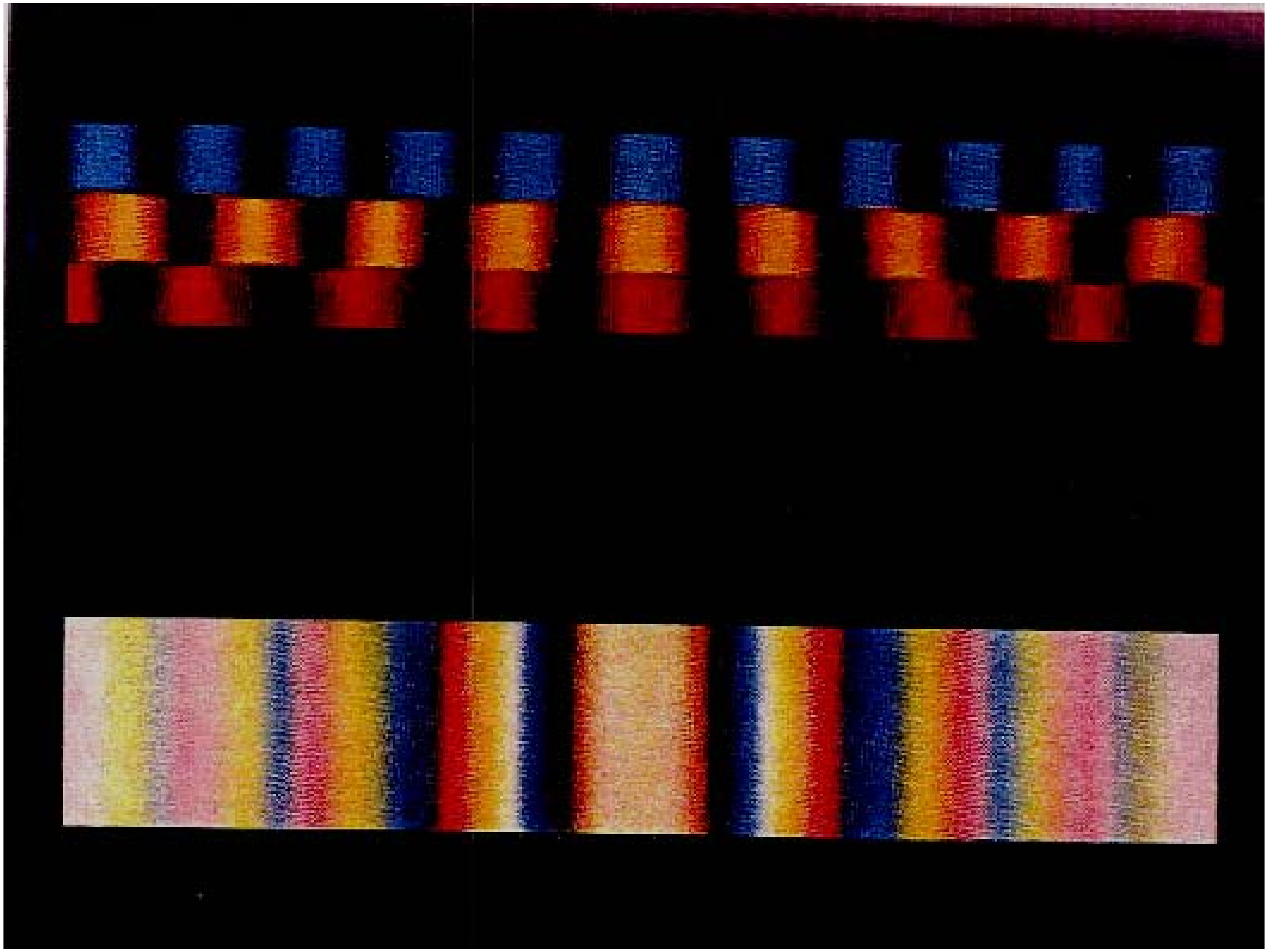
and 1 pixel each for $I_\pm$.



Channel spectrum.

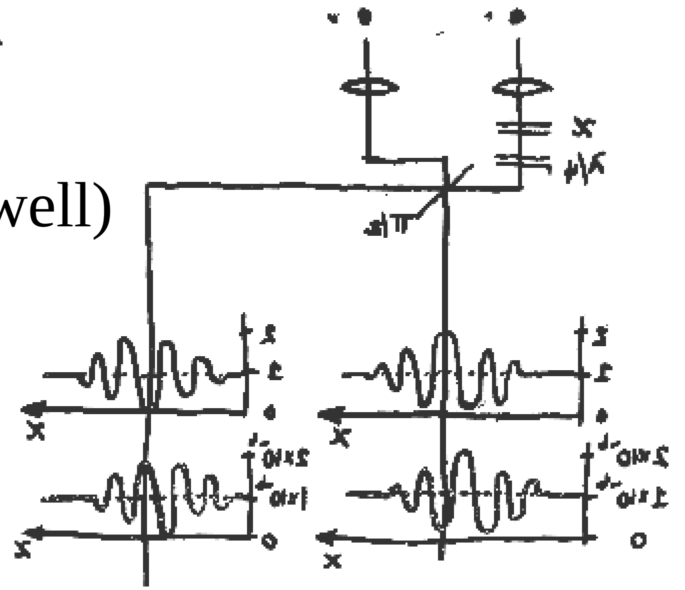
Spatially display each wavelength segment of I_{int}, with delay $\approx$ few λ.

