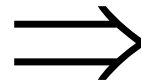


Measuring fundamental stellar properties using binaries

Christian A. Hummel (European Southern Observatory)



Courtesy of Tyler Nordgren

Overview

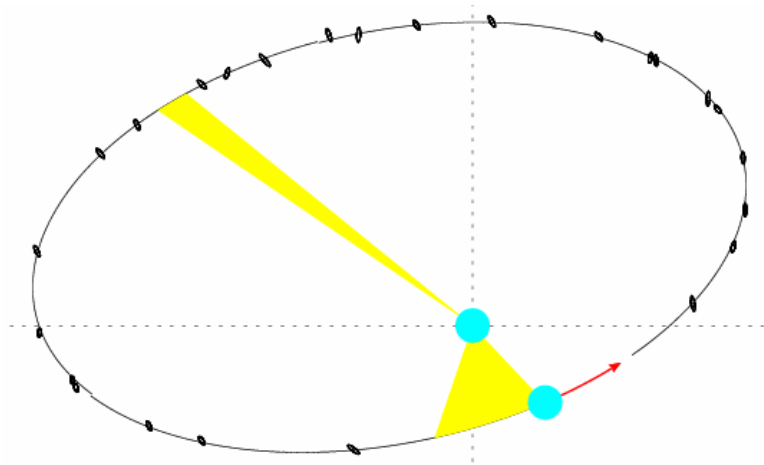
- Motivation
- Stellar evolution
- Fundamental parameters
- Visual binaries
- Eclipsing binaries
- Current examples

Mizar A, the movie

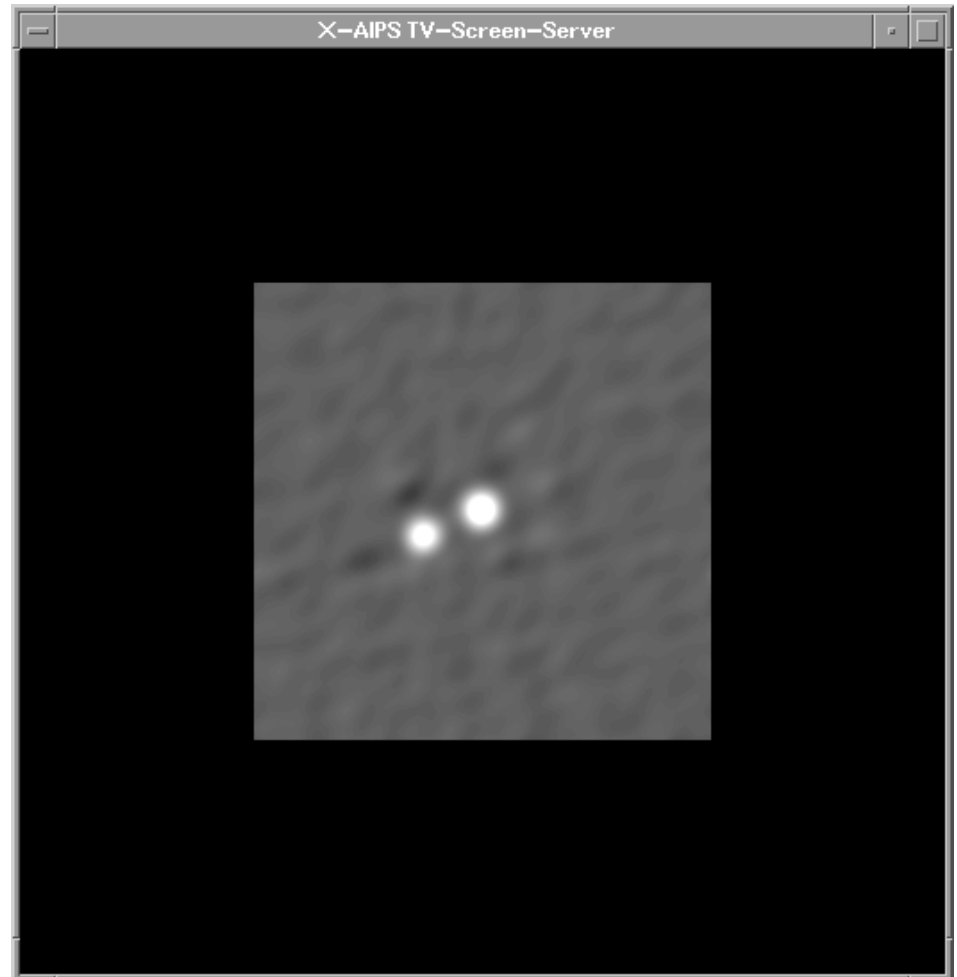
$D = 25.4 \text{ pc}$

$P = 20.5 \text{ days}$

$a = 9.8 \text{ mas}$



(NPOI)



The Hertzsprung-Russell Diagram

The observations

- Apparent visual magnitudes
- Trigonometric parallaxes
- Spectral type classification, a measure of the effective temperature

The result

- Stars occupy a limited parameter space in absolute (intrinsic) stellar brightness and spectral type
- Two branches exist, main sequence stars and giants. The narrow lines of the latter are indicative of a lower surface gravity. In combination with their greater brightness, this indicates the intrinsically larger sizes of the giants.

The interpretation

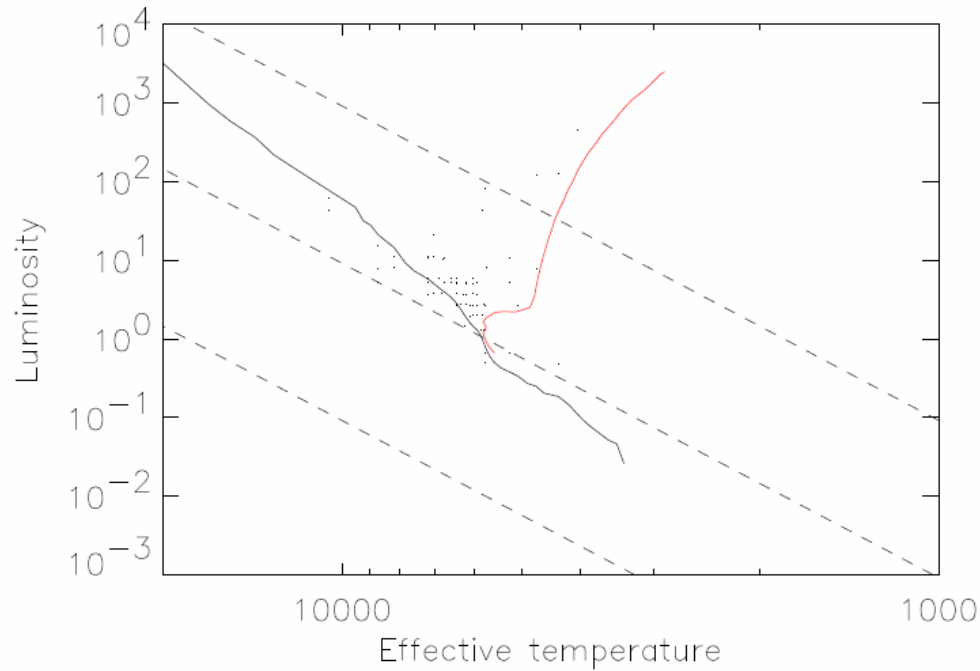
- A physical model of stellar structure based on nuclear fusion of matter with composition (X, Y, Z)
- Blackbody radiation
- Lifetime and main-sequence location of a star governed by its mass

Stellar model and evolution (I)

Fundamental stellar properties

- **Mass**
- **Chemical composition**
- **Angular momentum**
- Luminosity
- Effective temperature
- Radius

Stellar model and evolution (II)



Theoretical Hertzsprung-Russell diagram. Solid line is Zero-age-mainsequence (ZAMS). Dashed lines are blackbody stars of constant radius, the middle line corresponding to the solar radius. The red line is an example for an evolutionary track of a star of one solar mass.

Relationships between fundamental parameters

Stefan–Boltzmann law (black body radiation)

$$\log L/L_{\odot} = 2 \log R/R_{\odot} + 4 \log T_e/T_{e\odot}$$

Surface gravity g

$$\log M/M_{\odot} = 2 \log R/R_{\odot} + \log g/g_{\odot}$$

Solar values: $T_e = 5780$ K, $\log g_{\odot} = 4.44$ (cgs units).

Zero–age main sequence

$$\log L/L_{\odot} = 3.8 \log M/M_{\odot} + 0.08$$

$$\log R/R_{\odot} = \begin{cases} 0.917 \log M/M_{\odot} - 0.020 & (-1.0 < \log M/M_{\odot} < 0.12) \\ 0.640 \log M/M_{\odot} + 0.011 & (0.12 < \log M/M_{\odot} < 1.3) \end{cases}$$

Bolometric luminosity

$$M_{\text{bol}} \equiv M_V - BC = -2.5 \log(L/L_{\odot}) + 4.74$$

$$M_V = m_V + 5 + 5 \log \pi$$

Some useful calibrations (I)

Bolometric correction

Gubochkin, A. N. & Miroshnichenko, A. S. 1991, *Kin. and Phys. of Cel. Bodies*, 7, 59

$$BC = \begin{cases} -0.0508T_3^2 + 0.762T_3 - 2.831 & (4.7 < T_3 < 10, T_3 \equiv T/1000) \\ 0.0032T_3^2 - 0.260T_3 + 1.978 & (10 < T_3) \end{cases}$$

Flower, P. J. 1996, *ApJ*, 469, 355 (corrected)

$$BC = \begin{cases} -19053.73 + 15514.49 \log T - 4212.788(\log T)^2 + 381.4763(\log T)^3 & (\log T < 3.70) \\ -37051.02 + 38567.26 \log T - 15065.149(\log T)^2 + 2617.2464(\log T)^3 & \\ \quad -170.62381(\log T)^4 & (3.70 < \log T < 3.90) \\ -118115.45 + 137145.97 \log T - 63623.381(\log T)^2 + 14741.2924(\log T)^3 & \\ \quad -1705.87278(\log T)^4 + 78.873172(\log T)^5 & (3.90 < \log T) \end{cases}$$

Interstellar extinction

Schmidt-Kaler, Th. 1982, *Landolt-Börnstein series VI/2b*

$$\frac{A_V}{E_{B-V}} = 3.30 + 0.28(B - V)_0 + 0.04E_{B-V}, \quad A_V = V - V_0, \quad E_{B-V} = (B - V) - (B - V)_0$$

Some useful calibrations (II)

Effective temperature

Flower, P. J. 1996, ApJ, 469, 355 (corrected)

$$\log T = \begin{cases} 3.9791 - 0.6550(B - V) + 1.7407(B - V)^2 - 4.6088(B - V)^3 \\ \quad + 6.7926(B - V)^4 - 5.3969(B - V)^5 + 2.1930(B - V)^6 \\ \quad - 0.3595(B - V)^7 & \text{MS, Subgiants, Giants} \\ 4.0126 - 1.0550(B - V) + 2.1333(B - V)^2 - 2.4598(B - V)^3 \\ \quad + 1.3494(B - V)^4 - 0.2839(B - V)^5 & \text{Supergiants} \end{cases}$$

