Orbital Estimation of Binary Stars

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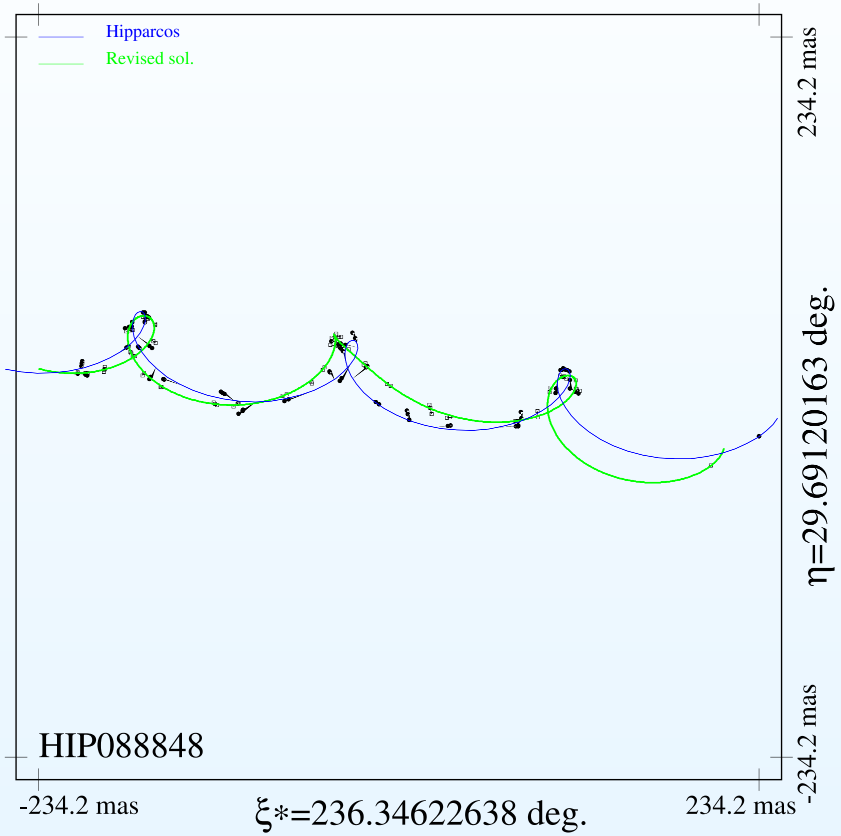
1 cent piece of advice

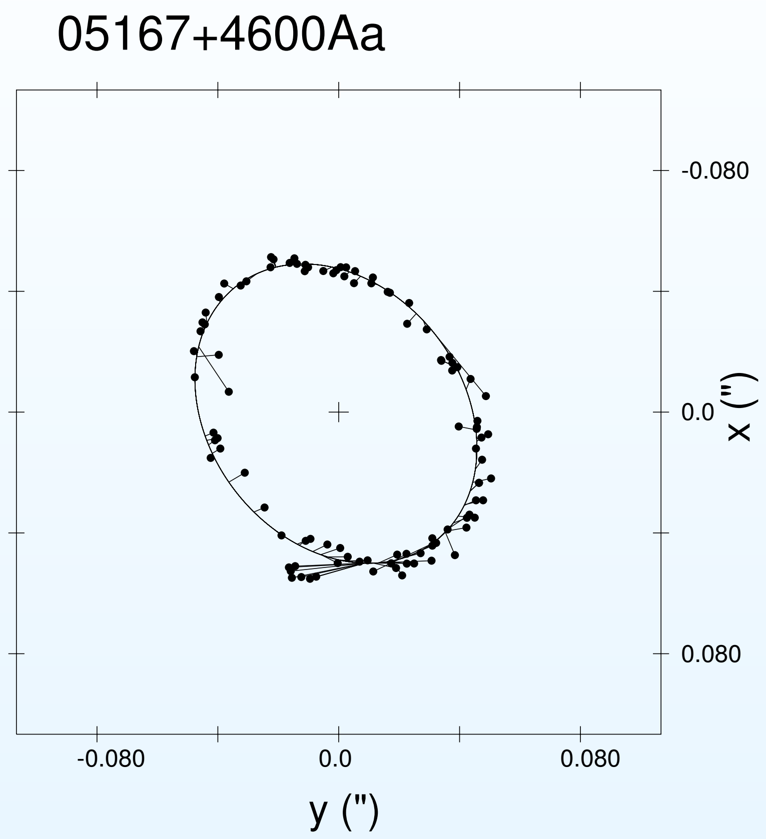
Become an expert in something nobody is willing to learn but a lot of people need (e.g. data processing and handling, especially of binaries and other strange objects, for a space astrometry mission).

