Transforming Measured to Standard Coordinates:

Models for wide-field astrographs

and simplifications for long-focus telescopes

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References

- A. König, 1962, "Astronomical Techniques", Edited by W. A. Hiltner, [University of Chicago Press] (Chapter 20)
- P. van de Kamp, 1962, "Astronomical Techniques", Edited by W. A. Hiltner, [University of Chicago Press] (Chapter 21)
- P. van de Kamp, 1967, "Principles of Astrometry", [Freeman & Company], (Chapters 5 and 6)
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- personal class notes, Astro 575a, 1987 (taught by W. van Altena)

Wide-Field vs. Long-Focus Telescopes

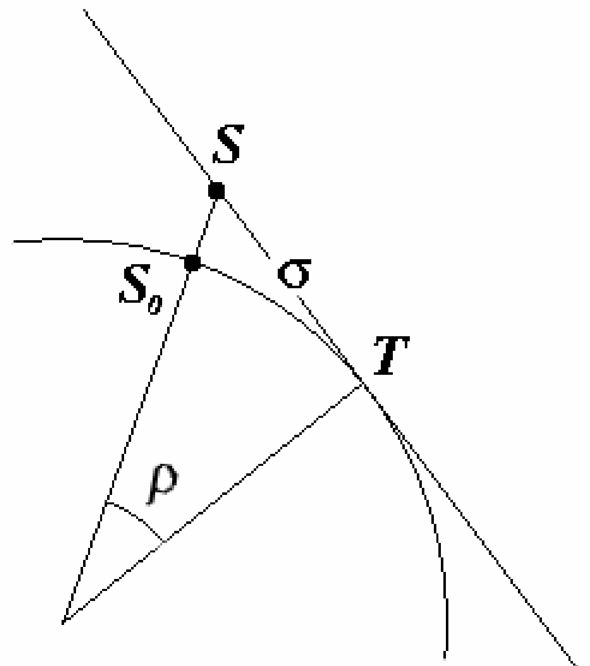
	Wide-field	Long-focus
field of view	2° → 10°	< 2°
focal length	2 → 4 m	> 10 m
f-ratio	f/4 → f/10	f/15 → f/20
scale	50 → 100 ″/mm	10 → 20 ″/mm
uses	positions, absolute proper motions	parallaxes, binary-star motion, relative proper motions

A Typical Astrometric Reduction

The goal is the determination of celestial coordinates (α,δ) for a star or stars of interest on a plate or other detector.

1. Extract reference stars from a suitable reference catalog.
2. Identify and measure target stars and reference stars on the plate.
3. Transform reference-star coordinates to standard coordinates.
4. Determine the plate model (*e.g.,* polynomial coefficients) that transforms the measured *x,y's* to standard coordinates. Use the reference stars, knowing their measures and catalog coordinates, to determine the model.
5. Apply the model to the target stars.
6. Transform the newly-determined standard coordinates into celestial coordinates.

Relation Between Equatorial and Standard Coordinates



Standard coordinates

(*aka* tangential coordinates, *aka* ideal coordinates)

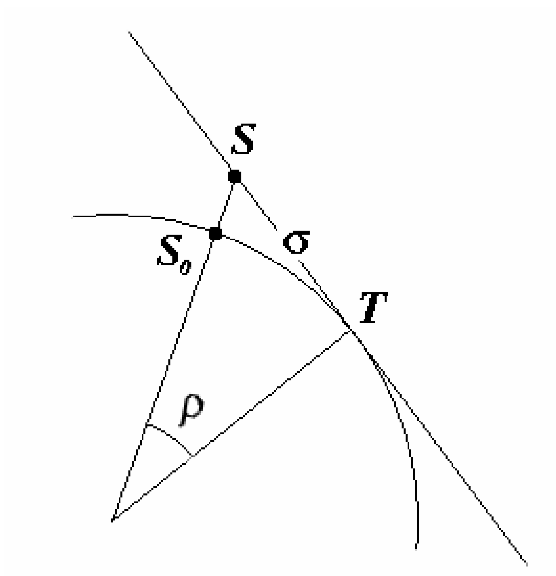
A. The coordinate system lies in a plane tangent to the celestial sphere, with the tangent point ***T*** at the origin, (0,0).