Zero-magnitude bolometric flux outside the Earth's atmosphere

$$(m_{\text{bol}} = 0) \text{ star} \equiv 2.48 \cdot 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1}$$

Zero-magnitude fluxes for the Johnson system

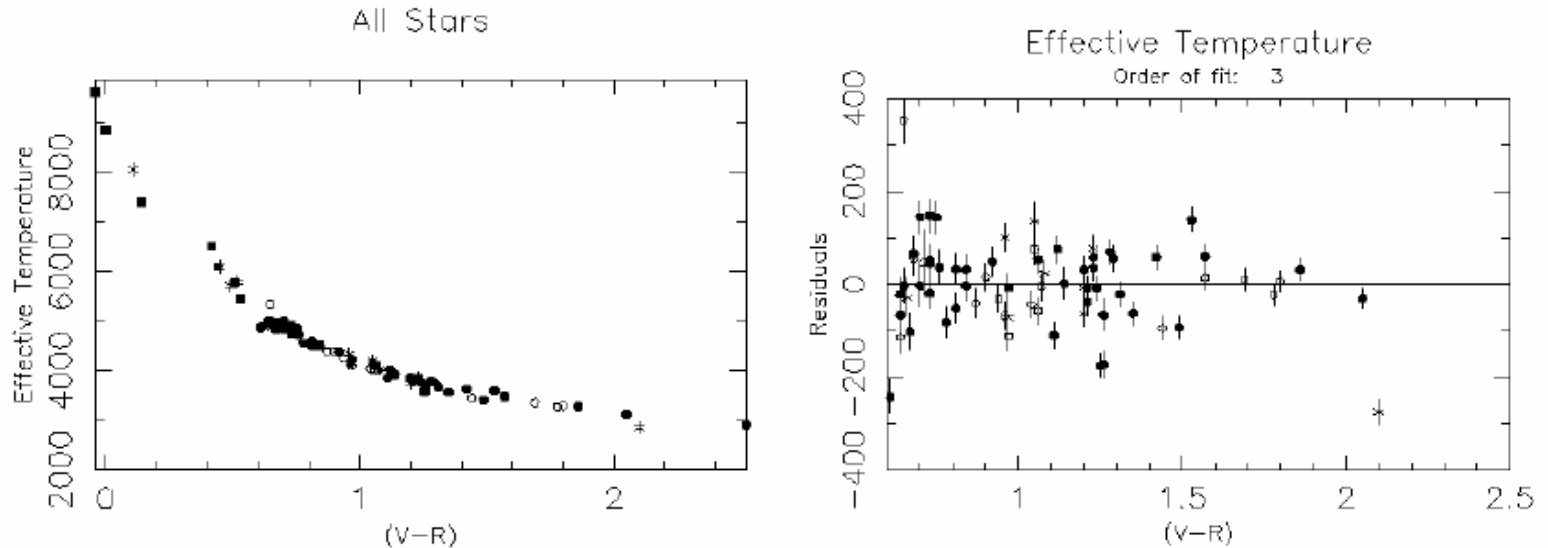
| Filter | λ_{eff} [μm] | $F_{\nu,0}$ [$\text{W}/(\text{m}^2\text{Hz})$] | $F_{\lambda,0}$ [$\text{erg}/(\text{cm}^2\text{s}\text{\AA})$] |
|--------|--|--|--|
| U | 0.36 | $1.88 \cdot 10^{-23}$ | $4.22 \cdot 10^{-9}$ |
| B | 0.44 | $4.65 \cdot 10^{-23}$ | $6.40 \cdot 10^{-9}$ |
| V | 0.55 | $3.95 \cdot 10^{-23}$ | $3.75 \cdot 10^{-9}$ |
| R | 0.70 | $2.87 \cdot 10^{-23}$ | $1.75 \cdot 10^{-9}$ |
| I | 0.90 | $2.24 \cdot 10^{-23}$ | $8.4 \cdot 10^{-10}$ |

Effective temperature

Effective temperatures are derived from angular diameters and bolometric fluxes.

$$F_{\text{bol}} = \sigma(\theta/2)^2 T_{\text{eff}}^4,$$

where F is the flux measured above the atmosphere, σ is the Stefan-Boltzmann constant ($\sigma = 5.67 \cdot 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$), θ is the limb-darkened angular diameter in radians.



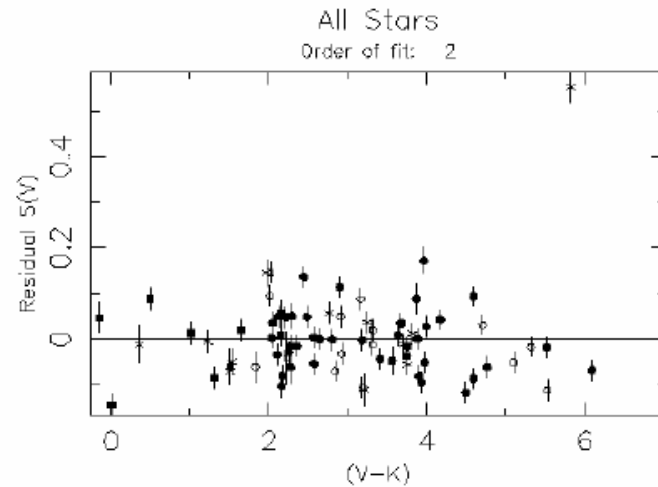
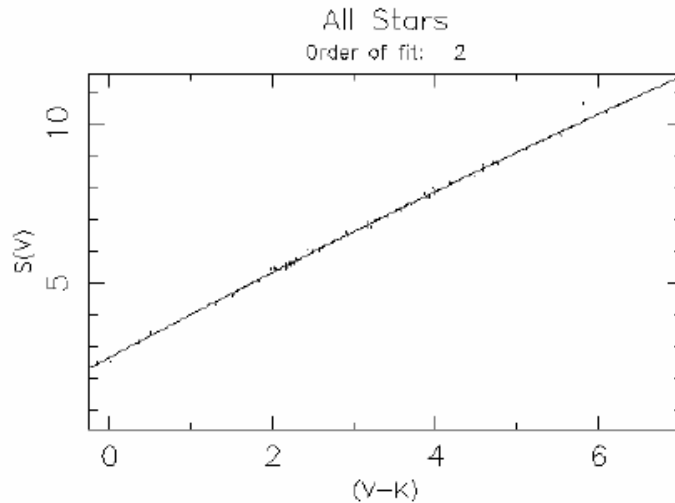
(Mozurkewich, D., et al. *in prep*, based on data of the Mark III stellar interferometer.)

Surface brightness calibration

The stellar surface brightness is defined through

$$S_V = V + 5 \log(\theta),$$

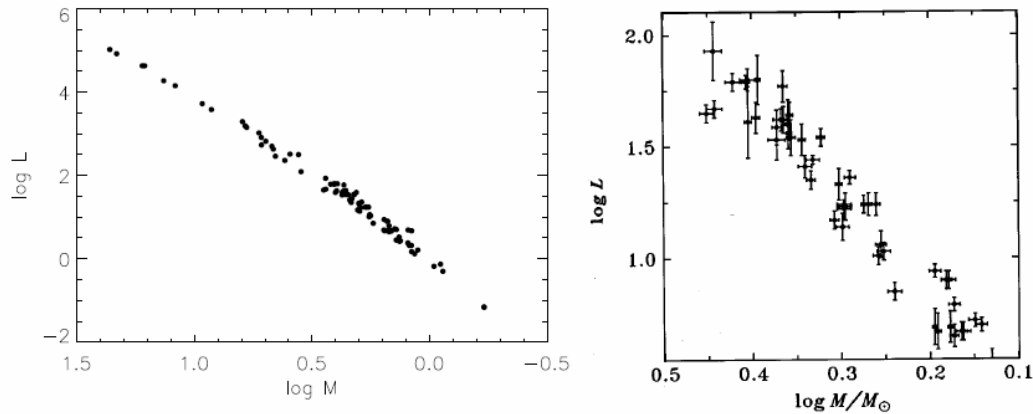
where θ is measured in milliarcseconds.



These calibrations can be used to determine limb-darkened diameters of stars of any luminosity class.

$$S_V = \begin{cases} 2.646 + 4.235(V - R) - 0.022(V - R)^2 - 0.102(V - R)^3 \\ 2.680 + 1.361(V - K) - 0.015(V - K)^2 \end{cases}$$

Andersen's bar



The mass-luminosity relationship after Andersen, 1991, based on observations of eclipsing binaries. On the right is a zoom-in on a section of the diagram, showing that the scatter considerably exceeds the observational errors.

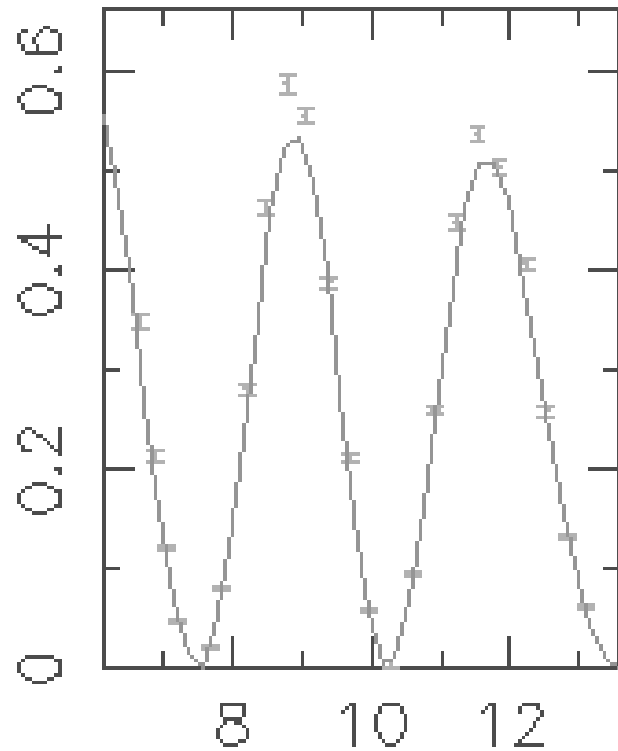
Typical properties of a well studied binary system (after Andersen 1995, IAU 166)