Overview

1. Visual, astrometric binaries and extrasolar astrometric orbit fitting
   - What does one see and how to model it?
   - Two-body problem
   - Why does one care? Stellar and planetary systems quite similar
   - How to guess the orbit?
   - How to efficiently fit it?
   - 3+ stellar and planetary systems not so similar after all
   - Preliminary Conclusions
2. Spectroscopic watch dog
3. Blind fit

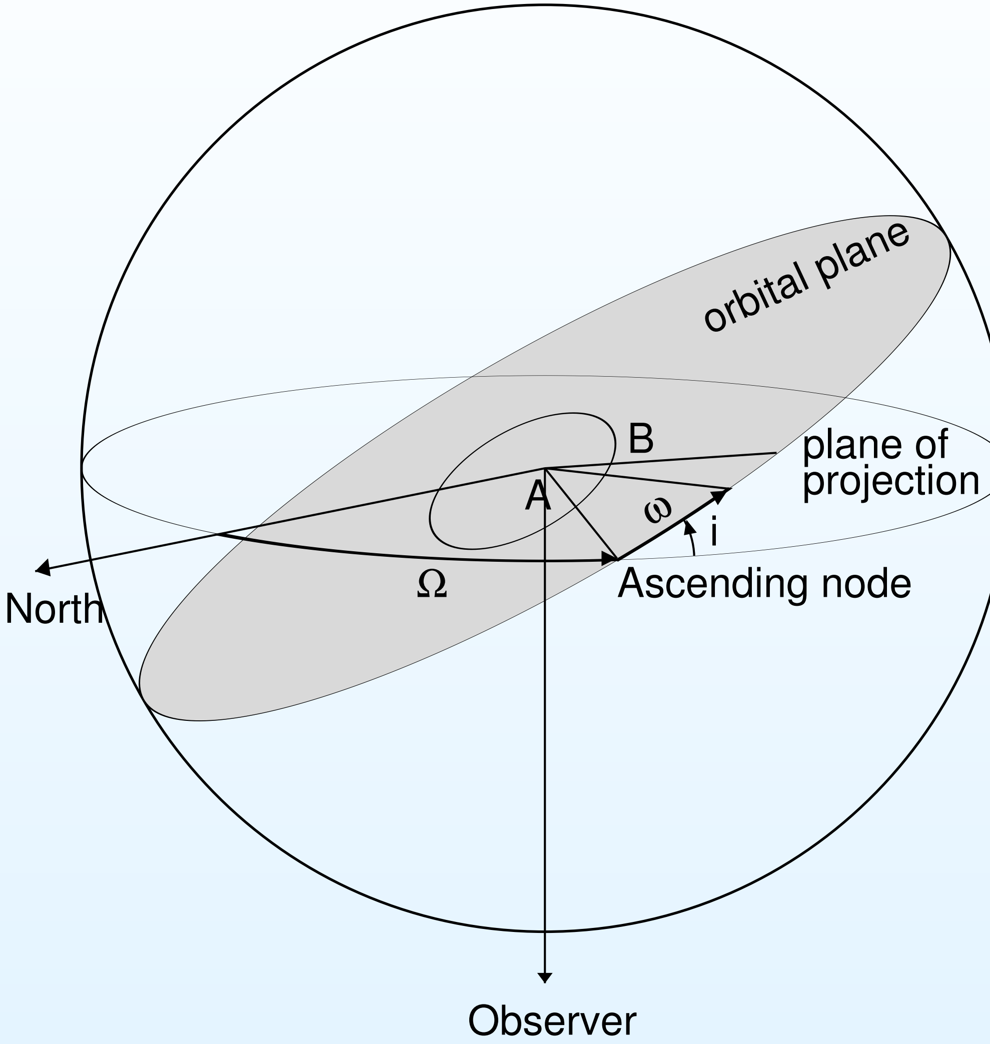
Absolute path versus relative orbit





The relative visual orbit requires 7 parameters only.