B. The "y" axis, η, is tangent to the declination circle that passes through ***T***, (+ toward NCP).

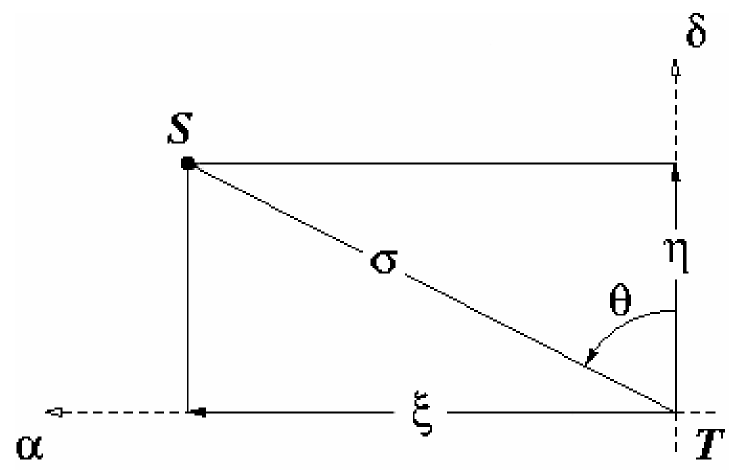
C. The "x" axis, ξ, is perpendicular to η, (+ toward increasing R.A.)

D. The unit of length is the radius of the celestial sphere or that of its image - the focal length. (In practice, arcseconds are commonly used.)

Equatorial & Standard Coordinates *(cont.)*



$$\sigma = \tan\rho$$

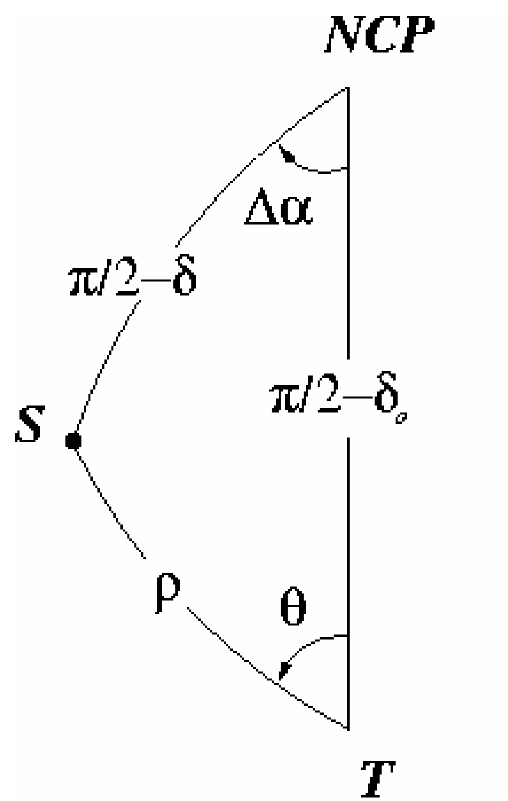


$$\xi = \tan\rho\sin\theta$$
$$\eta = \tan\rho\cos\theta$$

Equatorial & Standard Coordinates *(cont.)*

Tangent point is at α_o,δ_o
Star is at α,δ

$$\Delta\alpha = \alpha - \alpha_o$$
$$\Delta\delta = \delta - \delta_o$$



Spherical triangle formed by star ***S***, tangent point ***T***, and north celestial pole ***NCP***.

$$\cos\rho = \sin\delta\sin\delta_o + \cos\delta\cos\delta_o\cos\Delta\alpha$$
$$\sin\rho\sin\theta = \cos\delta\sin\Delta\alpha$$
$$\sin\rho\cos\theta = \sin\delta\cos\delta_o - \cos\delta\sin\delta_o\cos\Delta\alpha$$