- Mass: 1%
- Radius: 1%
- Luminosity: 10%
- Distance: 3%

Visibility amplitude of a binary

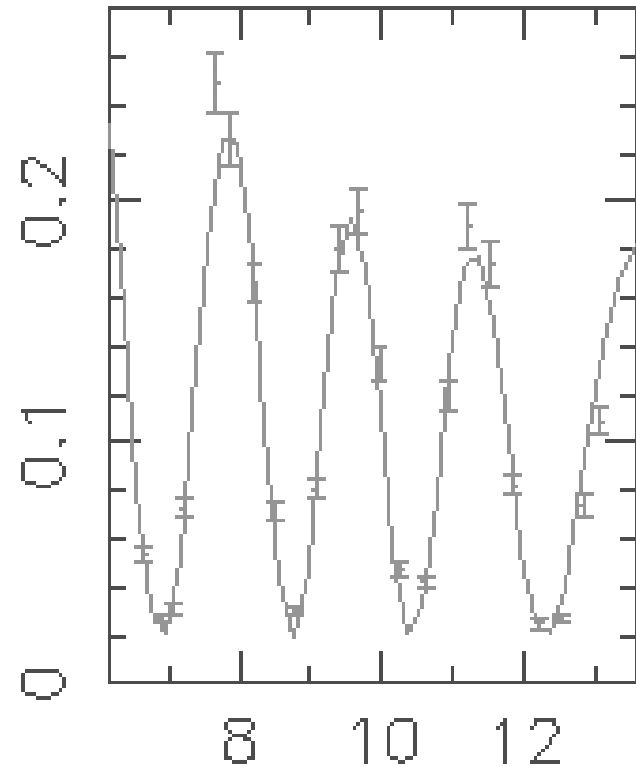
FK5 193 801nm

$$\chi^2/\nu = 11.21$$



BI 1 501nm

$$\chi^2/\nu = 6.10$$

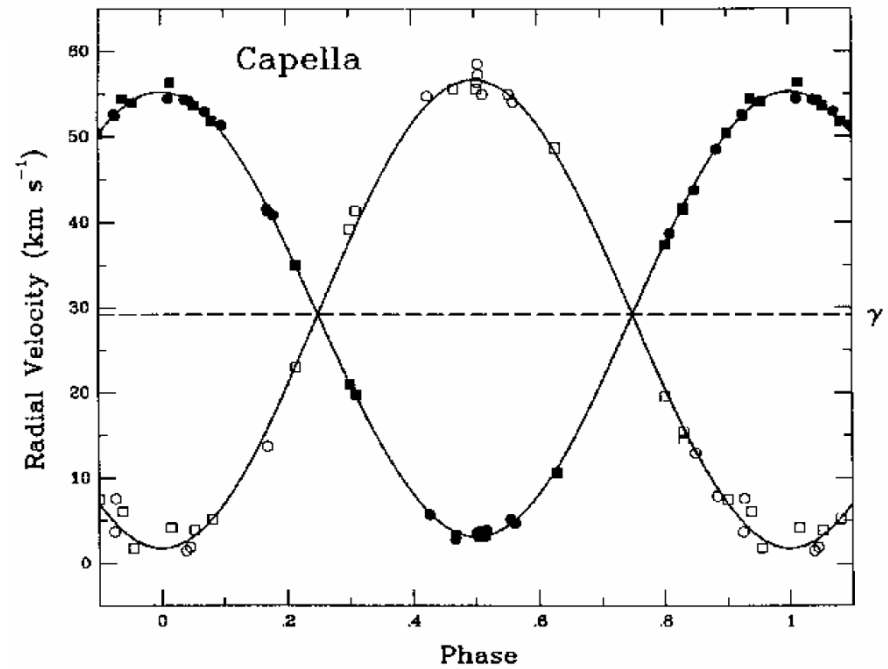
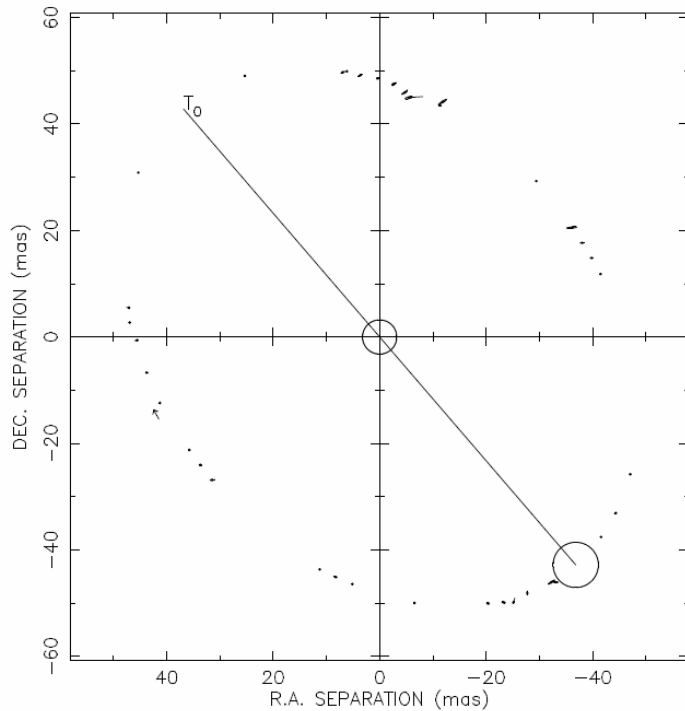


Mark III

State of the art: visual (I)

Observational parameters:

- Spectroscopy: $K_{1,2} \pm 0.1$ km/s,
 $M_{1,2}/M_{\odot} = 1.0385 \cdot 10^{-7} (1 - e^2)^{3/2} (K_1 + K_2)^2 K_{2,1} P$, with $K_{1,2}$ in km/s and P in days.
- Interferometry: $i \pm 0.1^\circ$, $a \pm 0.1$ mas, $\Delta m_{\lambda} \pm 0.05$ mag, $D_{1,2} \pm 0.1$ mas



State of the art: visual (II)

Steps in the analysis:

- derive masses: $\pm 2\%$
- derive distance (here 13.3 ± 0.1 pc), absolute magnitudes, and colors
- derive linear radius, $R/R_{\odot} = 215(D/2)/\pi$, where π is the orbital parallax
- derive T from absolute bolometric flux and diameters...or
- derive T from absolute magnitudes, bolometric correction and diameters
- derive total luminosity from integrated flux measurements and distance (here $L_{Aa+Ab}/L_{\odot} = 153$)
- compare total luminosity to luminosity based on T and diameters
- compare T and colors to calibrations

State of the art: eclipsing binaries (I)

Observational data:

- Photometry: i , $R/a \pm 1\%$, $\Delta m_\lambda \pm 0.05$ mag, $m_{1,2} \pm 0.02$ mag
- Spectroscopy: $M_{1,2}(\sin i)^3$, $K_{1,2} \pm 0.1$ km/s

| | | |
|----------------------------------|-------------------------------|---------------|
| K_A (km s ⁻¹) | 50.90±0.08 | 50.95±0.08 |
| K_B (km s ⁻¹) | 49.24±0.07 | 49.20±0.08 |
| γ_A (km s ⁻¹) | -1.76±0.06 | -1.76±0.06 |
| γ_B (km s ⁻¹) | -1.92±0.06 | -1.92±0.06 |
| e_A | 0.188±0.0020 | 0.1855±0.0016 |
| e_B | $e_B=e_A$ | 0.1895±0.0015 |
| ω_A | 109°9±0°6 | 111°0±0°5 |
| ω_B | $\omega_B=\omega_A+180^\circ$ | 289°6±0°4 |
| σ_A (km s ⁻¹) | 0.43 | 0.40 |
| σ_B (km s ⁻¹) | 0.37 | 0.37 |
| M_B/M_A | 1.034±0.002 | |
| $a \sin i$ (R_\odot) | 47.78±0.05 | |
| $M_A \sin^3 i$ (M_\odot) | 1.194±0.004 | |
| $M_B \sin^3 i$ (M_\odot) | 1.234±0.005 | |

| | | | | | |
|-----------------|---------|-------|-------|--------|-------|
| i | 88°45 | | r_A | 0.0380 | |
| | 5 | | | 5 | |
| $e \cos \omega$ | -0.0634 | | k | 1.613 | |
| | 3 | | | 10 | |
| $e \sin \omega$ | 0.178 | | r_B | 0.0613 | |
| | 10 | | | 10 | |
| e | 0.189 | | | | |
| | 10 | | | | |
| ω | 109°6 | | | | |
| | 1.0 | | | | |
| | U | B | V | R | I |
| J_B/J_A | 0.184 | 0.296 | 0.408 | 0.484 | 0.556 |
| L_B/L_A | 0.433 | 0.736 | 1.021 | 1.224 | 1.414 |
| | u | v | b | y | |
| J_B/J_A | 0.198 | 0.252 | 0.356 | 0.418 | |
| L_B/L_A | 0.472 | 0.621 | 0.872 | 1.038 | |

Example from: Andersen, J., et al. 1988, A&A, 196, 128
 "AI Phoenicis: a case study in stellar evolution"

State of the art: eclipsing binaries (II)

Steps in the analysis:

- derive masses ($\pm 1\%$)
- obtain effective temperature from multi-color photometry
- derive luminosities ($\pm 10\%$) from effective temperature and radius.
- derive distance luminosity, bolometric correction, and apparent magnitude