Astrometric or visual binaries and extrasolar planets



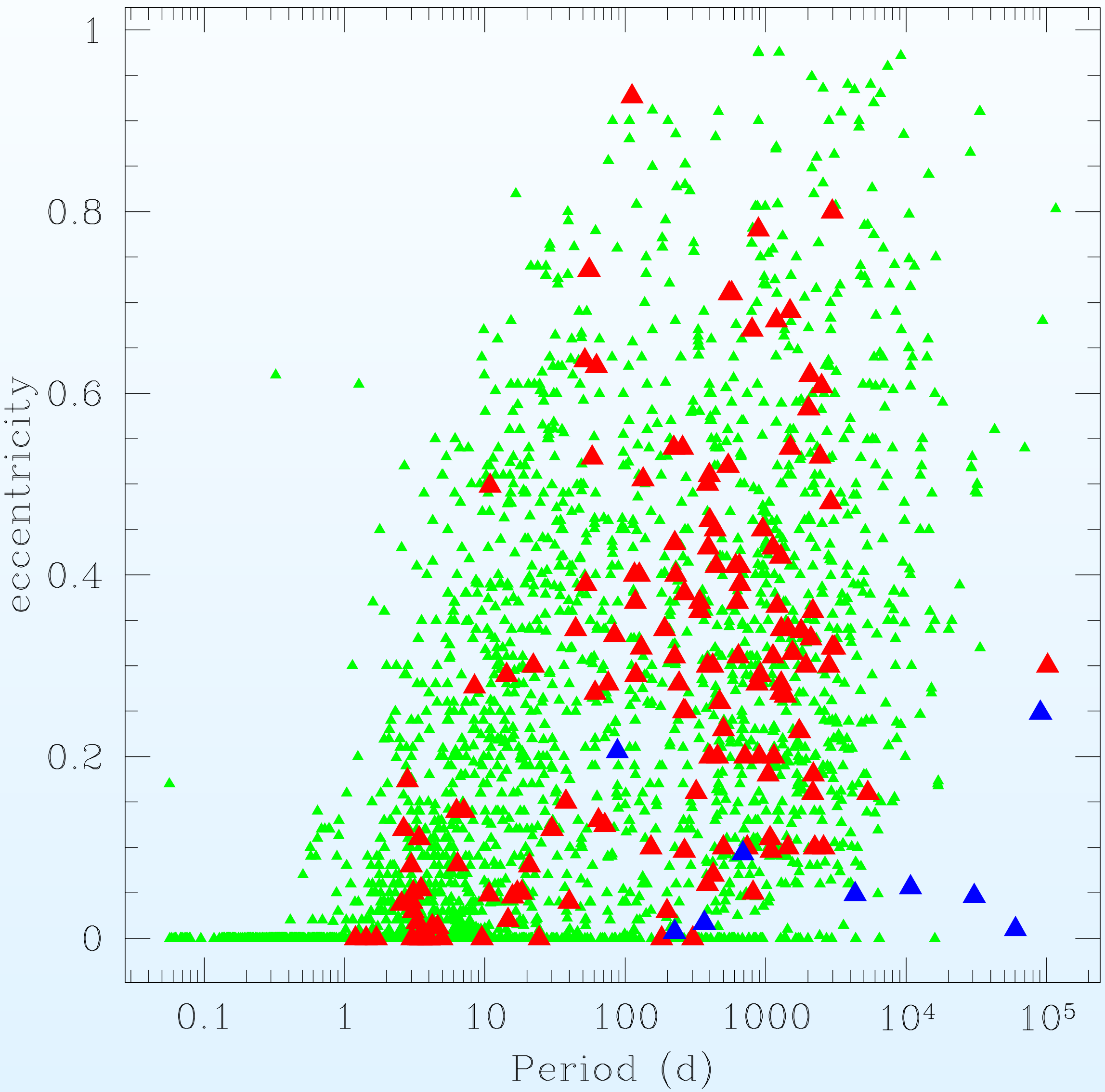
- a: semi-major axis of the orbit
- e: eccentricity
- P: orbital period
- T: epoch of the periastron passage
- i: inclination
- ω: argument of the periastron
- Ω: longitude of the ascending node