Equatorial & Standard Coordinates *(cont.)*

Standard from Equatorial:

$$\xi = \frac{\cos\delta \sin\Delta\alpha}{\sin\delta \sin\delta_o + \cos\delta \cos\delta_o \cos\Delta\alpha}$$

$$\eta = \frac{\sin\delta \cos\delta_o - \cos\delta \sin\delta_o \cos\Delta\alpha}{\sin\delta \sin\delta_o + \cos\delta \cos\delta_o \cos\Delta\alpha}$$

Equatorial from Standard:

$$\tan\Delta\alpha = \frac{\xi}{\cos\delta_o - \eta \sin\delta_o}$$

$$\sin\delta = \frac{\sin\delta_o + \eta\cos\delta_o}{\sqrt{1+\xi^2+\eta^2}}$$

Corrections to Measured Coordinates

A. Correct for known "measuring machine" errors – repeatable deviations (offsets and rotation) from an ideal Cartesian system.

Direct and reverse measures can be used to calibrate such effects.

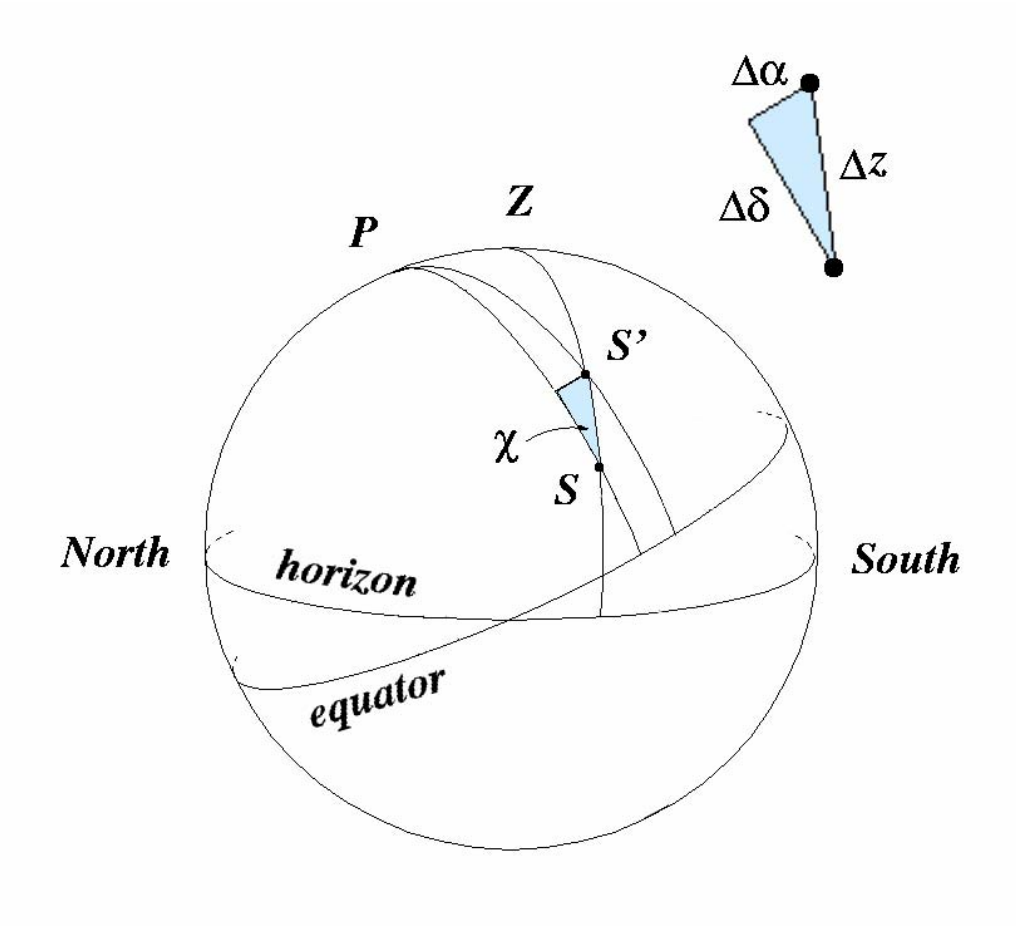
B. Correct for instrumental errors – plate scale, orientation, zero-point, plate tilt, higher-order plate constants, magnitude equation, color equation, etc.