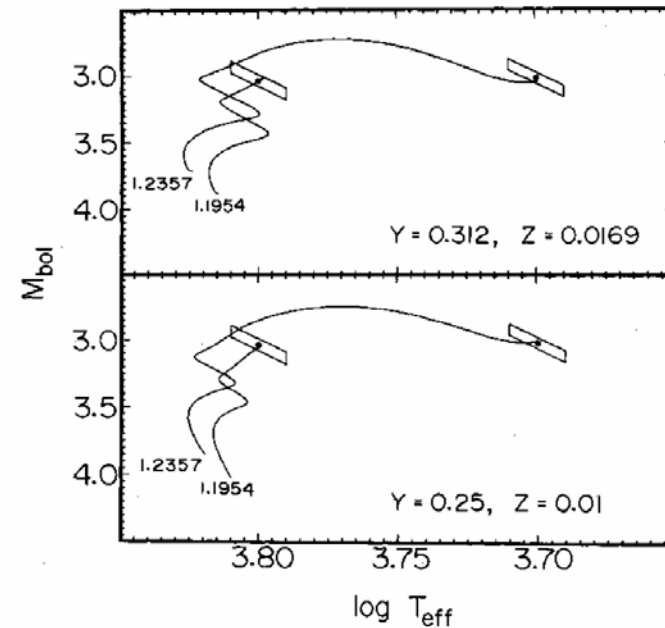
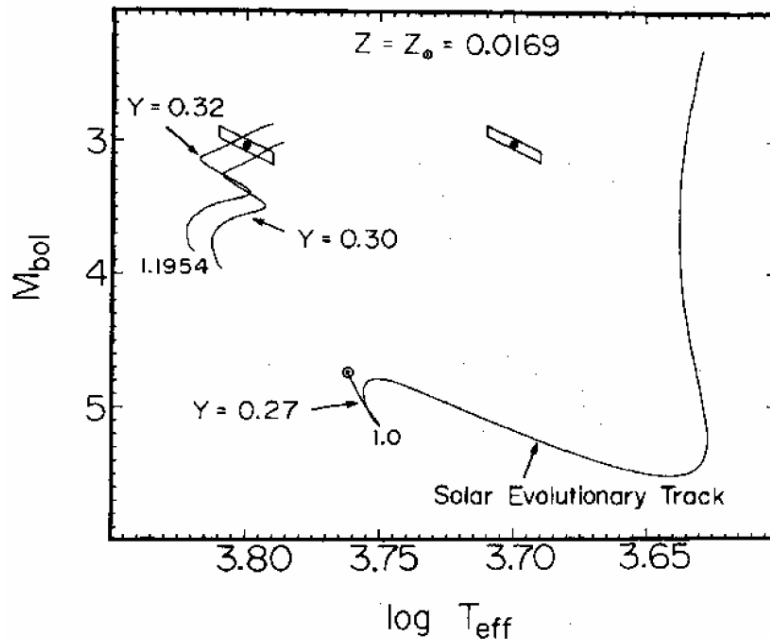
| Absolute dimensions: | | |
|---|---------------------|---------------------|
| M/M_{\odot} | 1.1954 ± 0.0041 | 1.2357 ± 0.0045 |
| R/R_{\odot} | 1.816 ± 0.024 | 2.930 ± 0.048 |
| $\log g$ (cgs) | 3.997 ± 0.012 | 3.596 ± 0.014 |
| $v \sin i$ (km s^{-1}) | 4 ± 1 | 6 ± 1 |
| [Fe/H] | -0.14 ± 0.1 | |
| Photometric data: | | |
| T_{e} (K) ^a | 6310 ± 150 | 5010 ± 120 |
| $M_{\text{bol}}^{\text{b}}$ | 3.06 ± 0.11 | 3.03 ± 0.11 |
| $\log L/L_{\odot}$ | 0.67 ± 0.04 | 0.69 ± 0.04 |
| $B.C.^{\text{b}}$ | -0.06 | -0.26 |
| M_{v} | 3.12 ± 0.11 | 3.29 ± 0.11 |
| Distance (pc) | 162 ± 6 | (no reddening) |
| $(U, V, W)^{\text{c}}$ (km s^{-1}) | $(+30, -23, +5)$ | |

^a Vandenberg and Hrivnak (1985)
^b Popper (1980, Table 1), assuming $M_{\text{bol}, \odot} = 4.75$
^c proper motions from SAO

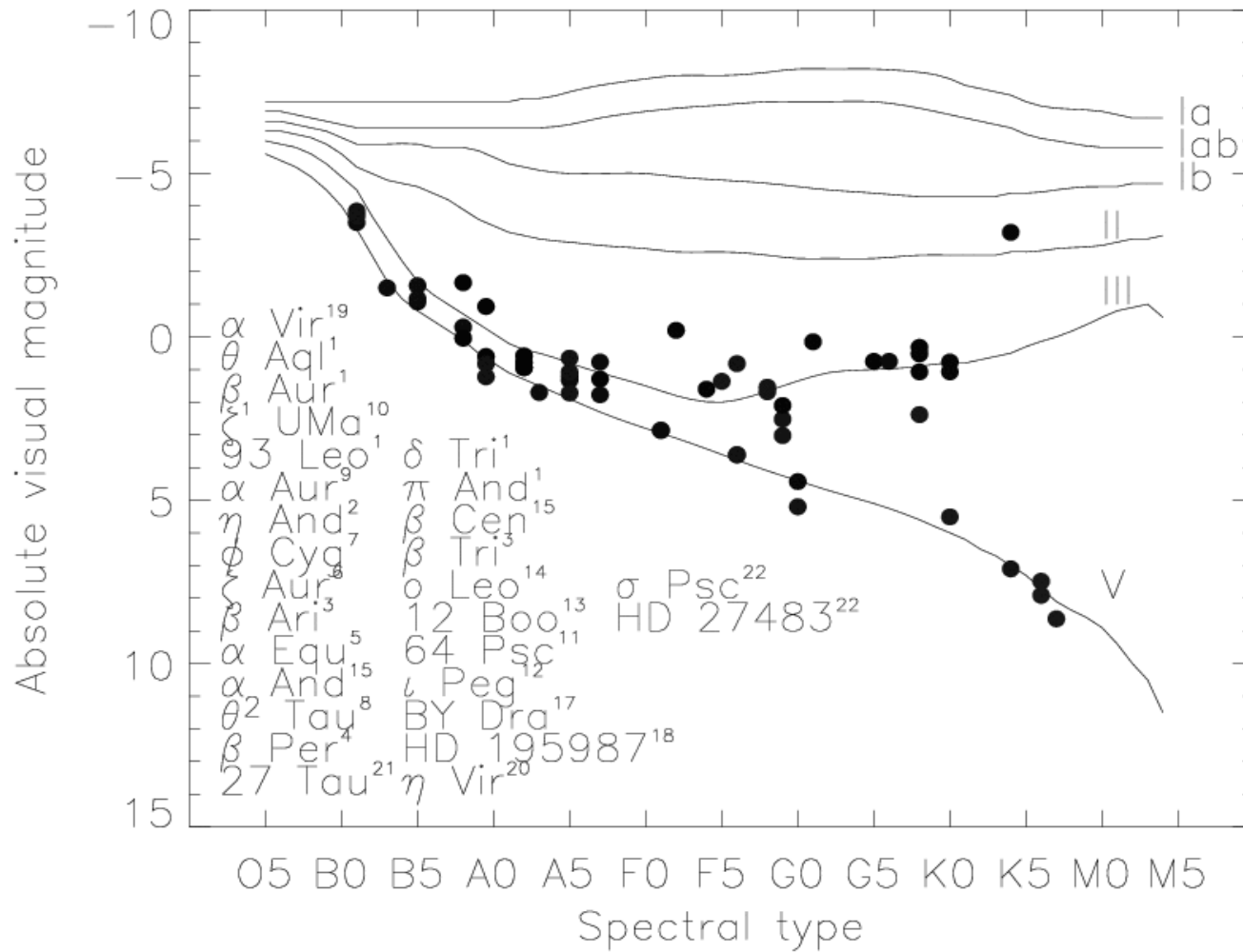
State of the art: eclipsing binaries (III)

Steps in the comparison with evolutionary models

- Determine metal abundance from spectroscopic analysis, here (Al Phe) $Z = 0.012 \pm 0.003$
- Adjust helium abundance, a free parameter, by matching the luminosity, here $Y = 0.27 \pm 0.02$
- Compute track for less evolved component for stellar masses and determine age
- Compute evolutionary state for more evolved component and compare with observations. Note that there are no free parameters.



Footprints in the HR diagram



Orbital Parallaxes

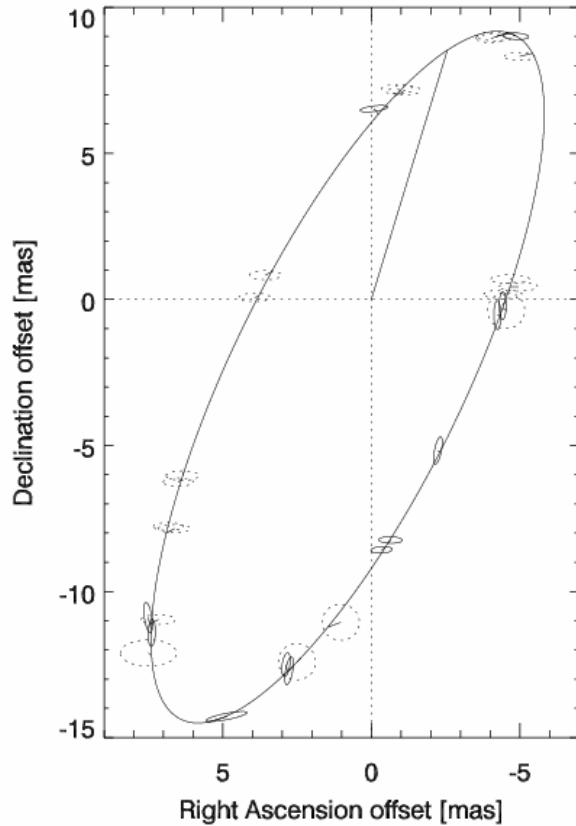
Comparison of Hipparcos parallaxes with orbital parallaxes [mas]

| Star | HIC | M3/PTI/NPOI | \pm | Hipparcos | \pm |
|----------------|--------|-------------|-------|-----------|-------|
| θ Aql | 99473 | 13 | 1 | 11.4 | 0.9 |
| β Aur | 28360 | 40 | 1 | 39.7 | 0.8 |
| 93 Leo | 57565 | 13.8 | 0.5 | 14.4 | 0.9 |
| η And | 4463 | 13.1 | 0.3 | 13.4 | 0.7 |
| β Ari | 8903 | 53 | 2 | 54.7 | 0.8 |
| β Per | 14576 | 35.4 | 1.1 | 35.1 | 0.9 |
| α Equ | 104987 | 18.1 | 0.8 | 17.5 | 0.9 |
| ζ Aur | 23453 | 3.8 | 0.1 | 4.1 | 0.8 |
| ϕ Cyg | 96683 | 12.4 | 0.3 | 13.0 | 0.6 |
| θ^2 Tau | 20894 | 21.2 | 0.8 | 21.9 | 0.8 |
| α Aur | 24608 | 75.1 | 0.5 | 77.3 | 0.9 |
| Mizar A | 65378 | 39.4 | 0.3 | 41.7 | 0.6 |
| 64 Psc | 3810 | 43.3 | 0.5 | 41.8 | 0.8 |
| ι Peg | 109176 | 86.9 | 1.0 | 85.1 | 0.7 |
| 12 Boo | 69226 | 27.1 | 0.4 | 27.3 | 0.8 |
| o Leo | 47508 | 24.2 | 0.1 | 24.1 | 1.0 |

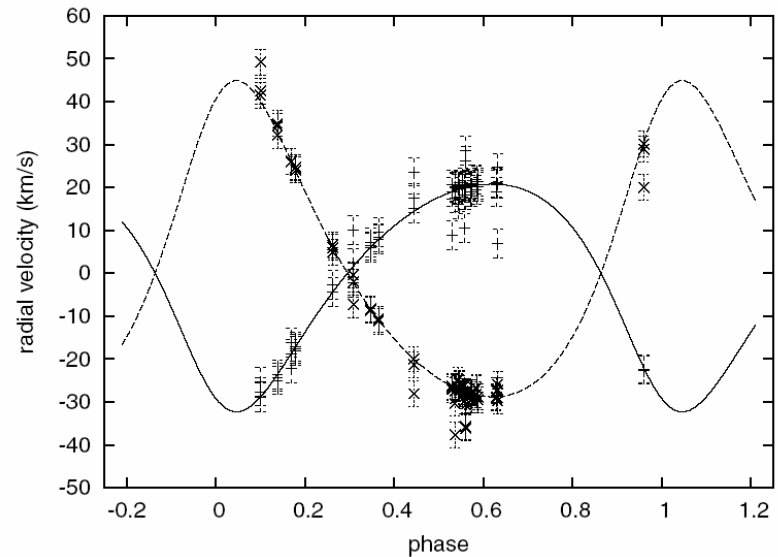
Atlas (Pleiades)