Colors in interferogram



Nulling

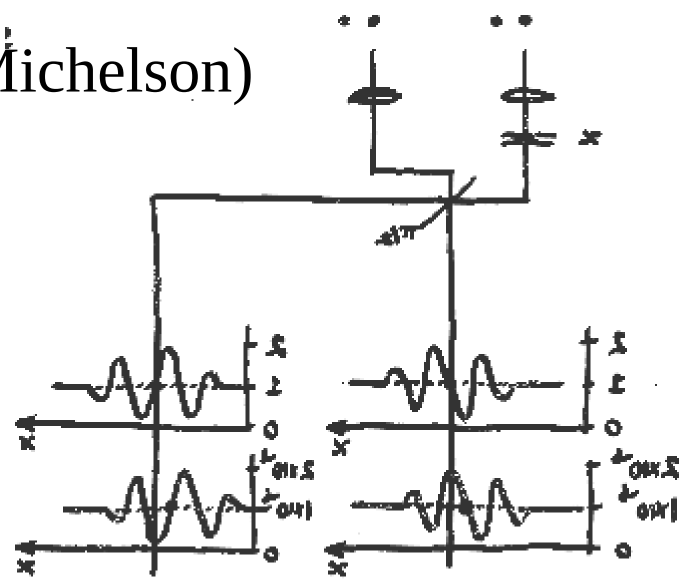
Nulling interferometer (Bracewell)



Star at x=0,
I=0

Planet at x=0,
$I=I_{planet}$

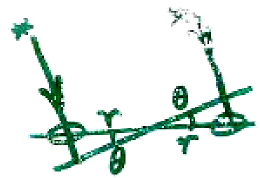
Stellar interferometer (Michelson)



Star at x=0,
I=Istar

Planet at x=0,
I=Iplanet

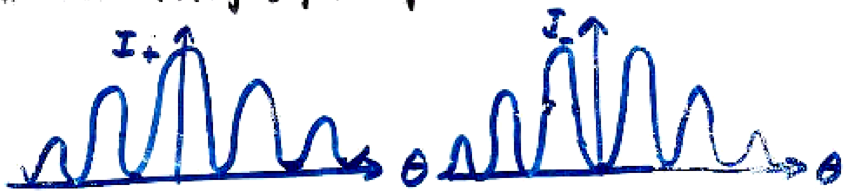
Standard Nulling



Assume that an ideal achromatic null can be arranged. Then the intensity is

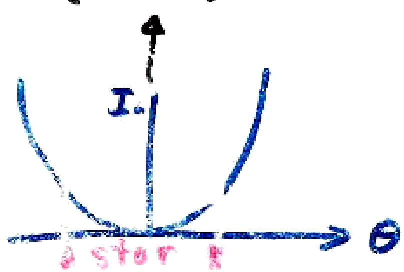
$$I_\pm = \left| e^{ik\theta r} \pm e^{-ik\theta r} \right|^2 = 2\left[1 \pm \cos 2k\theta r\right]$$

The complementary outputs are the bright & null fringes, resp.

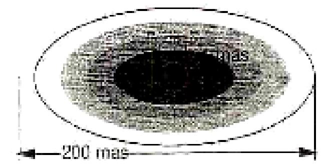


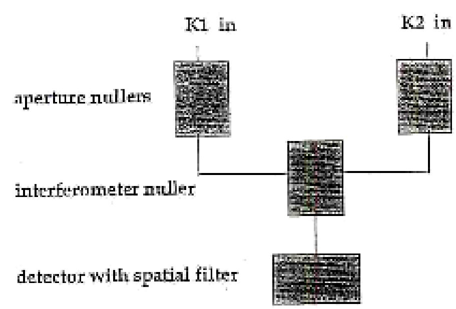
Near the null, the intensity is quadratic

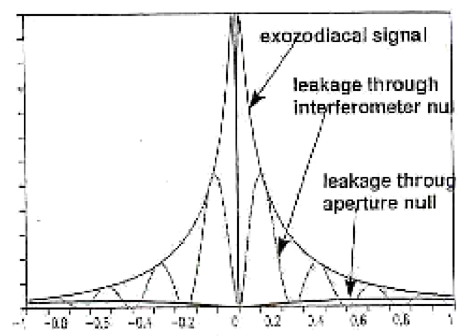
$$I_-(\theta) \approx 2\left[1 - \left(1 - \frac{(2k\theta r)^2}{2!} + \cdots\right)\right] \approx (2k\theta r)^2 - \cdots$$



Theta^2 nulling



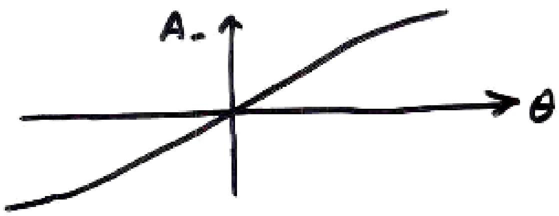




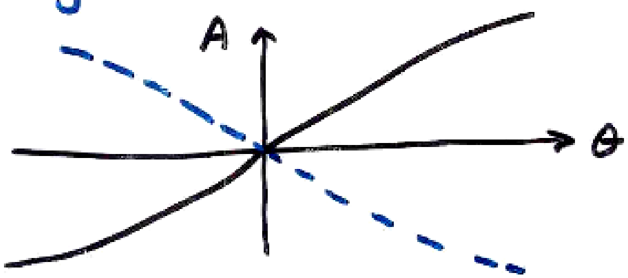
Theta^4 nulling

The amplitude (electric vector) in a standard null is

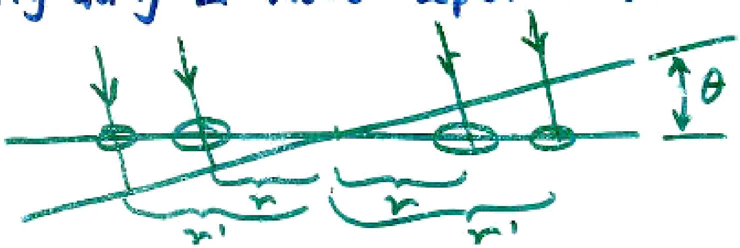
$$A_- = e^{+ik\theta r} - e^{-ik\theta r}$$
$$= 2i \cdot \sin(k\theta r).$$



If we could cancel this amplitude with one of opposite sign, we could make a very wide null.