(To be discussed.)

C. Correct for "spherical" errors – refraction, stellar aberration, precession, nutation.

(To be discussed.)

"Spherical" Errors: Atmospheric Refraction



$$\Delta z = \beta \tan \zeta,$$

$$\beta = \beta_o + \beta' \tan^2 \zeta$$

where ζ is the true zenith distance, i.e., the arclength *ZS*

and χ is the parallactic angle.

“Spherical” Errors: Atmospheric Refraction *(cont.)*

Refraction varies by observing site, and with atmospheric pressure and temperature.

$$\beta(P,T_F) \approx \frac{17P}{460+T_F}\beta_o, \qquad \beta_o = 58.2''$$

See R. C. Stone 1996, *PASP* 108, 1051 for an accurate method of determining refraction, based on a relatively simple model.

Importantly, refraction also varies with *wavelength!*

Differential Color Refraction (DCR) can introduce *color equation*, an unwelcome correlation between stellar color and measured position.

See R. C. Stone 2002, *PASP* 114, 1070 for a discussion of DCR and a detailed model for its determination.

(In practice, it is sometimes incorporated into the plate model.)

"Spherical" Errors: Stellar Aberration

If θ is the angle between the star and the apex of the Earth's motion,

$$\Delta\theta = \frac{v_\oplus}{c}\sin\theta,$$

where

$$\frac{v_\oplus}{c} = 20.5''$$

Thus, differentially across a field of size $\delta\theta$,

$$\Delta(\theta + \delta\theta) = 20.5''\left\{\delta\theta\cos\theta - \frac{(\delta\theta)^2}{2}\sin\theta + \ldots\right\}$$

Note: For $\delta\theta = 5°$, the maximum quadratic effect has amplitude ~100 mas.

In practice, this would be absorbed by general quadratic terms in the plate model, which would almost certainly be present for such a large field.

“Spherical” Errors: Precession & Nutation

As both precession and nutation represent simple rotations, these are almost never applied explicitly.

They are effectively absorbed by the rotation terms in the plate model, which are always present.

Note: The equinox of the reference system is therefore, in a practical sense, arbitrary. It is typically chosen to be that of the reference catalog for convenience. Of course, the tangent point must be specified in whatever equinox is chosen.

The Plate Model

Often, a polynomial model is used to represent the transformation from measured coordinates, *(x,y),* to standard coordinates, (ξ,η).

$$\xi = a_1 x + a_2 y + a_3 + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3$$
$$+ a_{11} m + a_{12} CI + \ldots$$
$$\eta = b_1 y + b_2 x + b_3 + b_4 y^2 + b_5 yx + b_6 x^2 + b_7 y^3 + b_8 y^2 x + b_9 yx^2 + b_{10} x^3$$
$$+ b_{11} m + b_{12} CI + \ldots$$

Note: The various terms can be identified with common corrections...

For each reference star, i, calculate deviation, $(\Delta\xi,\Delta\eta)$. Minimize Δ^2.

$$\Delta\xi_i = \xi_{ref\ i} - \xi_i(a_k, x_i, y_i, m_i, CI_i)$$
$$\Delta\eta_i = \eta_{ref\ i} - \eta_i(b_k, x_i, y_i, m_i, CI_i)$$

The Plate Model *(cont.)*

Correction	ξ **- *solution***	η **- *solution***
scale	x	y
orientation	y	x
zero point	*constant*	*constant*
plate tilt	$x*(px+qy)$	$y*(px+qy)$
cubic distortion	$x*(x^2+y^2)$	$y*(x^2+y^2)$
magnitude equation	$m, (m^2, m^3 ...)$	$m, (m^2, m^3 ...)$
coma	$x*m$	$y*m$
color equation	*CI*	*CI*
color magnification	$x*CI$	$y*CI$
higher order terms...	...	...