Hipparcos distance of Pleiades: 118.3 ± 3.5 pc

Atlas from orbital parallax: 132 ± 4 pc



(MARK III, NPOI)

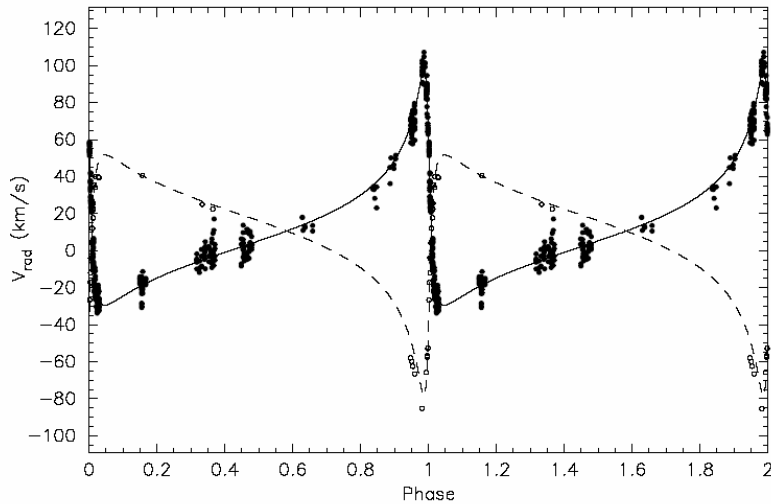


(ELODIE, CORALIE, FEROS + KOREL)

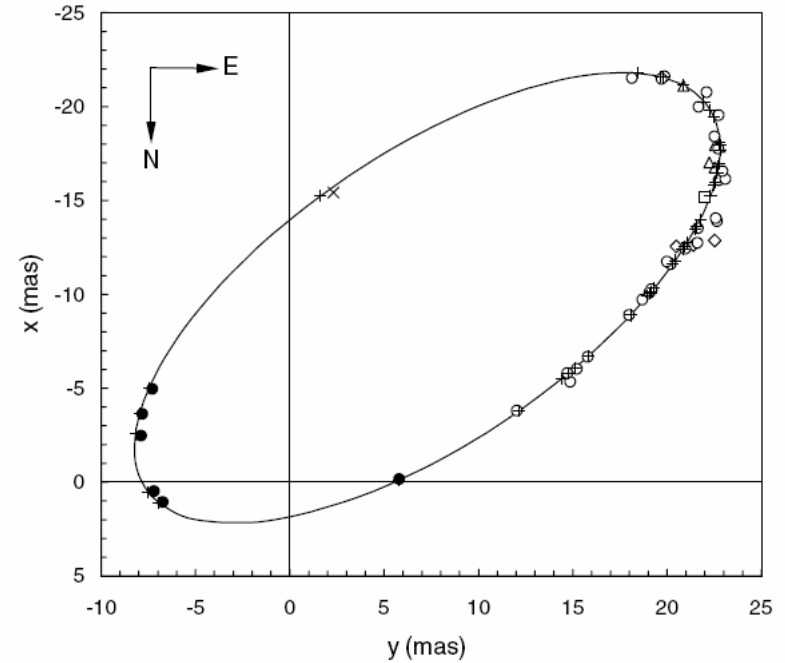
Zwahlen et al. 2004

β Centauri (B1III + B1III)

$$\mathcal{M} = 9.1 \text{ (14.4)}$$

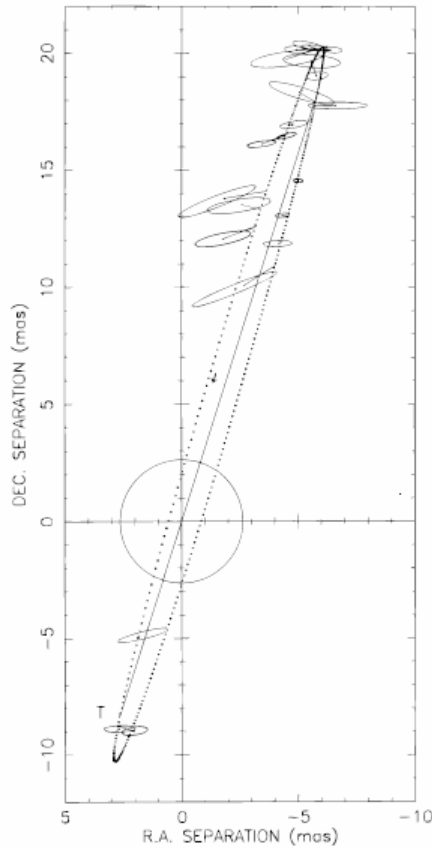


Ausseloos et al. 2002

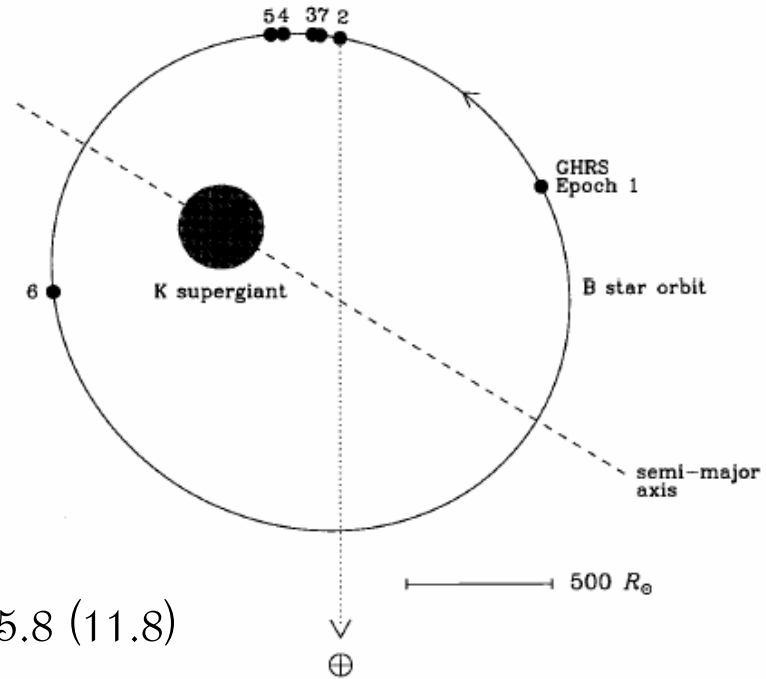


Davis et al. 2005
(SUSI)

ζ Aurigae (K4Ia + B5V)



Mark III

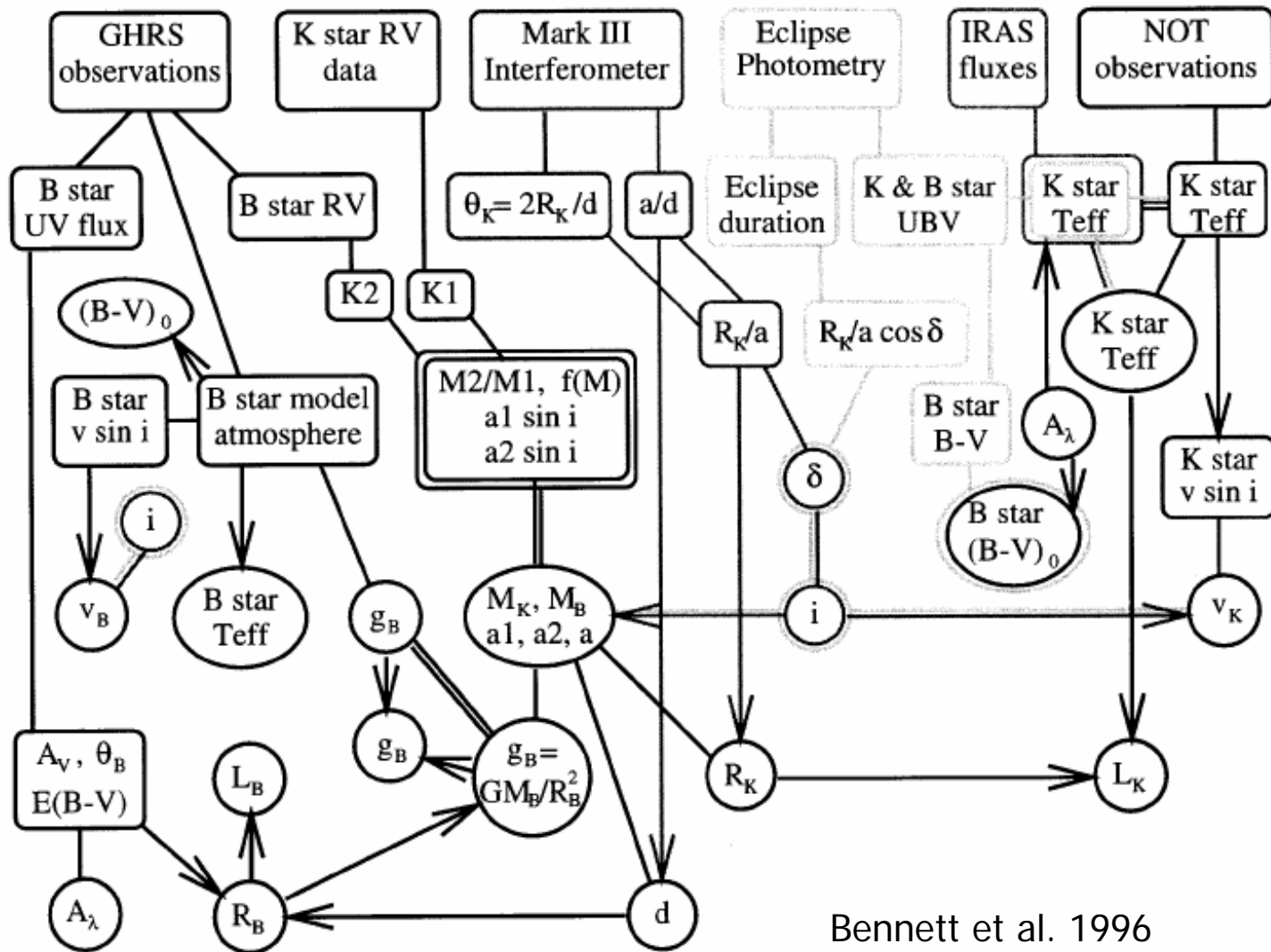


$$\mathcal{M}_{K4Ia} = 5.8 \text{ (11.8)}$$

$$\mathcal{M}_{B5V} = 4.8 \text{ (4.5)}$$

Bennett et al. 1996

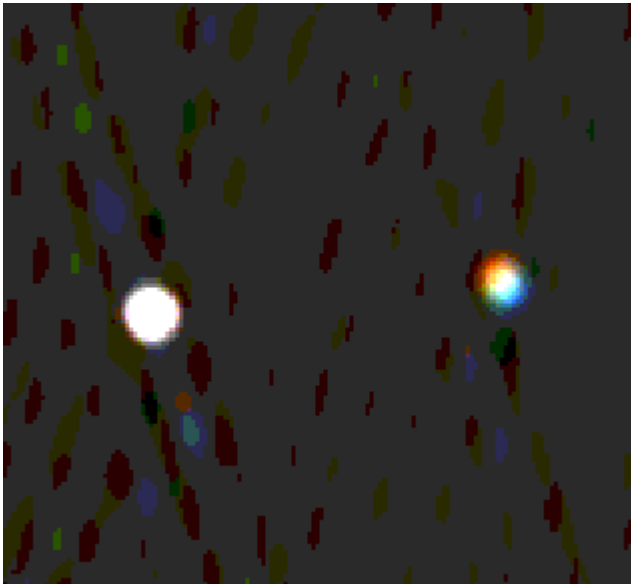
Fundamental stellar parameters (ζ Aurigae)