Try using 2 more apertures:



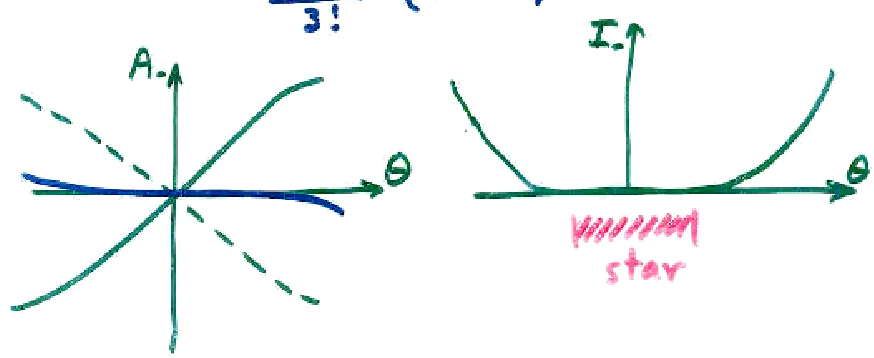
The amplitude from all 4 apertures is, assuming the outer ones have relative weight ε:

$$A_- = \varepsilon e^{+ik\theta r'} + e^{+ik\theta r} - e^{-ik\theta r} - (\varepsilon e^{-ik\theta r'})$$
$$= 2i \cdot \left\{ \sin(k\theta r) - \varepsilon \sin(k\theta r') \right\}$$

where we have added exactly π retardation to flip the sign.

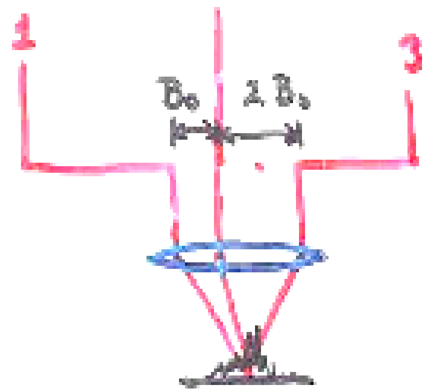
$$A_- \simeq \underbrace{k\theta(r - \varepsilon r')}_{=0 \text{ if } \varepsilon = r/r'} - \frac{(k\theta)^3}{3!}(r^3 - \varepsilon r'^3) + \cdots$$

Then $A_- \simeq -\frac{(k\theta r)^3}{3!} \cdot (1 - 1/\varepsilon^2) + \cdots$



So 2 extra mirrors, or petals, will work, provided that phase control exists.

Multiplexing in the image plane

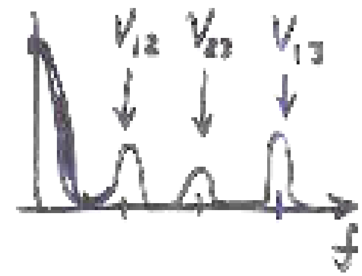


Use minimum redundancy array at combiner lens.

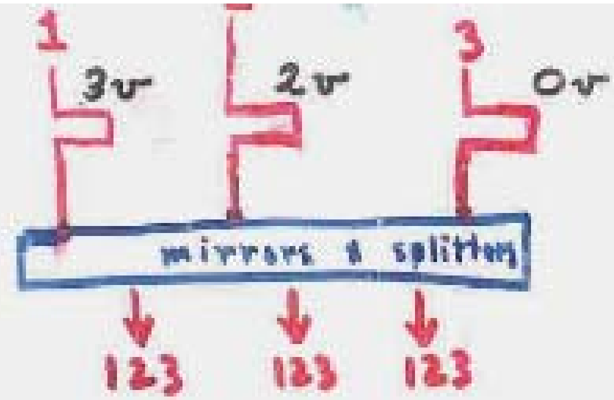
Get different spatial frequency for each baseline.

Fizeau type.

$$|FFT\,(\text{fringe pattern})|^2 = \text{power spectral density}$$



Multiplexing in the pupil plane

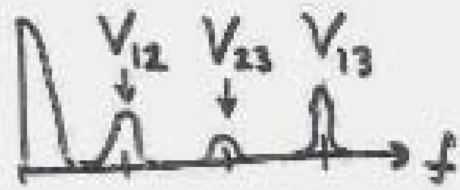


Use different delay-line speeds.

Mix beams & get different time frequencies in each.

Michelson type.

$|\text{FFT (each time sequence)}|^2 = \text{power}$



FOV. The schemes above have a FOV $\gtrsim \theta_{tel}$, i.e. small, because output pupils $\neq$ scaled input pupils.