The Plate Model *(cont.)*

Some Helpful Hints

The simplest possible form should be used. The modeling error is thus kept minimal. Reference stars are usually at a premium!

(Rule of thumb: $N_{ref} > 3*N_{terms}$.)

Pre-correct measures for known (spherical) errors.

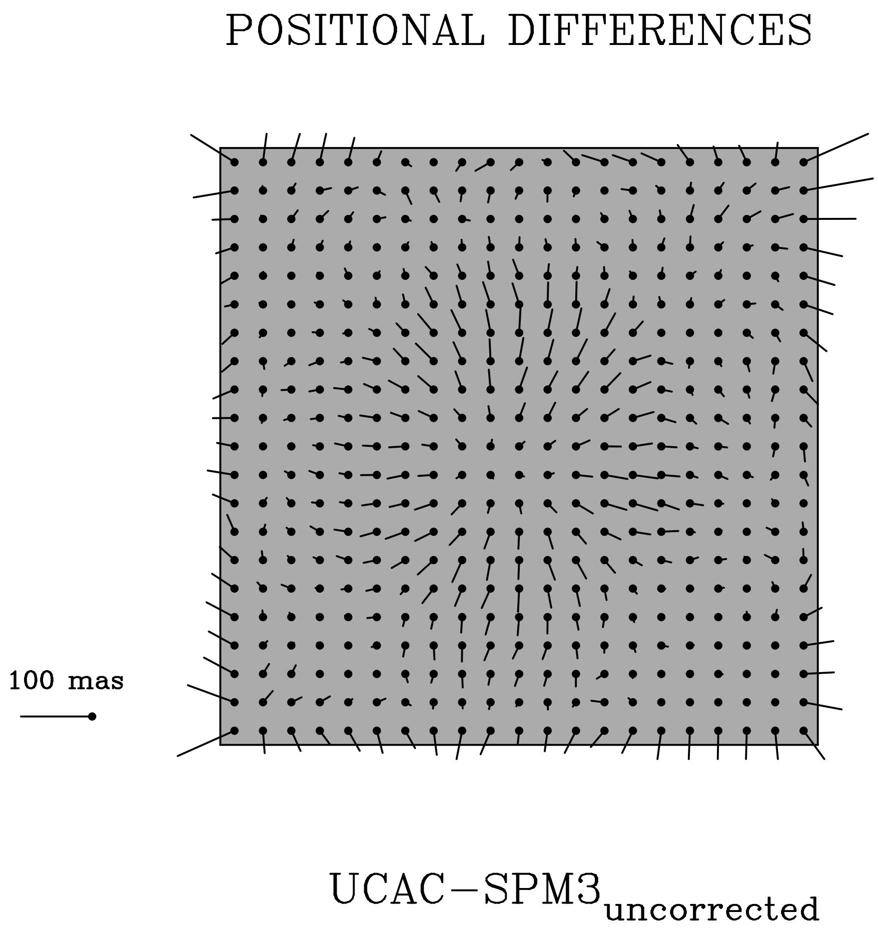
Update reference star catalog positions to epoch of plate material, *i.e.,* apply proper motions when available.

Uniform distribution of reference stars is best. Avoid extrapolation.

Iterate to exclude outliers, but trim with care.

Plot residuals versus everything you can think of!

When The Plate Model Is Just Not Enough



“Stacked” differences *wrt* an external catalog can uncover residual systematics.

A comparison between preliminary SPM3 positions (derived using a plate model with cubic field terms) and the UCAC.

The resulting “mask” was used to adjust the SPM3 data, field by field.

