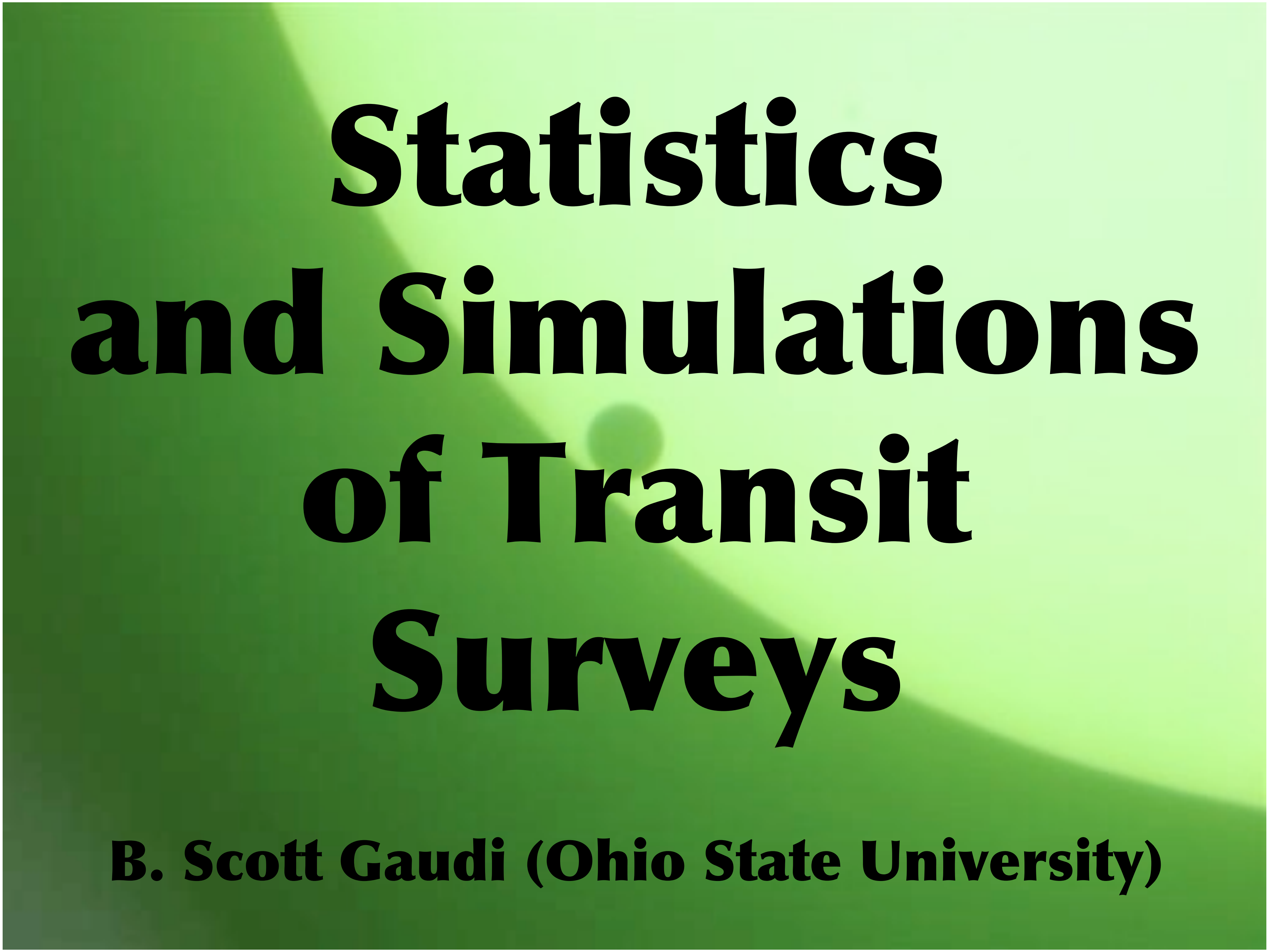


Statistics and Simulations of Transit Surveys

B. Scott Gaudi (Ohio State University)

Thanks to…

- Thomas Beatty
- Chris Burke
- Andrew Gould
- Gabriella Mallen-Ornelas
- Josh Pepper
- Frederic Pont
- Sara Seager
- Andrzej Udalski

Why Transit Surveys?