Golden rule

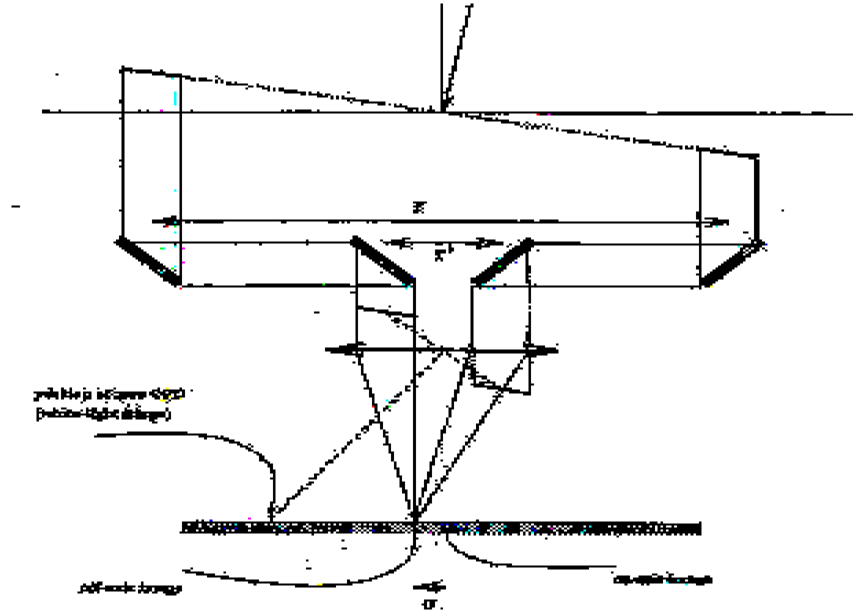
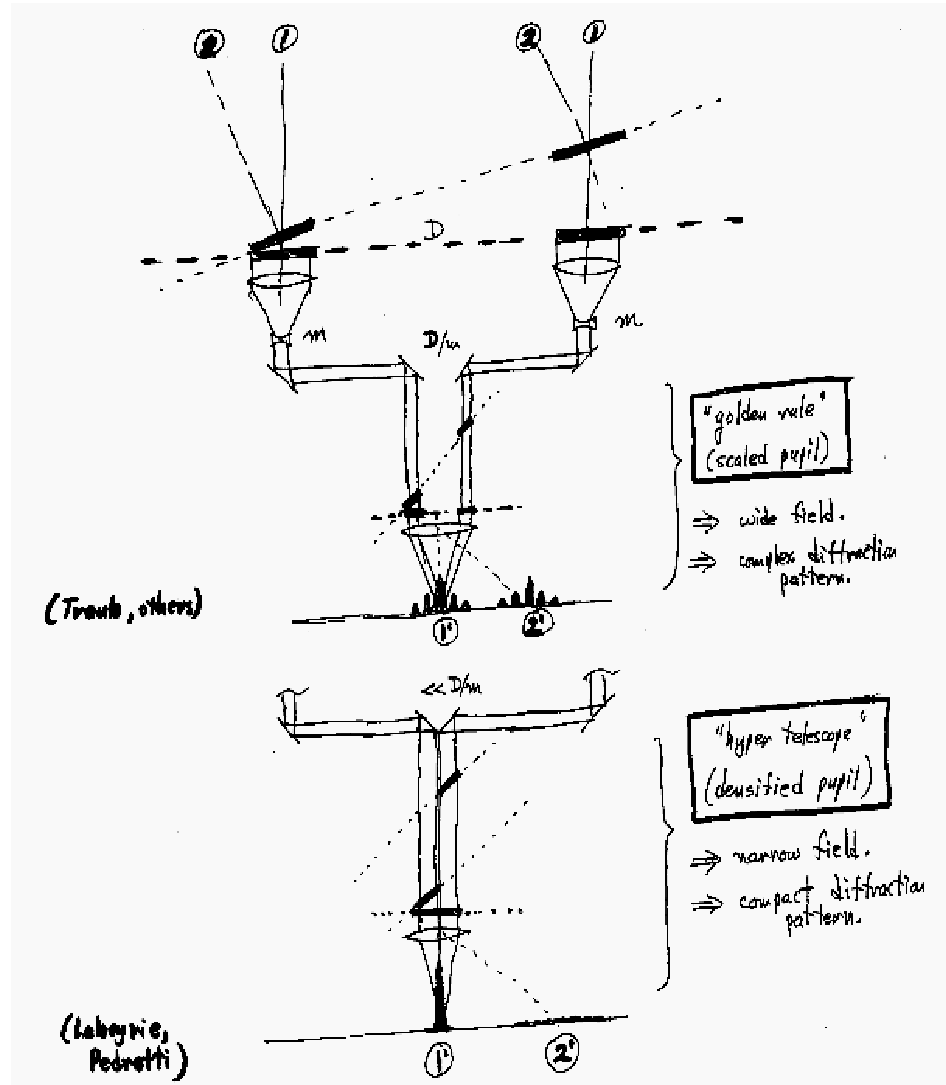


Figure 4.9: Geometry for 2 element Michelson interferometer.

Output pupil must be a scaled version of input pupil in order to obtain a wide field of view.



Pupil densification

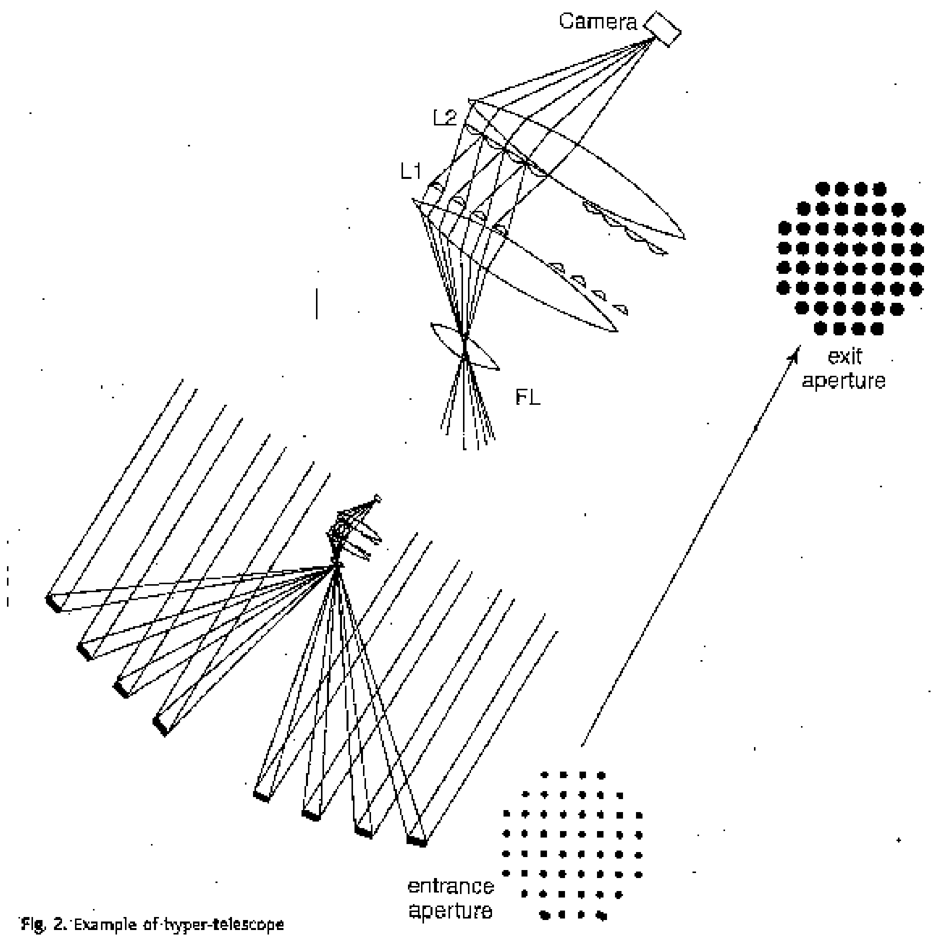


Fig. 2. Example of hyper-telescope optics. Multiple mirrors form a sparse paraboloidal giant mirror, and a lens (FL) in the focal plane forms a pupil image on a pair of lens arrays L1 and L2, having short and long focal lengths, respectively. This setup enlarges the subpupils in the exit aperture compared to those in the entrance aperture, producing a usable image on the camera.