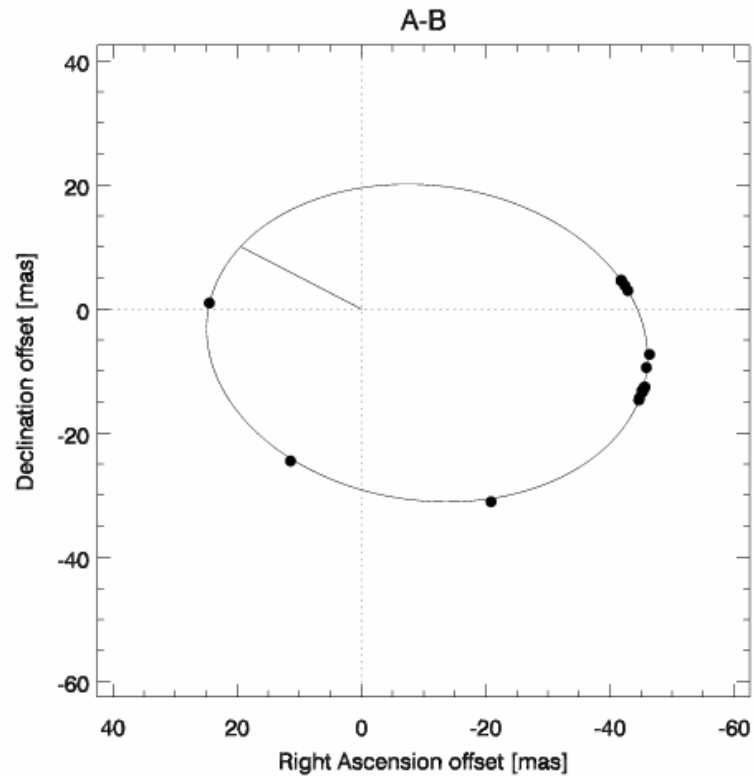
Bennett et al. 1996

Zeta Orionis (O9.5I + OIV)

$P=7.2$ y
 $r=40$ mas

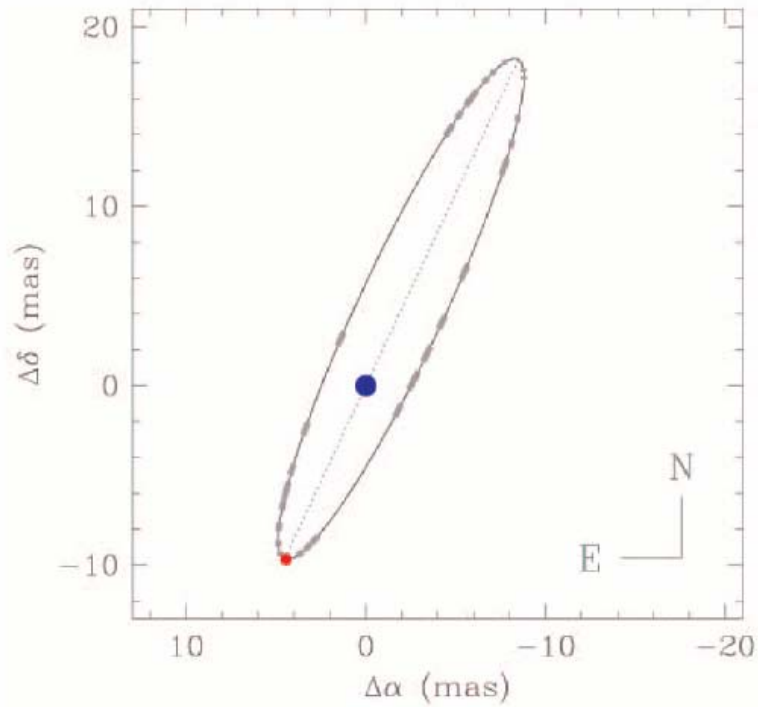


Zeta Orionis A



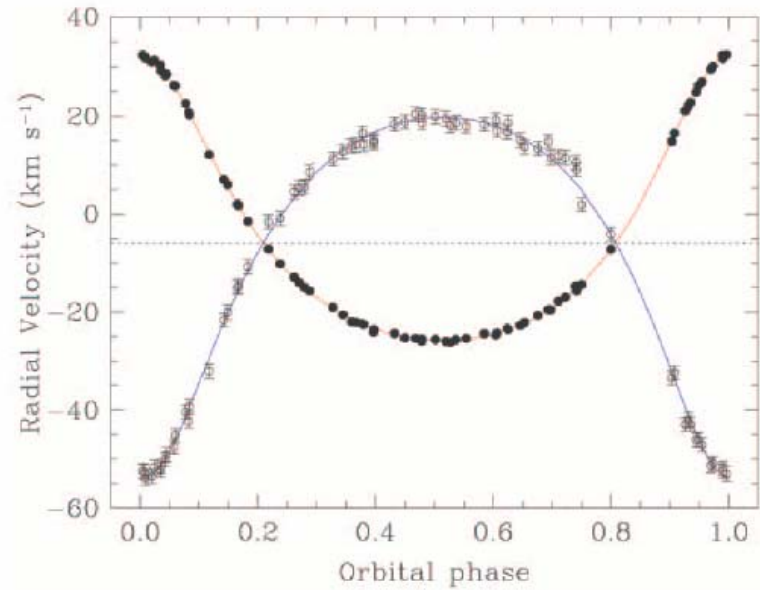
Hummel et al. 2000

Metal poor HD 195987



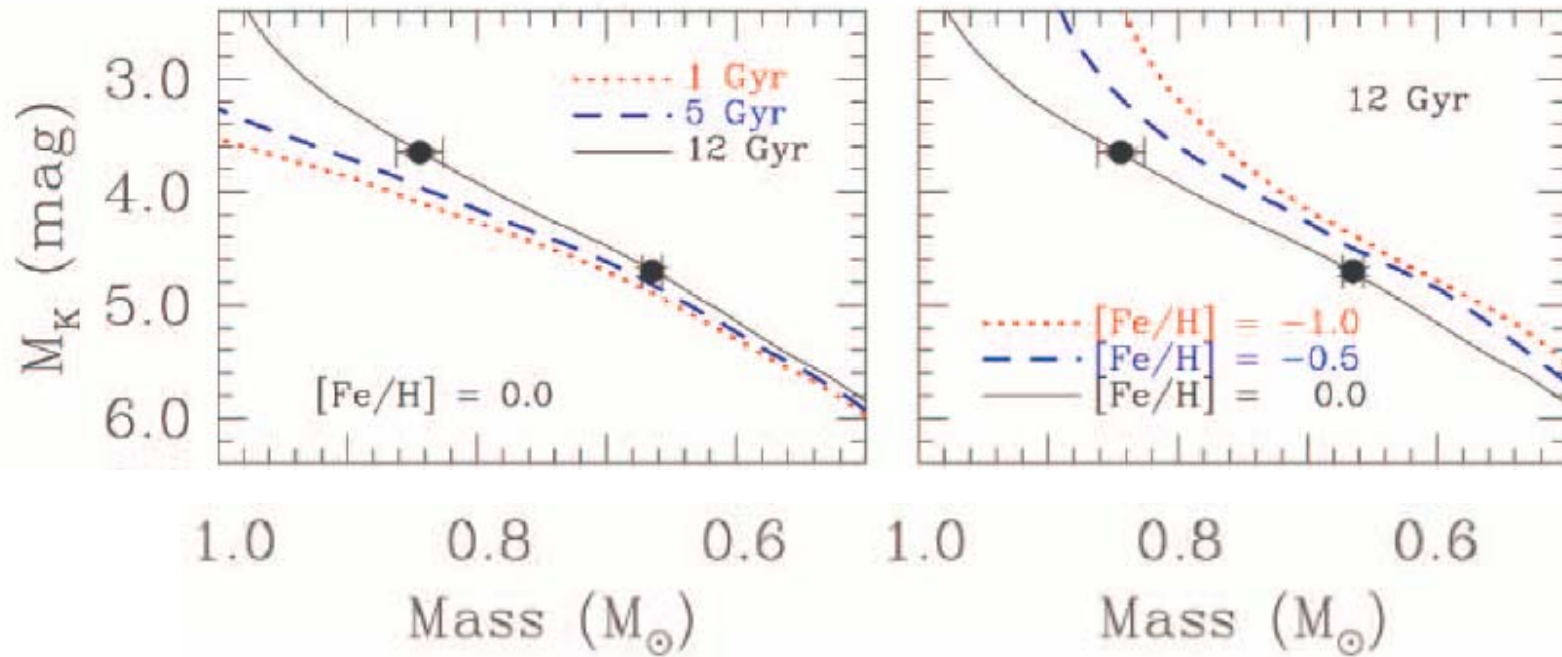
(PTI)