1. Find Planets!

2. Unusual/Extreme Populations

3. Statistics of Close-In Planets

Statistics of Transit Surveys

- Statistics ⇔ Predicting planet yields
- Planning surveys ⇔ Planet yields

Predicting Planet Yields

Two approaches:

- **Backward**

$$\langle N\rangle = \sum_k P_{\text{planet,k}}(M,[\text{Fe/H}],P,r,\ldots)P_{\text{transit,k}}(R,P,i)P_{\text{detect,k}}(R,F,r,P\ldots)$$

- **Forward**

$$\frac{d^n\langle N\rangle}{dMdRdPdrdld\Omega d\ldots} = \frac{dn(l)}{dMdRdL}\frac{df}{dPdr}P_{\text{transit}}(R,P,i)P_{\text{detect}}(R,F,r,P\ldots)l^2$$

- **Hybrid: Forward constrained by # of stars**

Backward Approach

$$\langle N\rangle = \sum_k \underbrace{P_{\text{planet,k}}(M,[\text{Fe/H}],P,r,\ldots)}_{\text{probability of hosting a planet}}\underbrace{P_{\text{transit,k}}(R,P,i)}_{\text{probability planet will transit}}\underbrace{P_{\text{detect,k}}(R,F,r,P\ldots)}_{\text{probability planet will be detected}}$$

- **Most Exact**

- **Difficult to implement**
  - **Requires knowledge of stellar properties**
  - **Not widely applicable**

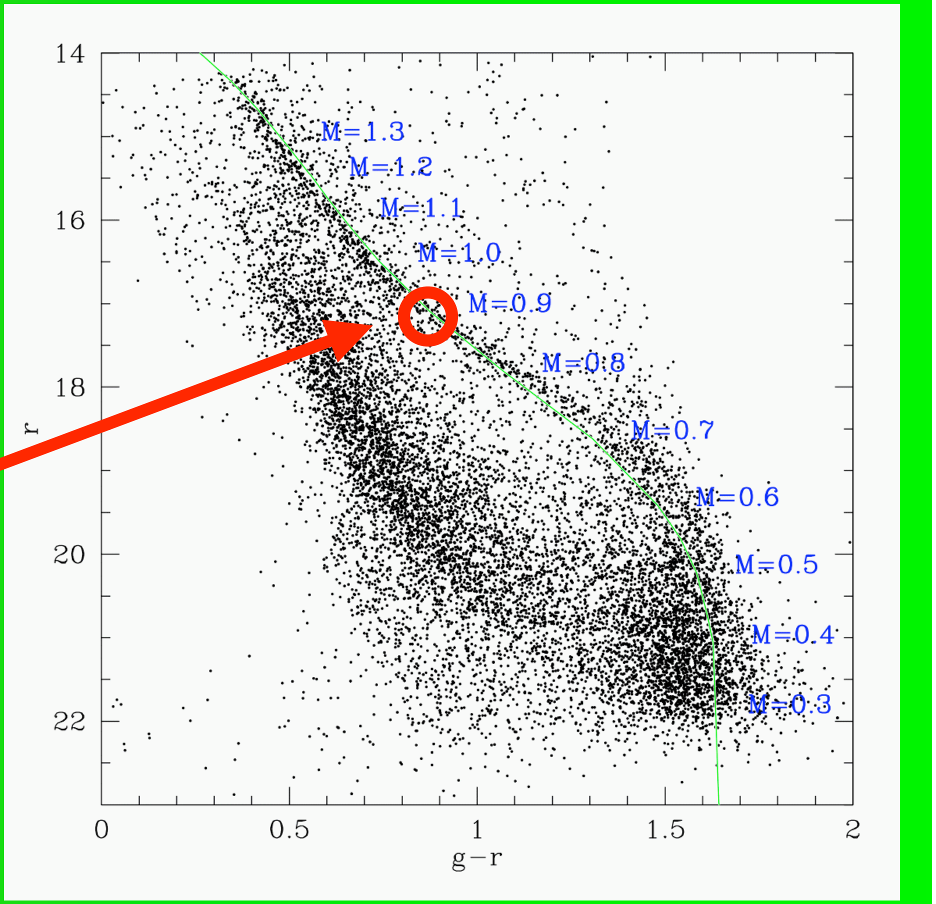
- **Only robust for stellar systems**

Yields for Stellar Systems

- **General theory**

  Janes (1996), Gaudi (2000), von Braun et al. (2005), Pepper & Gaudi (2005,2006)