Two-body absolute motion

$$
\begin{aligned}
\xi &= \alpha_0^* + P_\alpha \varpi + (t - t_0)\mu_{\alpha*} + B(\cos E - e) + G\sqrt{1-e^2}\sin E, \\
\eta &= \delta_0 + P_\delta \varpi + (t - t_0)\mu_\delta + A(\cos E - e) + F\sqrt{1-e^2}\sin E, \\
E &= \frac{2P}{\pi}(t - T_0) + e\sin E, \\
\alpha* &\equiv \alpha\cos\delta, \\
A &\equiv a(\cos\omega_1\cos\Omega - \sin\omega_1\sin\Omega\cos i), \\
B &\equiv a(\cos\omega_1\sin\Omega + \sin\omega_1\cos\Omega\cos i), \\
F &\equiv a(-\sin\omega_1\cos\Omega - \cos\omega_1\sin\Omega\cos i), \\
G &\equiv a(-\sin\omega_1\sin\Omega + \cos\omega_1\cos\Omega\cos i).
\end{aligned}
$$

12-parameter nonlinear model

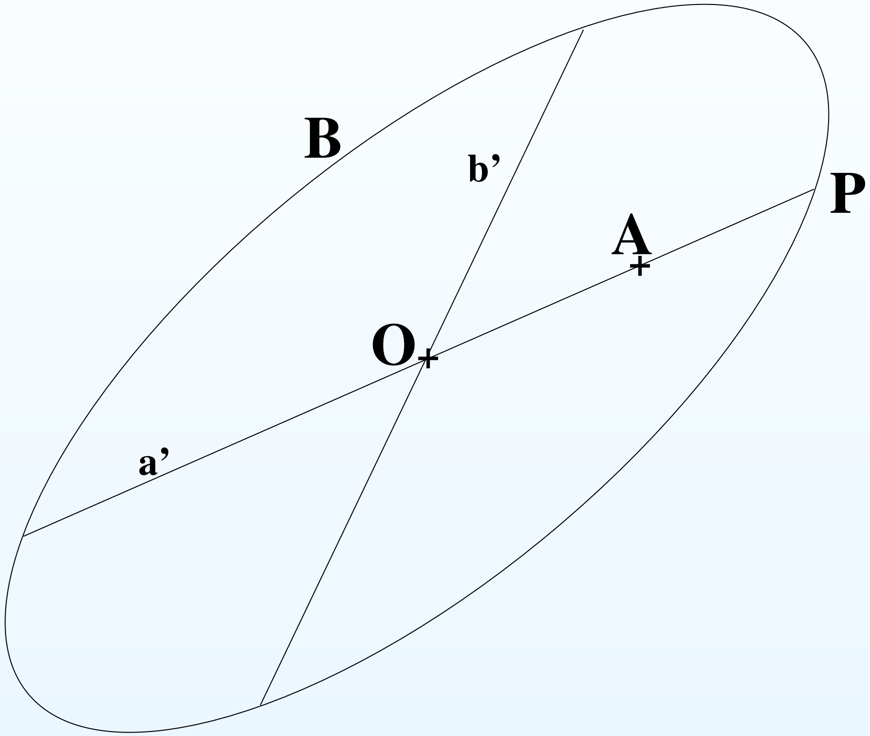
Spectroscopy-based e-log P



No clear difference between stellar and planetary systems.

Codes written for ones can be used for the others.

Old goodies: geometric method – Zwiers' method



Orbital solution derived from the apparent ellipse.