Torres et al. 2002

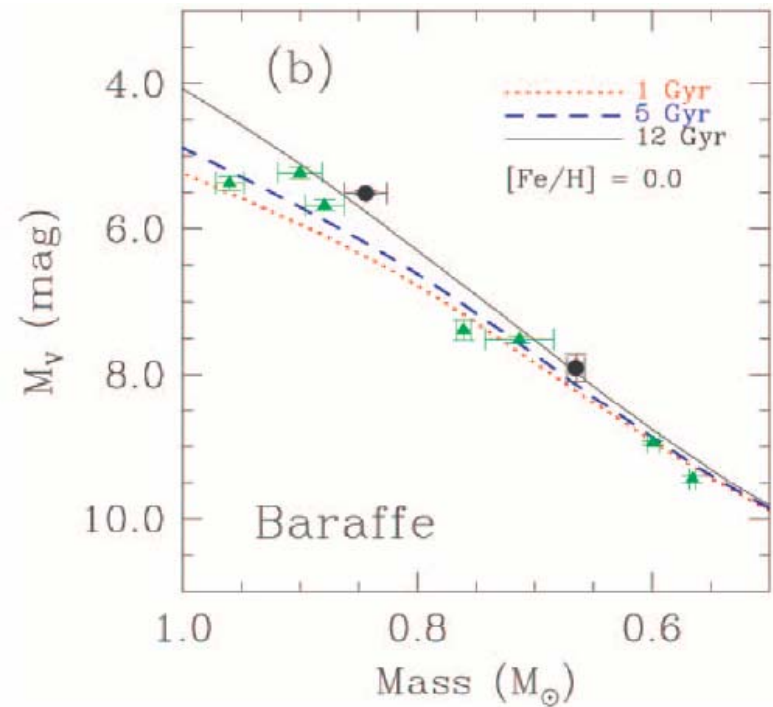
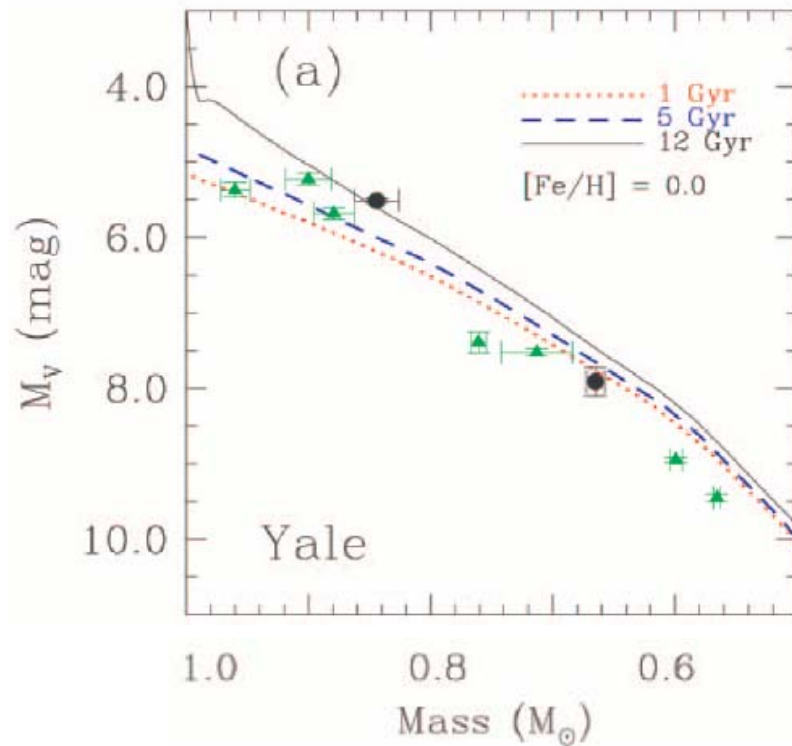


(Oak Ridge Obs.)

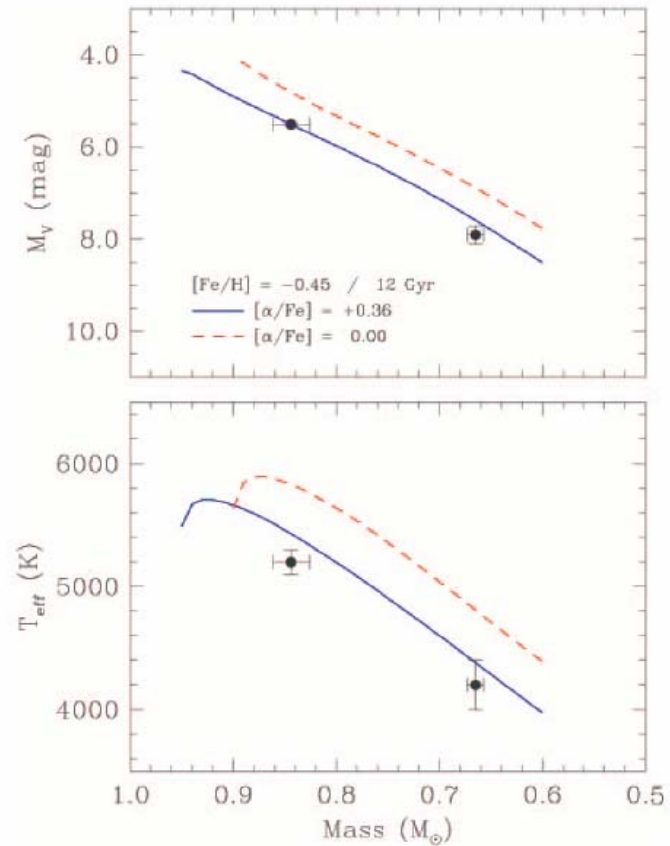
Isochrone fitting



Model atmospheres

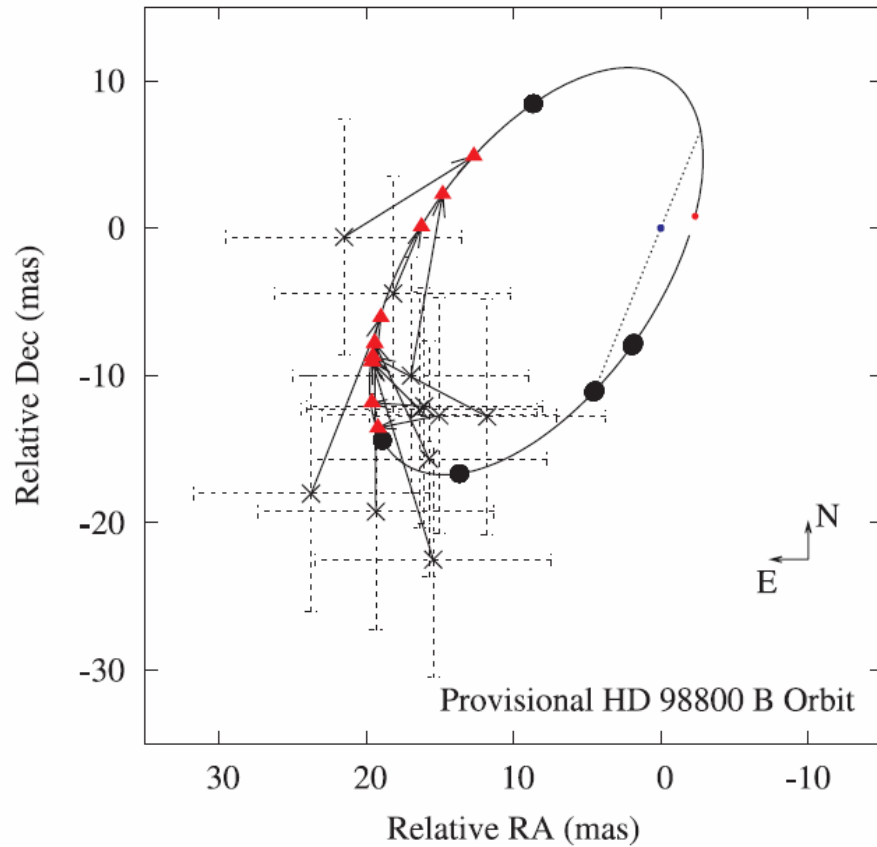


Detailed abundance analysis



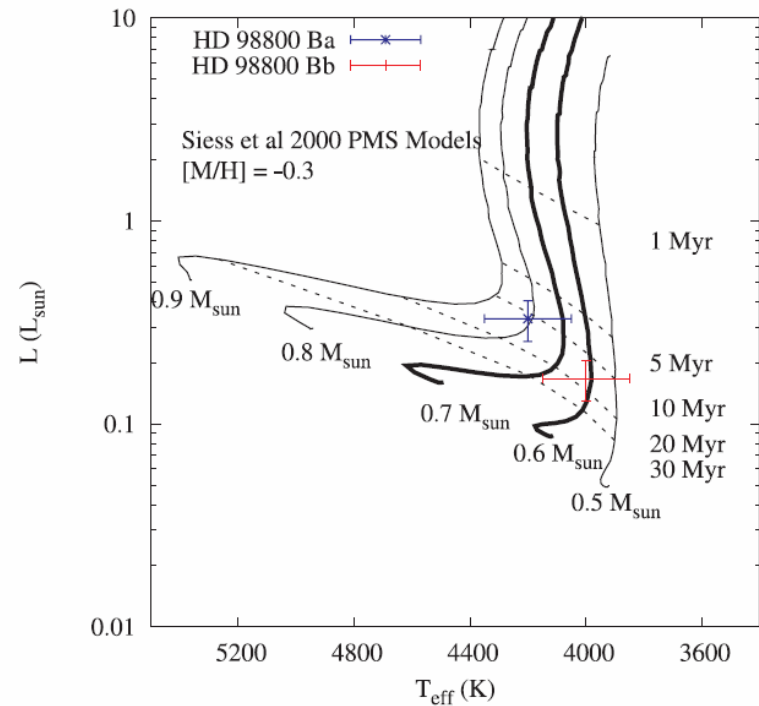
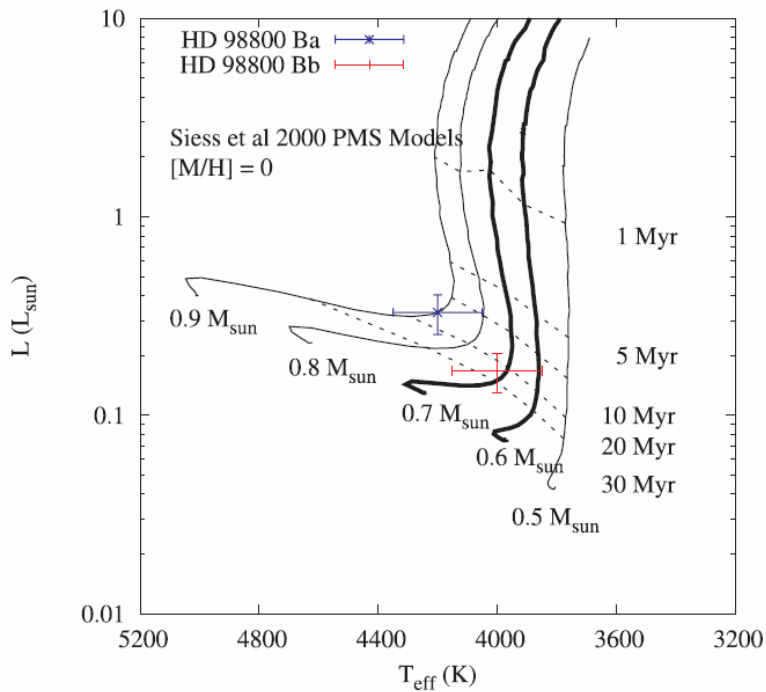
Torres et al. 2002