- **One Parameter - Mass**



(Hartman et al., in prep)

Yields for Field Surveys

- **General theory**

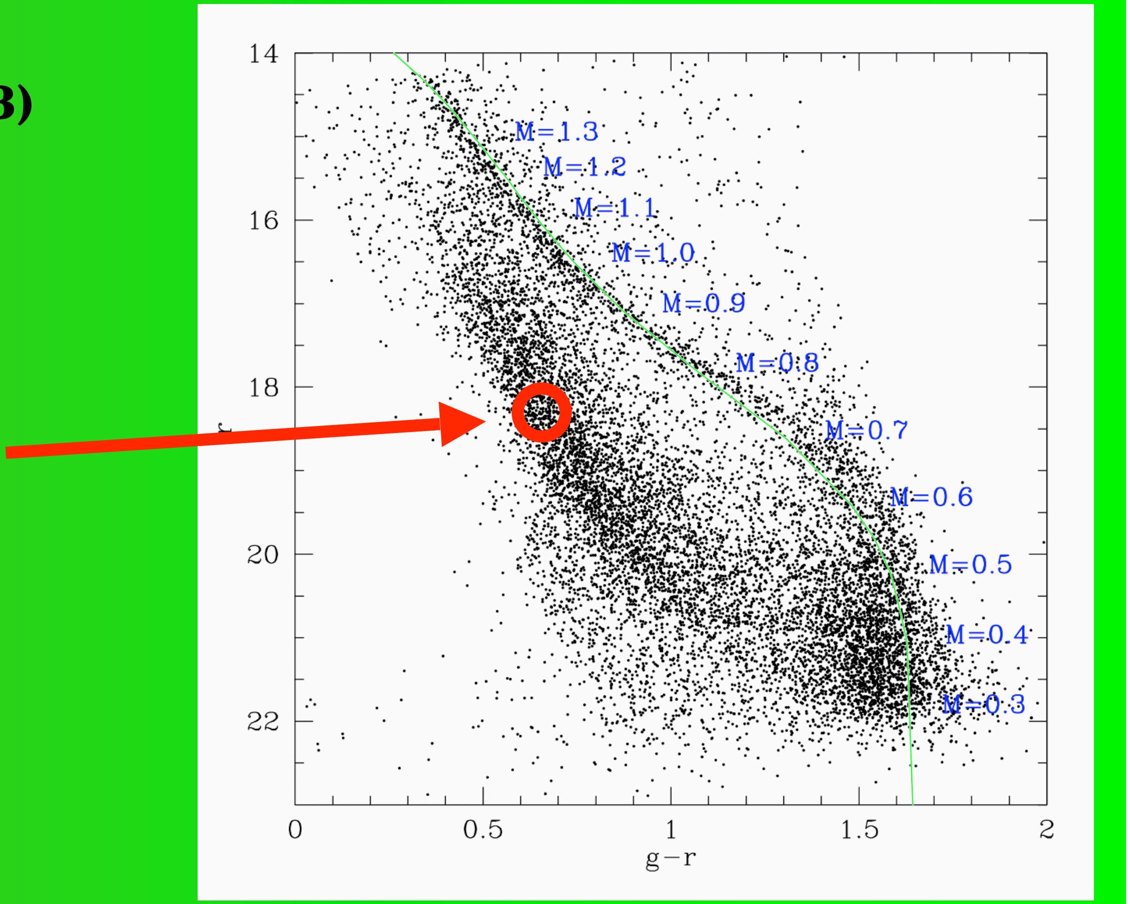
  Pepper, Gould & DePoy (2003)

- **Unknown Properties**

  (distance, reddening)