Instead of drawing it, fit

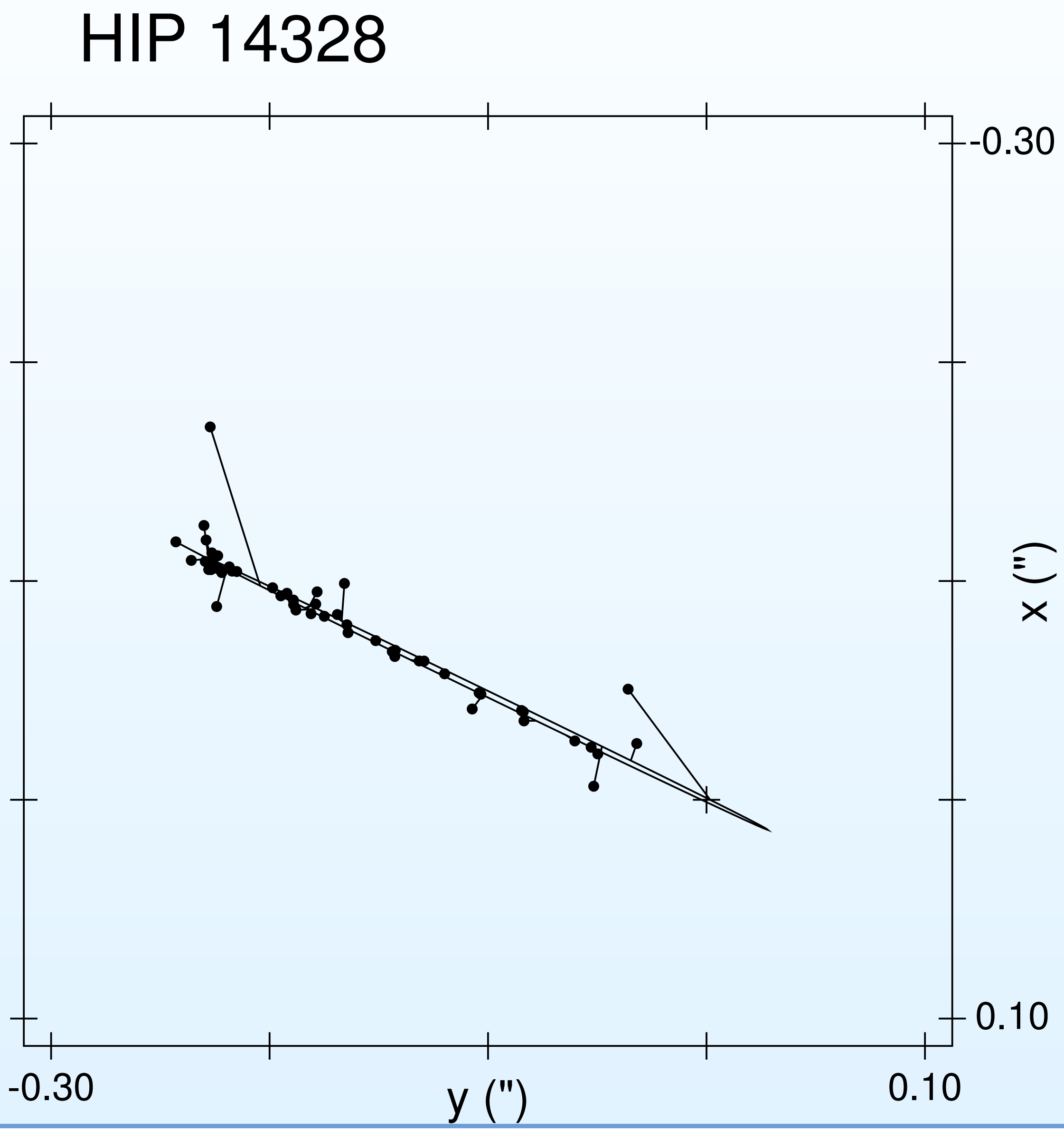
$$Ax^2+2Hxy+By^2+2Gx+2Fy+J=0.$$

Avoid assuming $J=1$ (numerical stability). The constants are **not** the Thiele-Innes ones.