Pre-mainsequence stars



Boden et al. 2006

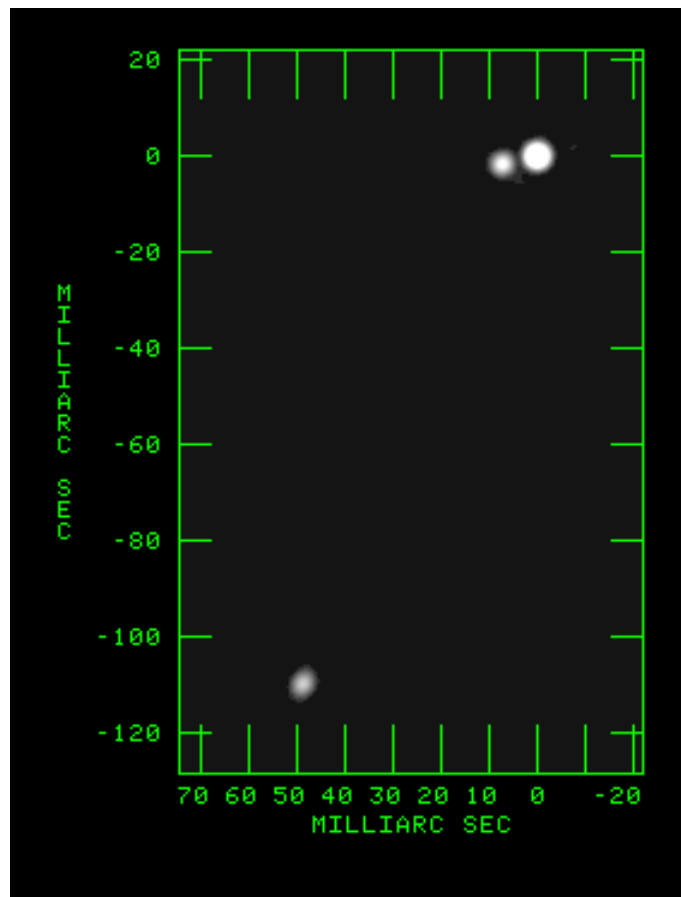
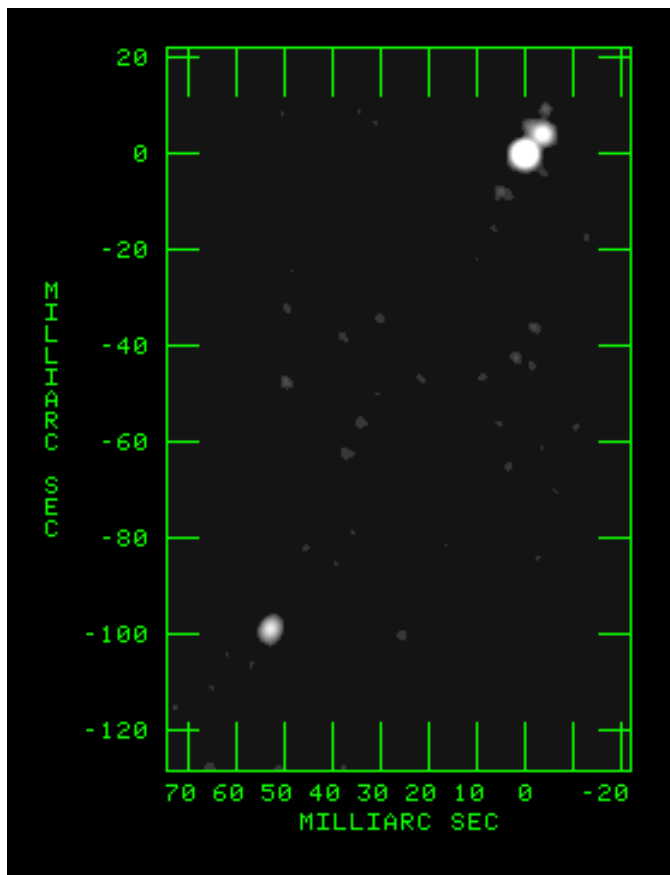
PMS models for HD 98800 B



Siess et al. 2000

Triple star

η Virginis

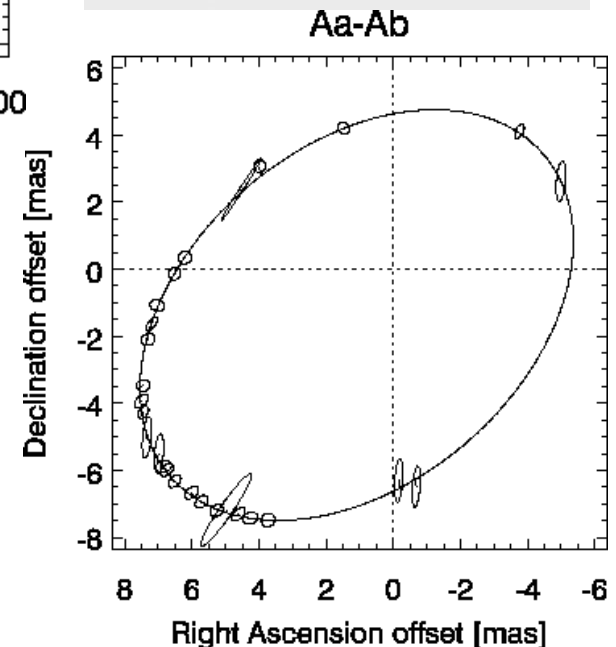
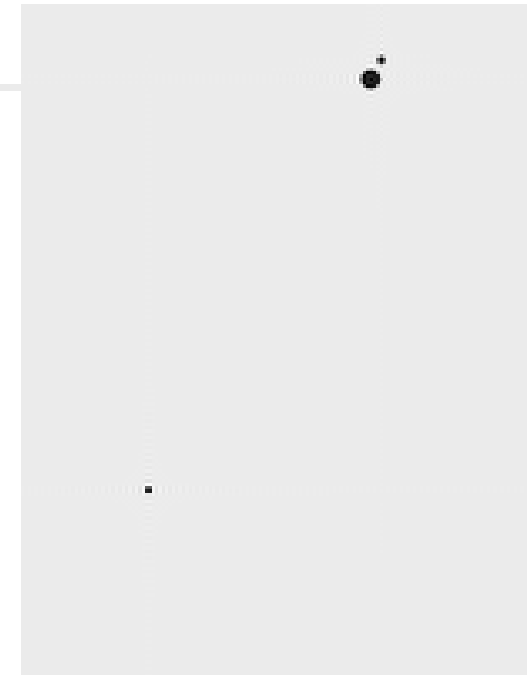
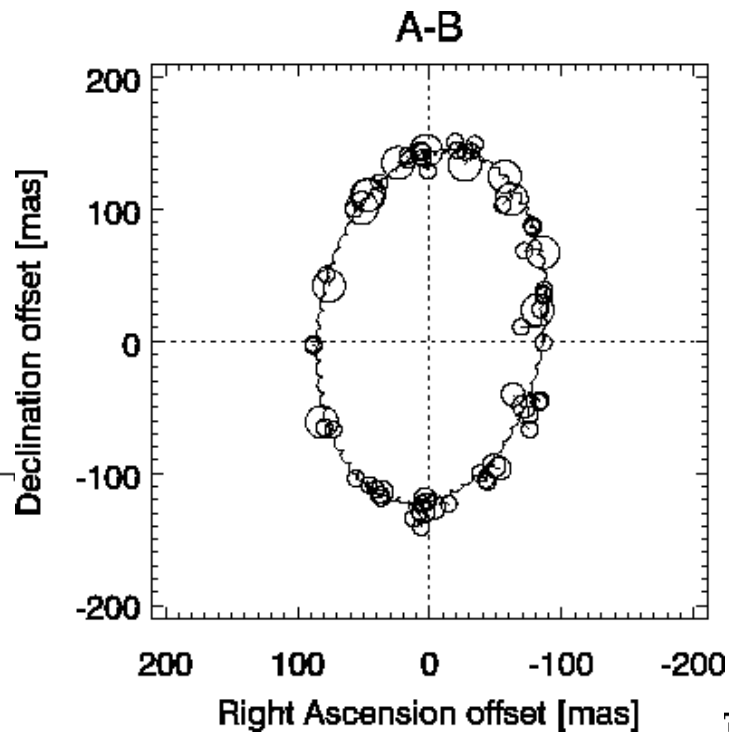
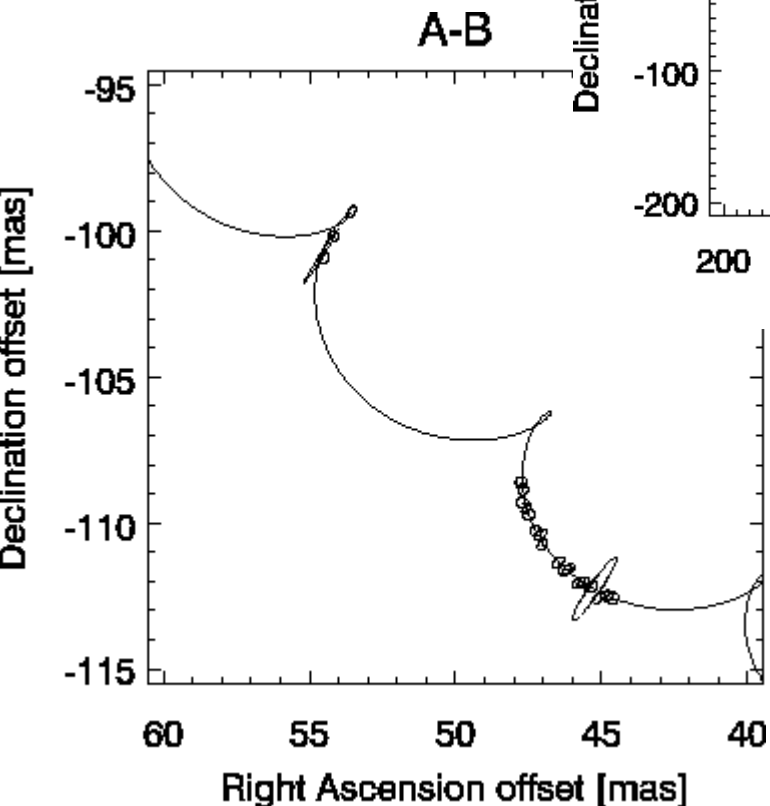


(NPOI)

Orbits in η Vir

$P = 4794$ d

$P = 71$ d



The End