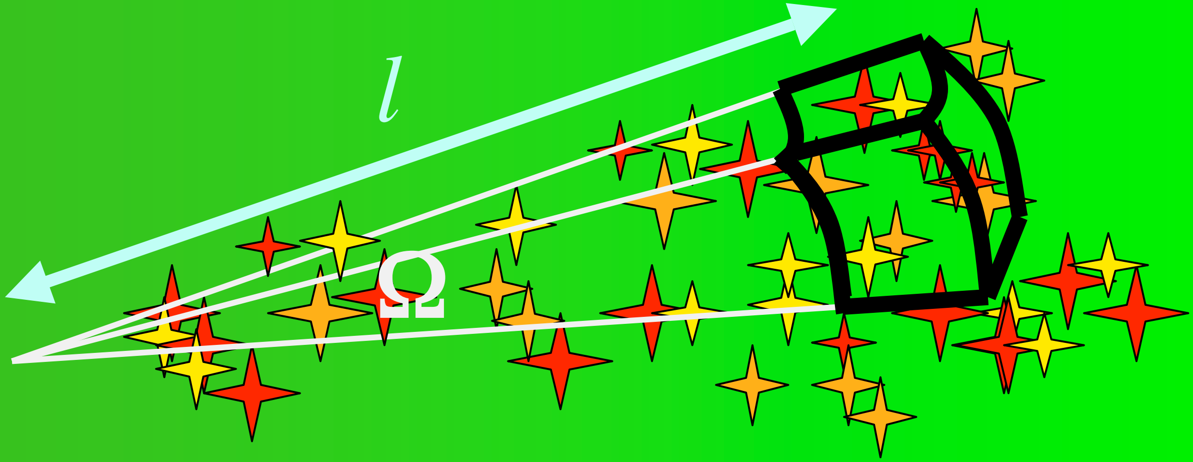
  (mass, radius)

- **Backward modeling hard**



(Hartman et al., in prep)

Forward Modeling



$$\frac{d^n\langle N\rangle}{dMdRdPdrdld\Omega d\ldots} = \underbrace{\frac{dn(l)}{dMdRdL}}_{\text{distribution of stellar properties}} \underbrace{\frac{df}{dPdr}}_{\text{distribution of planet properties}} \underbrace{P_{\text{transit}}(R,P,i)}_{\text{probability planet will transit}} \underbrace{P_{\text{detect}}(R,L,r,P\ldots)}_{\text{probability planet will be detected}} \underbrace{l^2}_{\text{volume element}}$$

Ingredients

- Distribution of stellar properties
  - Mass, radius, luminosity, metallicity dist.
  - Variation along the line of sight
- Distribution of planet properties
  - Period, radius distribution, metallicity, correlations
  - Dependence on mass?
- Transit probability
- Detection probability
  - Probability that a planet passes all cuts
  - Confirmed by high-precision RV follow-up

Detection Probability

Must account for *all* cuts consistently

- Algorithmic cuts (BLS: α, SDE)
  - Limb darkening?
  - Ingress/Egress?
- Number of transits
- By-eye selection
- Magnitude limit/RMS cut
- Color cut
- RV follow-up
- Saturation

Lessons I: Simple Estimates Fail

- **Naïve estimate:**

$$\langle N \rangle \approx f P_{transit} N_{<9} = 1\% \times 10\% \times 10^3 = 1$$

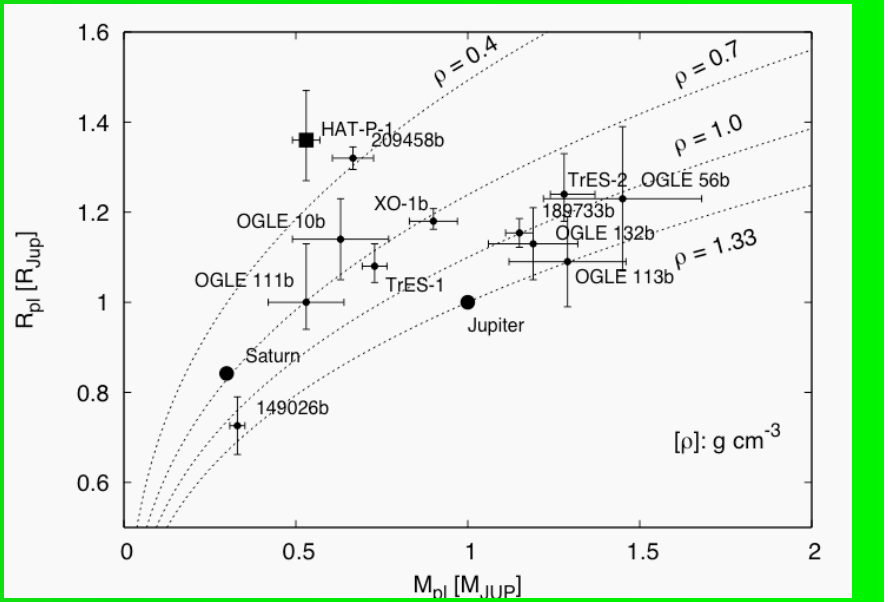
WRONG!