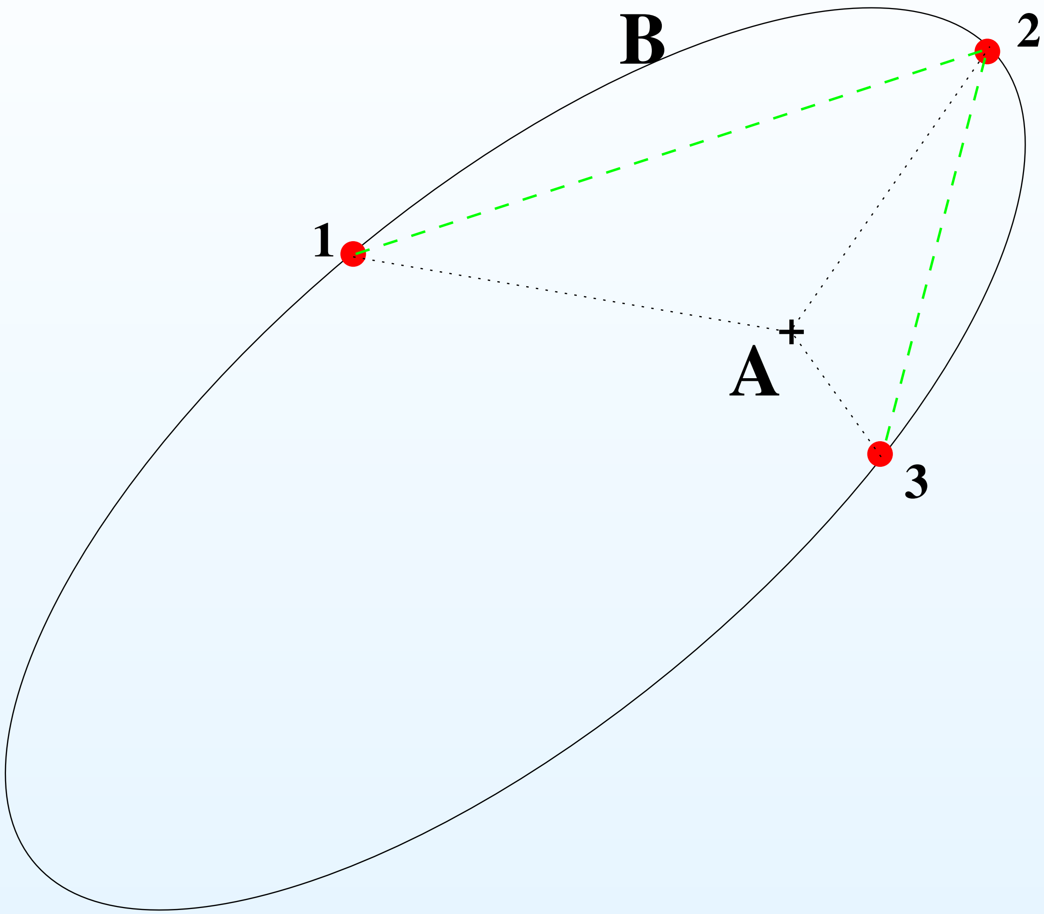
Weaknesses:

- The apparent orbit needs to be drawn.
- Cannot be applied when $i=90^\circ$ (degeneracy).

Independent term



Old goodies: Thiele-Innes-Van Den Bos' method



Exact solution of the equations for 3 *normal* points and the projected constant of areas ($\rho^2 \frac{d\theta}{dt}$). A better be in triangle(1,2,3).

Alternative: replacing $\rho^2 \frac{d\theta}{dt}$ with a 4th point.

High sensitivity of the overall solution on those favored 3 or 4 points.

What if 1D-observations?

Fourier

- Keplerian orbit = periodic phenomenon.
- Fourier expansion of the positions (Monet 1979, ApJ, 234, 275).

$$
\begin{aligned}
O(t_i) &= \sum_{n=0}^{\infty} a_n \cos(n\mathcal{M}_i) + \sum_{n=1}^{\infty} b_n \sin(n\mathcal{M}_i) \\
\mathcal{M}_i &= \frac{2\pi}{P}(t_i - t_0) \\
O(t_i) &= \sum_{n=0}^{\infty} \alpha_n \cos(nM_i) + \sum_{n=1}^{\infty} \beta_n \sin(nM_i) \\
\Delta &= \frac{2\pi}{P}(T - t_0) \\
M_i &= \frac{2\pi}{P}(t_i - T) \\
&= \mathcal{M}_i - \Delta
\end{aligned}
$$

Fourier (cnt.)