A. Labeyrie, Science, 1999 or 2000

Instrumental effects: 1

Bandpass. If rectangular bandpass of width $\Delta\sigma$ (where $\sigma = \frac{1}{\lambda}$),

then $V_{bandpass} = \frac{\sin \pi \cdot x \cdot \Delta\sigma}{\pi \cdot x \cdot \Delta\sigma}$ where $x \equiv X_{delay} - X_{star}$.

Wavefront tilt. If wavefronts are tilted by angle α, then

$V_{tilt} = \frac{\sin \pi \cdot D \cdot \alpha / \lambda}{\pi \cdot D \cdot \alpha / \lambda}$ or $\frac{2 J_1 (\pi D \alpha / \lambda)}{\pi D \alpha / \lambda}$

square circular

For $V_{tilt} > 0.90$ need $\alpha < 0.3 \lambda / D$.

Relative intensity. If the relay optics and/or beam combiner have intensity ratio ρ, then

$V_{rel\text{-}int.} = \frac{2}{\rho^{1/2} + \rho^{-1/2}}$.

Instrumental effects: 2

Non-flatness of surfaces. If the wavefronts have rms perturbations δ, then

$$V_{surfaces} \approx e^{-(2\pi\delta/\lambda)^2}.$$

If there are N surfaces of δ_0 each, then

$$\delta \approx N^{1/2} \cdot \delta_0 .$$

If $\delta = \lambda/20$ is rms of each beam, then

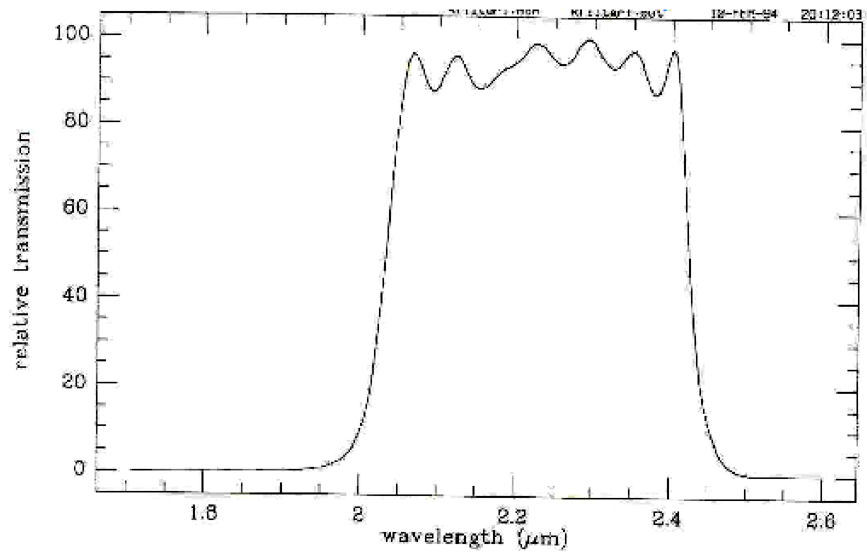
$$V(\lambda/20) \approx e^{-(\pi/10)^2} \approx 0.90 .$$

Shear. No effect, unless beams no longer overlap.

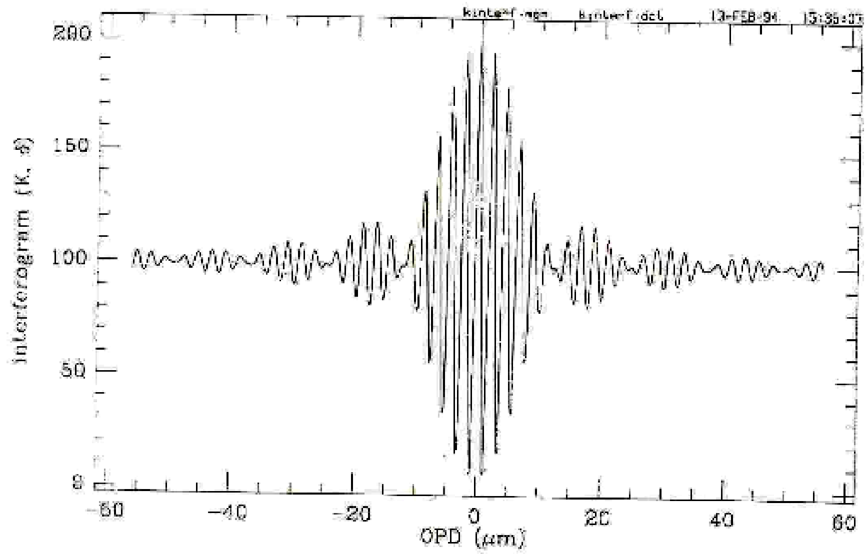
Different telescope diameters. See vol. int. calc.

Filter and interferogram shapes

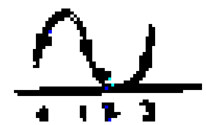
K-band filter transmission



K-band interferogram



Measuring visibility

λ/4 steps. Change x by $(\lambda/4)\cdot(0,1,2,3)$, measure $I(x)$,
calculate $V \sim \dfrac{0-2+1-3}{0+1+2+3}$  .

many λ sweep. Change $x = vt$ and repeat in triangle wave.
- fit theoretical wave packet to time data;
- calculate $|FFT|^2$ & ratio high freq. to low;
- wavelet analysis.

dispersed (channel) spectrum. Allow atmosphere to give few λ path variation of x
calculate peak-to-valley variations at each λ.