(See Girard et al. 2004, AJ 127, 3060)

Magnitude Equation – The Astrometrist's Bane

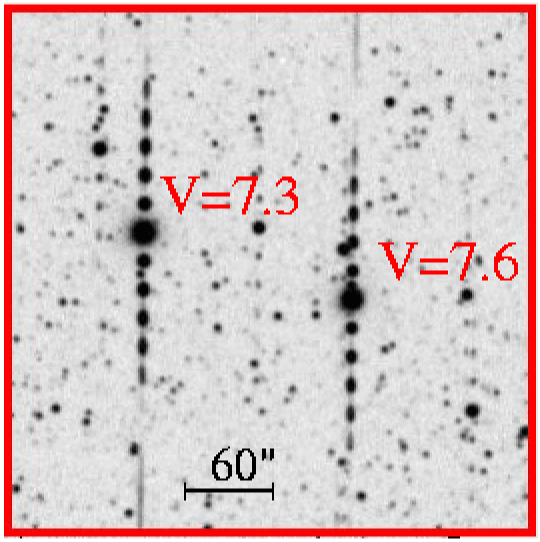
Magnitude equation = bias in the measured "center" of an image that is correlated with its apparent brightness.

It can be particularly acute on photographic plates, caused by the non-linearity of the detector combined with an asymmetric profile, (due to guiding errors, optical aberrations, etc.)

- Difficult to calibrate and correct internally
- Reference stars usually have insufficient magnitude range
- Beware of "cosmic" correlations in proper motions
- In clusters, magnitude and color are highly correlated

NOTE: Charge Transfer Efficiency (CTE) effects can induce a similar bias in CCD centers. (More often, the CTE effect mimics the classical coma term, *i.e., x*m*).

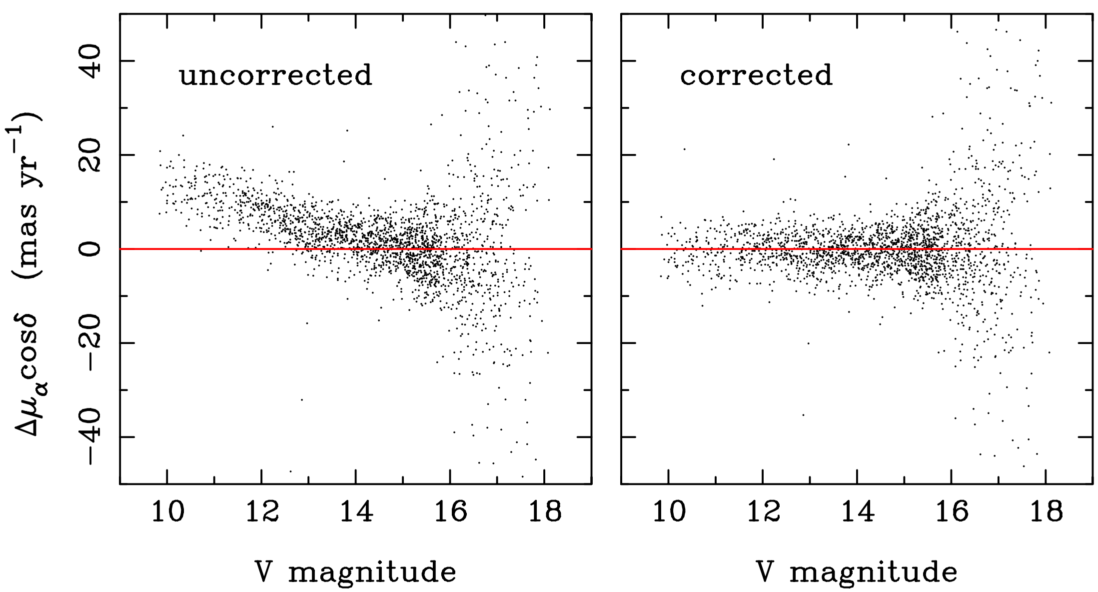
Magnitude Equation – The Astrometrist's Bane *(cont.)*



SPM (and NPM) plates use objective gratings, producing diffraction image pairs which can be compared to the central-order image to deduce the form of the magnitude equation.