- Reverse problem: deriving the orbital parameters from the Fourier coefficients (P assumed).

$$\sum_{n=0}^{\infty} \alpha_n \cos(nM_i) + \sum_{n=1}^{\infty} \beta_n \sin(nM_i) = f(o.e.) \sum_{n=0}^{\infty} F_n(e) \cos(nM_i) + g(o.e.) \sum_{n=1}^{\infty} G_n(e) \sin(nM_i)$$

- Tradeoff between solving the minimum number equations exactly or least squares on many equations. Usually the former.
- Can be applied to incomplete observations (e.g. ρ only).
- The lower the eccentricity the better.

Global minimization

Fitting the nonlinear model directly (rather a linear expansion) using

- Simulated Annealing,
  - Boltzmann probability, temperature.
  - Point generator.
- Genetic Algorithm
  - Encoding, elitism, crossover, mutations
  - Parallel computing
- Tabu search,
  - Diversification, intensification, tabu list, . . .

No guaranteed convergence to the global minimizer within a finite time.

Could be applied to a subset of parameters only.

A two-step approach

In space astrometry, single star solution is always derived anyway (GIS):

$$\Xi^2 = (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o)^{\mathsf{t}} V^{-1} (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o)$$

where

$$\frac{\partial v}{\partial o} = \frac{\partial v}{\partial \alpha_0^*}\frac{\partial \xi}{\partial o} + \frac{\partial v}{\partial \delta_0}\frac{\partial \eta}{\partial o}$$

Problem: $O(e^{12})$ local minima. A good initial guess of the solution is required.

$\forall\, e,\ P\ \&\ T, \exists!$ minimizer of $\Xi^2_{12}(\mathbf{p}, A, B, F, G, e, P, T)$

Instead $\Xi^2_3(e, P, T)$ min! where

$$\Xi^2_3(e, P, T) = \min \Xi^2_{12}(\mathbf{p}, A, B, F, G, e, P, T)$$

Speed up!

For any matrix symmetric positive definite matrix M, there is an upper triangular matrix R such that $M = R^t R$ (Cholesky decomposition).

So, instead of evaluating Ξ^2 using V^{-1}, find R such that $V^{-1} = R^t R$ and then

$$\begin{aligned}
\Xi^2 &= (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o)^{\mathsf{t}} V^{-1} (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o) \\
&= (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o)^{\mathsf{t}} R^t R (\Delta v - \sum_p \frac{\partial v}{\partial p}\Delta p - \frac{\partial v}{\partial o} o) \\
&= (R\Delta v - \sum_p R\frac{\partial v}{\partial p}\Delta p - R\frac{\partial v}{\partial o} o)^{\mathsf{t}} (R\Delta v - \sum_p R\frac{\partial v}{\partial p}\Delta p - R\frac{\partial v}{\partial o} o)
\end{aligned}$$

Each iteration becomes $O(N)$ instead of $O(N^2)$

3+ component model

Basic gravitational interaction, no tidal effect, no mass transfer, ...

$$\xi = \xi_S + \sum_k \left(B_k(\cos E_k - e_k) + G_k \sqrt{1 - e_k^2} \sin E_k \right),$$

$$\eta = \eta_S + \sum_k \left(A_k(\cos E_k - e_k) + F_k \sqrt{1 - e_k^2} \sin E_k \right),$$