Strehl: 1

Strehl ratio is approximately

$$S = e^{-\varphi^2}$$

where φ is the rms phase error across a wavefront.

Observed visibility is the product of 3 terms:

$$V_{observed} = S_{atmos} S_{instrum} V_{object}$$

Instrumental Strehl ratio is the product of many terms:

$$S_{instrum} = S_{servo} S_{flat} S_{align} S_{diffraction} S_{flux} S_{overlap} S_{vibration} S_{window} S_{polarization}$$

Strehl: 2

Atmospheric variance, with tip-tilt removed
by a servo system with bandwidth $v/\pi D$, is

$$\varphi^2 = (0.134 + 0.096)(D/r_0)^{5/3}(\lambda_0/\lambda)^2$$

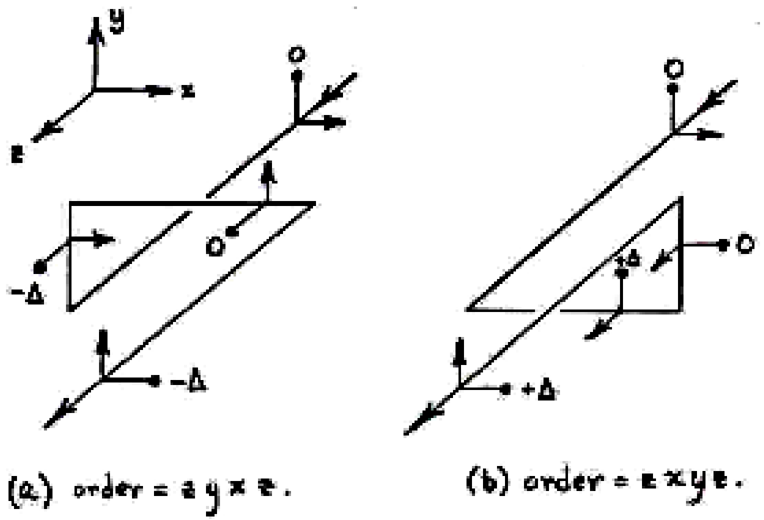
Wavefront flatness variance from mirror surfaces is

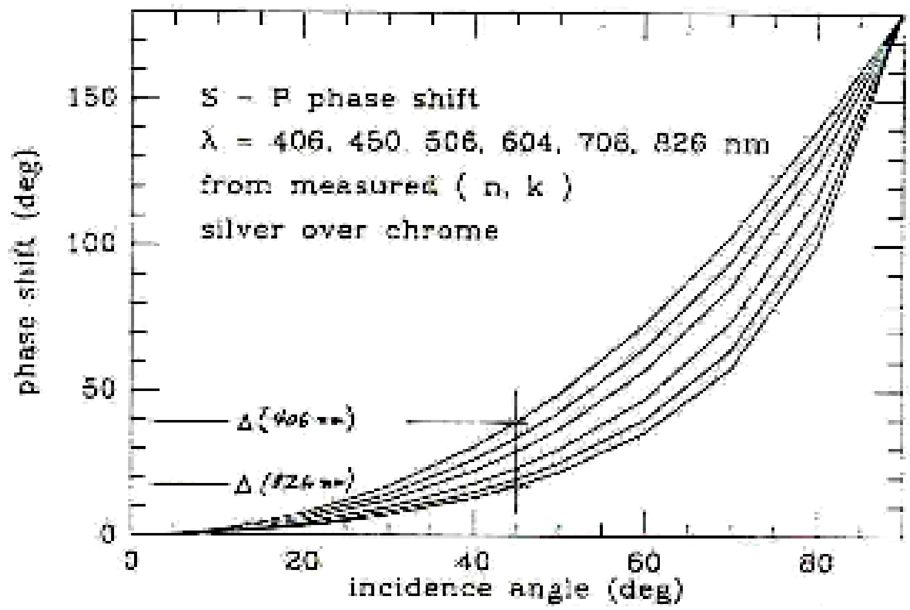
$$\varphi^2 = \varphi_1^2 + \ldots + \varphi_n^2$$

Mirrors are often specified in terms of surface peak-to-valley
where an empirical relation is

$$PV = 5.5\ RMS$$

Polarization and visibility





Visibility reduction factor

Beam 1: $\vec{A}_1 = (A_x, A_y)_1 = a_1 e^{ikz}(1, e^{i\phi_1})$

Beam 2: $\vec{A}_2 = (A_x, A_y)_2 = a_2 e^{ik(z+\ell)}(1, e^{i\phi_2})$

Combined: $I = |\vec{A}_1 + \vec{A}_2|^2$

$$I = \bar{I} \cdot \left[1 + \underbrace{\left(\frac{2a_1 a_2}{a_1^2 + a_2^2}\right)}_{\substack{\text{visibility} \\ \text{term}}} \cdot \underbrace{\cos\left(k\ell + \frac{\phi}{2}\right)}_{\substack{\text{modulation} \\ \text{term}}} \cdot \underbrace{\left|\cos\frac{\phi}{2}\right|}_{\substack{\text{polarization} \\ \text{term} \\ \equiv \text{constant}}} \right]$$

where $\phi \equiv \phi_2 - \phi_1$ = relative phase shift between 2 beams.

$\therefore$ In unpolarized light, the measured visibility can be permanently degraded by purely instrumental polarization effects, in the amount

$$\left|\cos\frac{\phi}{2}\right|$$

which is a function of wavelength only.

Single-mode fiber optics

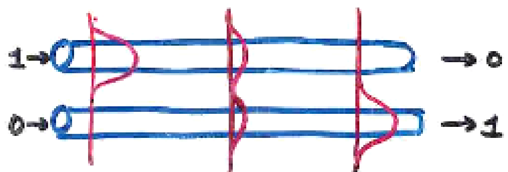
Core.
- Core index n_1, radius a $(n_1 \sim 1.48,\ a \sim 2\mu m)$.
- Cladding index n_2, radius b $(n_2 \sim 1.46,\ b \sim 60\mu m)$.
- Incident / exit cone $\sin i = (n_1^2 - n_2^2)^{1/2} \equiv NA$ $(i \sim 14°$ $f/2.0$

Dispersion.
- intermodal $= 0$ for SM fiber.
- material
- waveguide

$\}$ balance these to get zero.