Actual rates… one in 10^4 or 10^5

Why does the naïve estimate fail?

- **Typical planet size**
- **Large star contamination**

  Gould & Morgan 2003, Brown 2003
- **S/N requirements**
  - **1% not necessarily sufficient**
  - **Correlated errors & systematics**
- **Transit probability varies**
- **Multiple transits & aliasing**
- **Precision RV follow-up**



(Bakos et al 2006)

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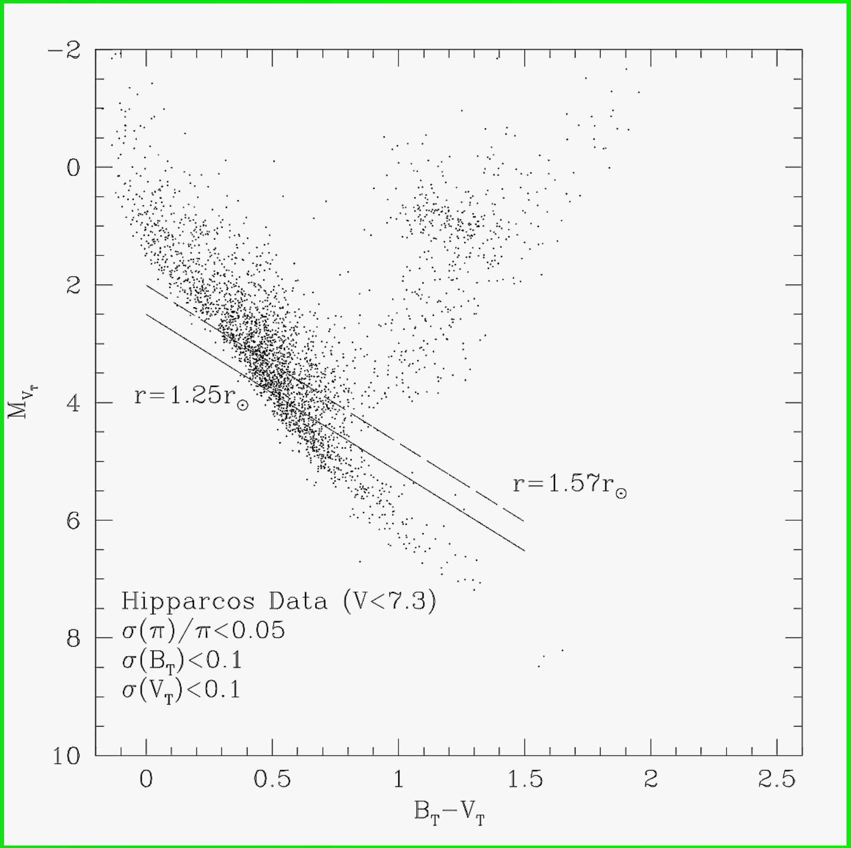

(Gould & Morgan 2003)

Why does the naïve estimate fail?