A comparison of proper motions derived from uncorrected SPM blue-plate pairs and yellow-plate pairs indicate a significant magnitude equation is present.

Using the grating images to correct each plate's individual magnitude equation, the proper motions are largely free of bias.



Long-Focus Telescope Astrometry

Traditional Simplifications - due to scale & small field of view

Plate tilt → often negligible, but should be checked

Distortion → can be significant for reflectors; usually can be ignored for refractors - constant over the FOV

Refraction → usually ignored unless at large zenith angle, or for plate sets with a large variation in HA

Aberration → small, ignored

Magnitude equation → usually present! Can be minimized by using a limited magnitude range.

Color equation → DCR will be present. Careful not to confuse with magnitude equation for cluster fields.

Color magnification → generally not a problem over the FOV

Coma → images are often affected, but variation across FOV is slight and can be neglected in general (but check)

Long-Focus Telescope Astrometry *(cont.)*

A Parallax and Binary-Motion Example: Mass of Procyon A & B
(Girard et al. 1999, AJ 119, 2428)

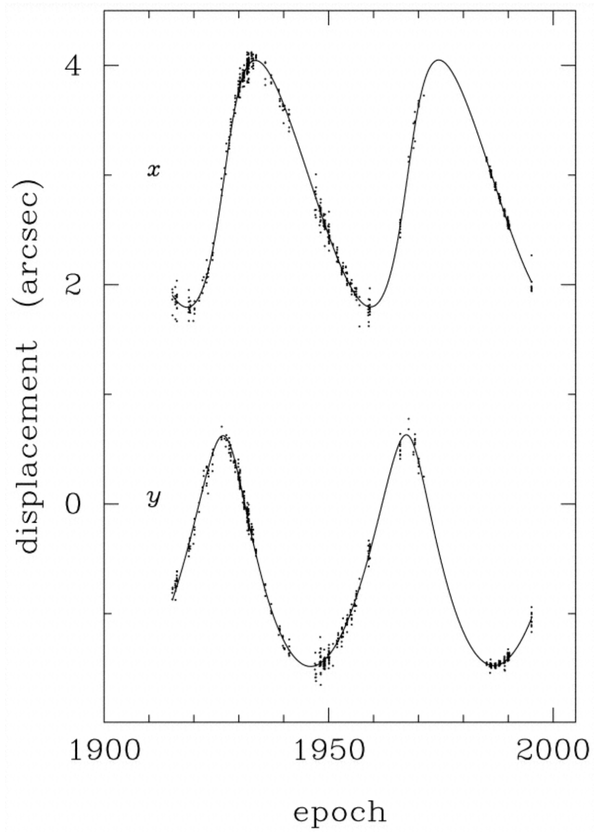
Overview:

The plate material consisted of 250 (primarily) long-focus plates, containing >600 exposures and spanning 83 years.

Magnitude-reduction methods were used during the exposures to bring Procyon's magnitude close to that of the reference stars.

Linear transformations between plates, putting all onto the same standard coordinate system.

Astrometric orbit and parallax were found.



Long-Focus Telescope Astrometry *(cont.)*

A Relative Proper-Motion Example: Open Cluster NGC 3680
(Kozhurina-Platais et al. 1995, AJ 109, 672)

Overview:

The plate material consisted of 12 Yale-Columbia 26-in. refractor plates, spanning 37 years.

Explicit refraction correction was needed as plates were taken at two observatories, Johannesburg and Mt. Stromlo.

Plates exhibited magnitude and color equation which affected the derived relative proper motions. The cluster's red giants were displaced in the proper-motion VPD.