Waveguide parameter. $V = \frac{2\pi}{\lambda_0} a \left(n_1^2 - n_2^2\right)^{1/2}$ $> 10 \Rightarrow$ geometric optics; $< 10 \Rightarrow$ wave optics.

Coupling.



$\}$ cyclic

z = coupling length $\approx 3\,mm$ typically.

Mfg. couplings.
- twist + melt (fused)
- polish + mate (polished).

Cutoff λ_c. For $\lambda > \lambda_c$ the fiber is single-mode.

$$\lambda_c = \frac{2\pi a}{2.4048}\left(n_1^2 - n_2^2\right)^{1/2}.$$

If $NA \sim .24$, $\lambda_c \sim 0.6a$.

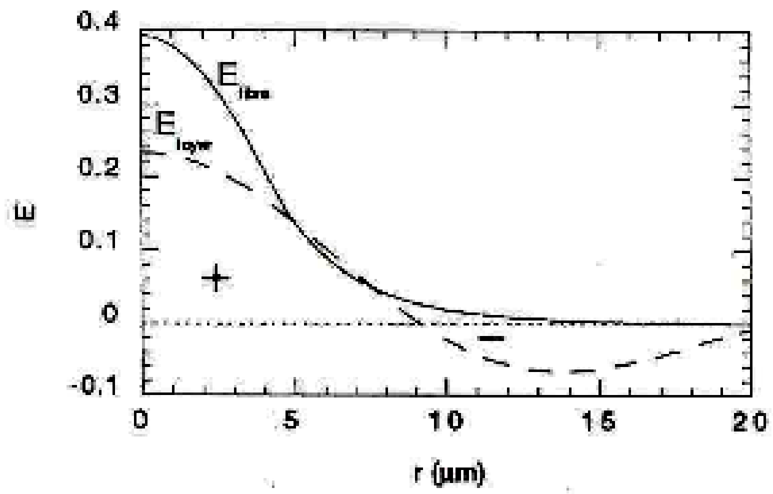
Etendue. $A\Omega = \pi a^2 \pi i^2 = \pi a^2 \pi (NA)^2 \approx (1.202\lambda_c)^2 \approx \lambda^2$.

Transmission. loss < 1 dB/km for silica + fluoride.

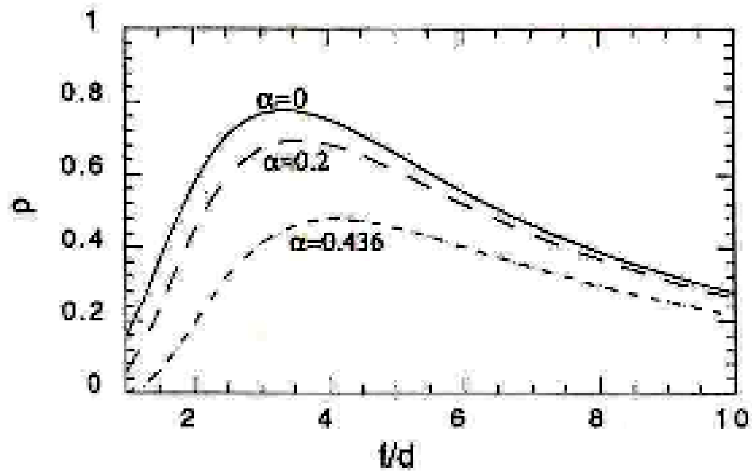
Integrated optics.

Fibers on a chip.

Injecting starlight into a fiber



- The optimal f/d is the one that maximizes the overlap integral
- Maximum possible efficiency: $\rho_{max} = 78\%$ (but less if the pupil has a central obstruction α)