$$E_k = \frac{2P_k}{\pi}(t - T_{0,k}) + e_k \sin E_k,$$

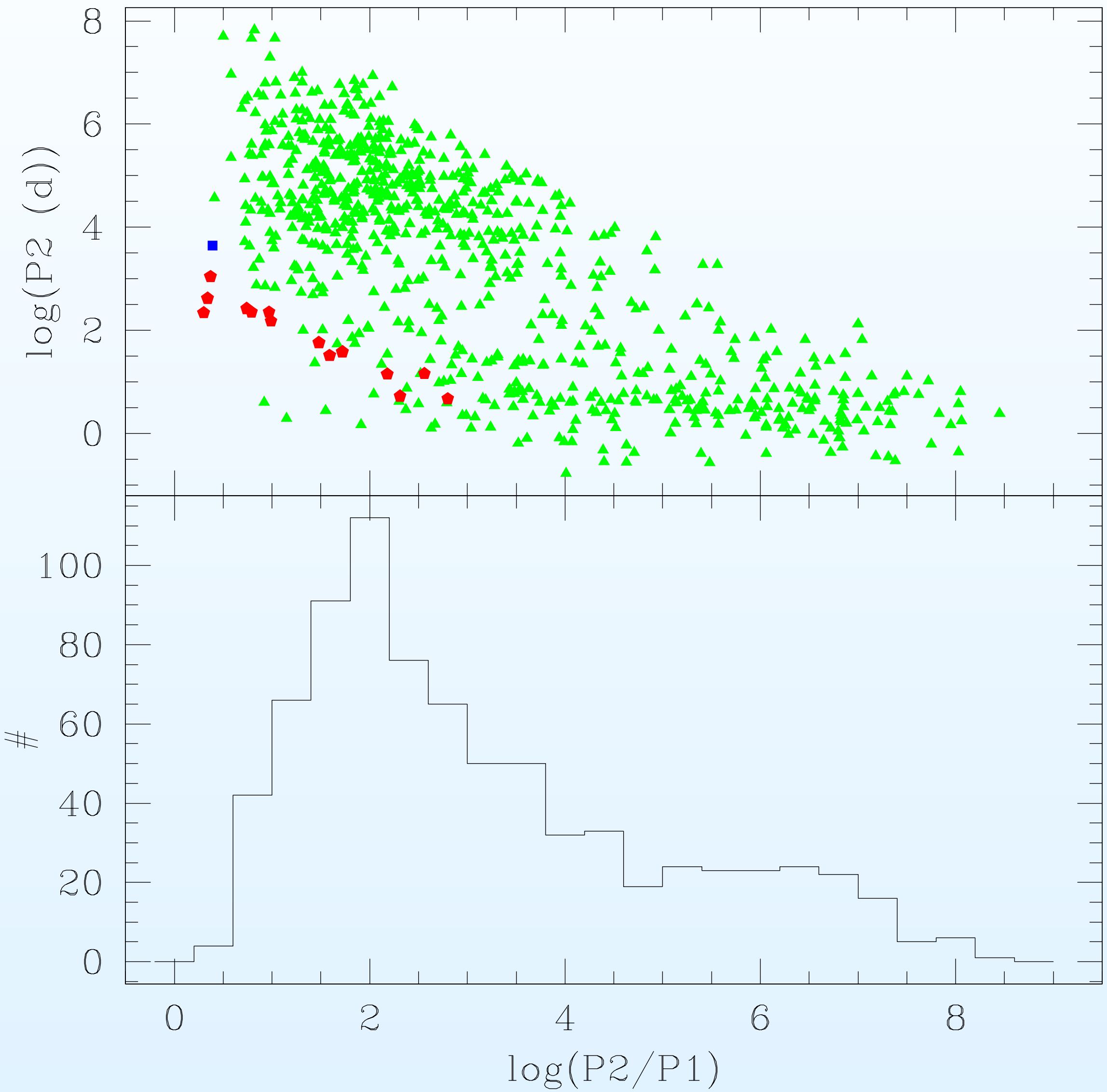
5+7*N-parameter nonlinear model.

Period ratio

Tokovinin (1997)
Scheinerd's enc.

Planetary periods are more similar than stellar ones.

No longer hierarchical (3!=2+2).



Conclusions

- Different interpretations but equations are essentially identical for visual and astrometric binary and extrasolar planetary orbits.
- Unique code for the two families.
- Fitting 3+ components takes longer for planetary than for stellar systems

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