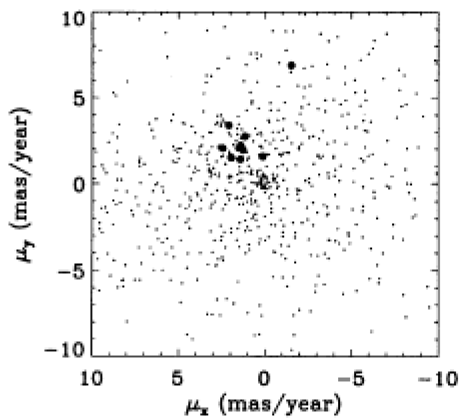


FIG. 3. The "first-attempt" observed proper motion vector-point diagram, which clearly indicates an uncorrected magnitude/color equation. The very probable red-giant cluster members in proper motion space are displaced from the rest of the cluster located approximately at (0,0) in the VPD.

Long-Focus Astrometry *(NGC 3680 Example cont.)*

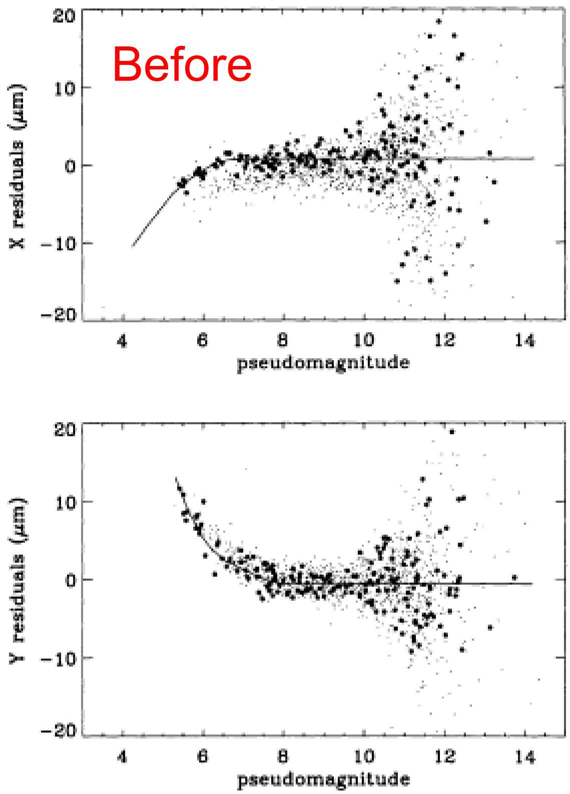


FIG. 2. Plate-pair #Y43−#Y36 positional differences illustrating the presence of magnitude and color equations. The bold dots are for red stars with $0.9<(B-V)<1.3$.

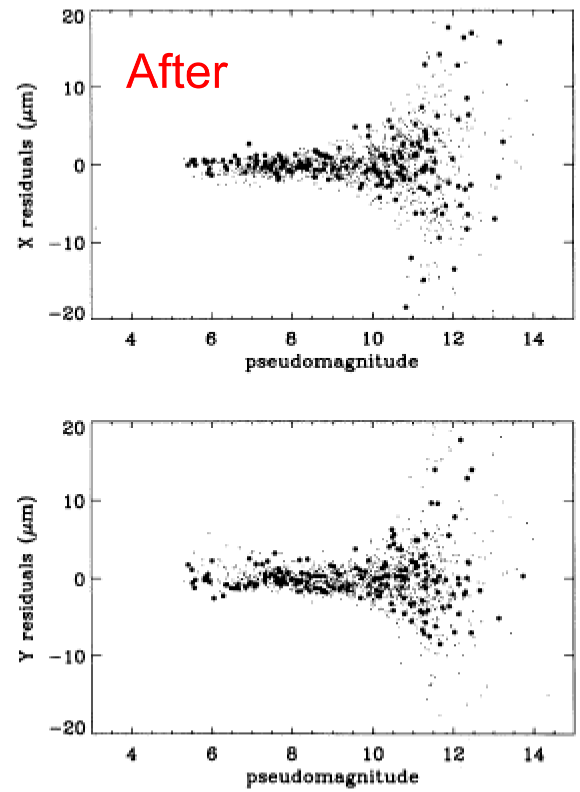


FIG. 4. Plate-pair #Y43−#Y36 residuals after correction for magnitude and color equations as described in Sec. *3.4*. Symbols are the same as in Fig. 2.

In lieu of a summary slide…

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