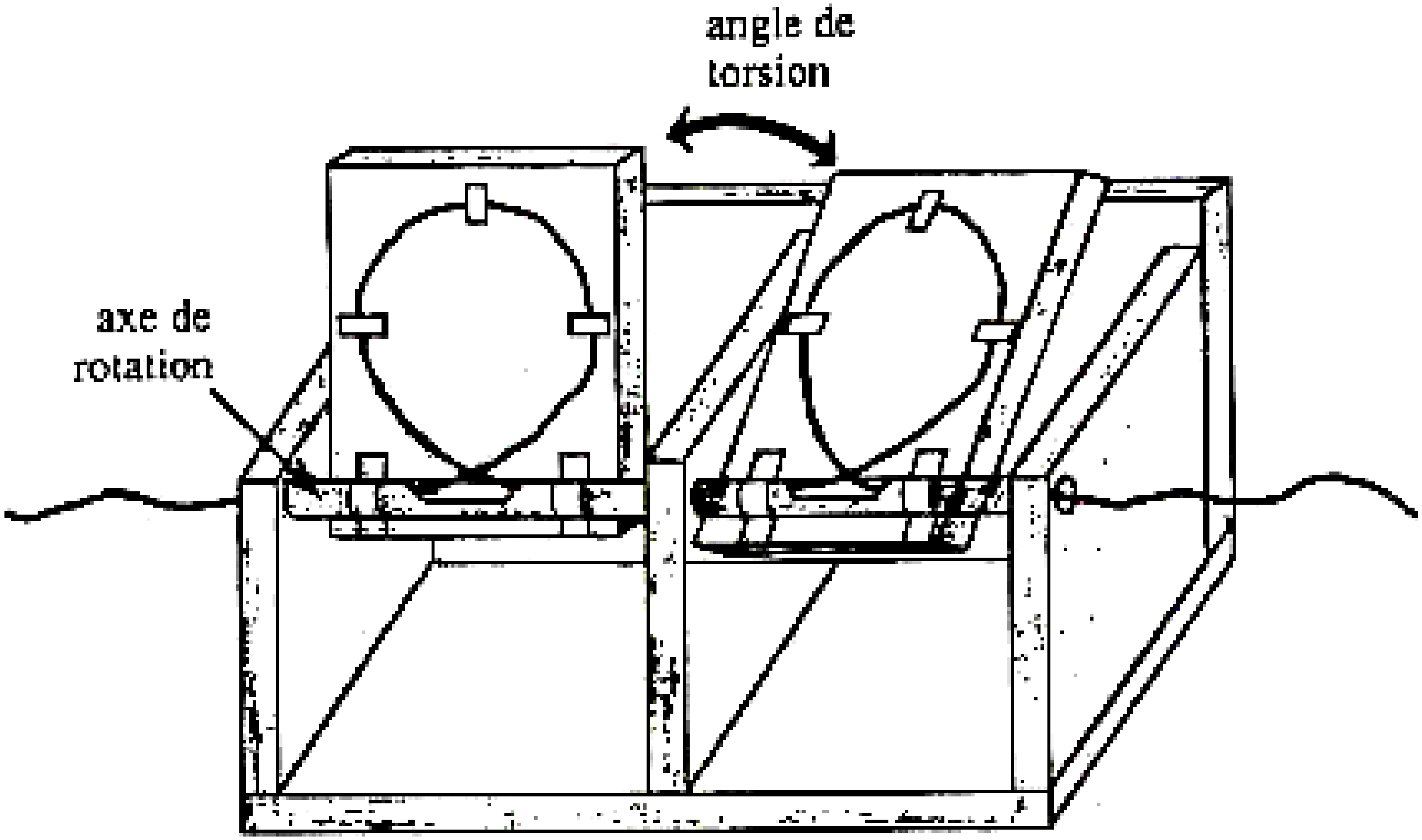


- Efficiency set by the overlap integral:

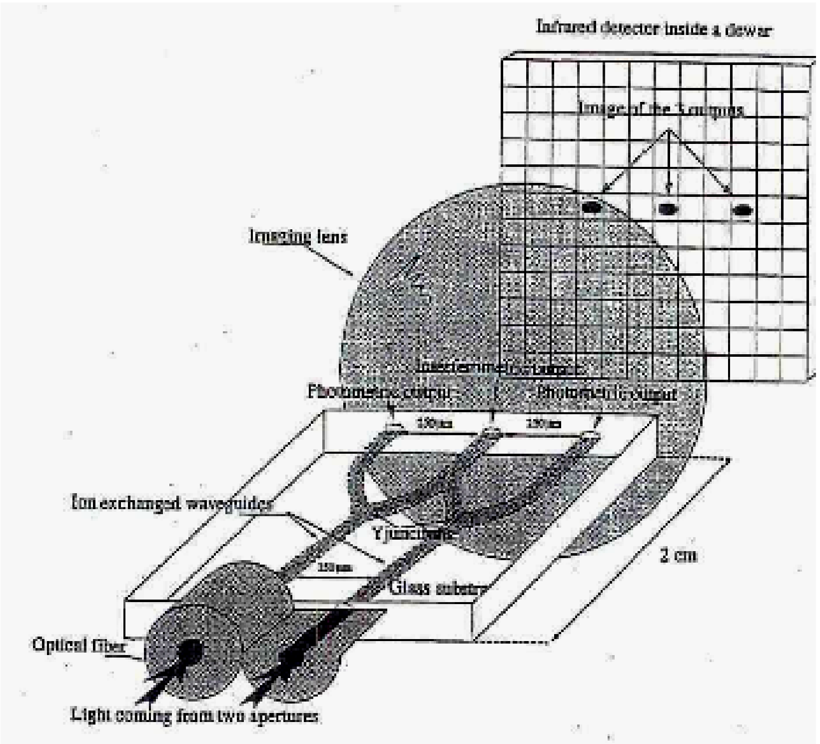
in the focal plane: $\rho = \left| \iint E_{tel}\, E^{*}_{fibre} \right|^2$

=> the field *amplitudes* must match

Electric vector control



Integrated optics: 1



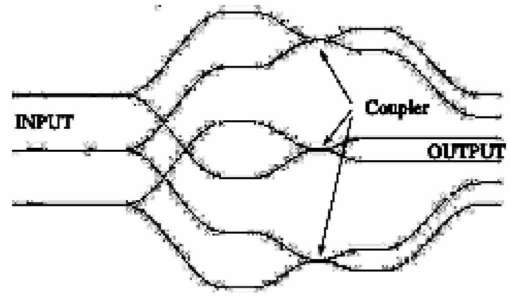


Figure 1- Description of the IONIC-IOTA three-way beam combiner. Three inputs are splitted with three "Y" junctions to provide a pairwise beam combination with another set of three couplers

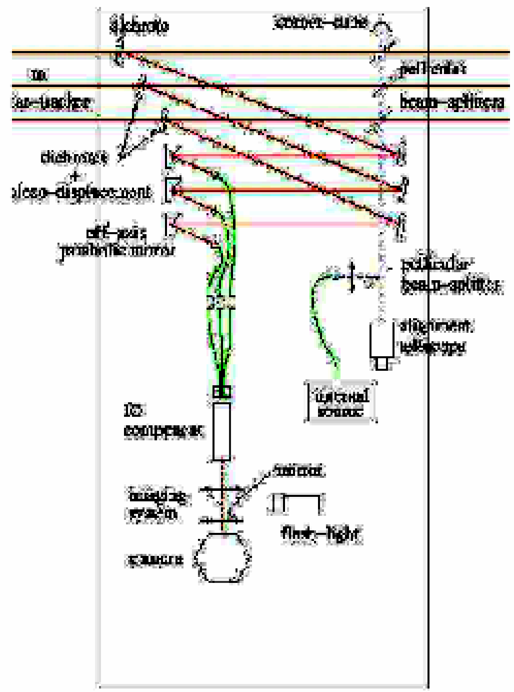


Figure 2- Schematic description of the IONIC bench (see text for details)

Integrated optics: 2