- Typical planet size
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  Gould & Morgan 2003, Brown 2003
- S/N requirements
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  - Correlated errors & systematics (Pont et al 2006)
- Transit probability varies
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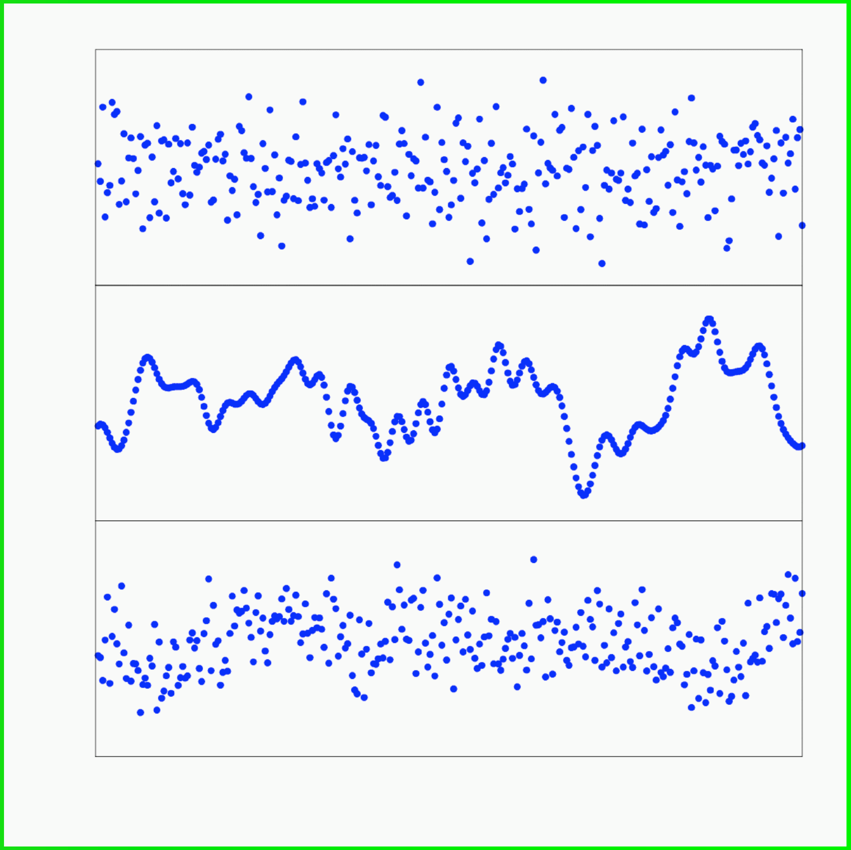

(Pont et al 2006)

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$$P_{\text{transit}} \approx \frac{R}{a} \propto M^{-1/3} R P^{-2/3}$$

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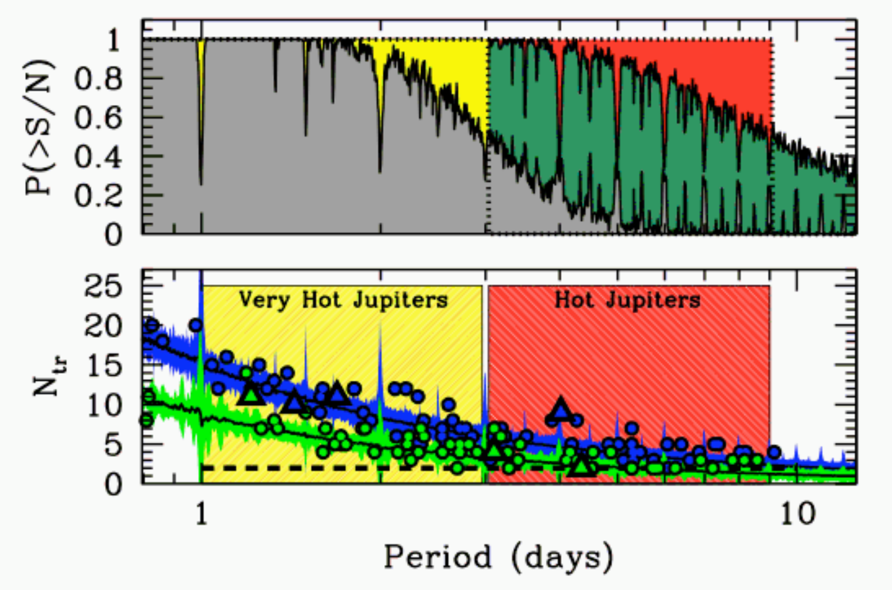

(Gaudi et al 2003)

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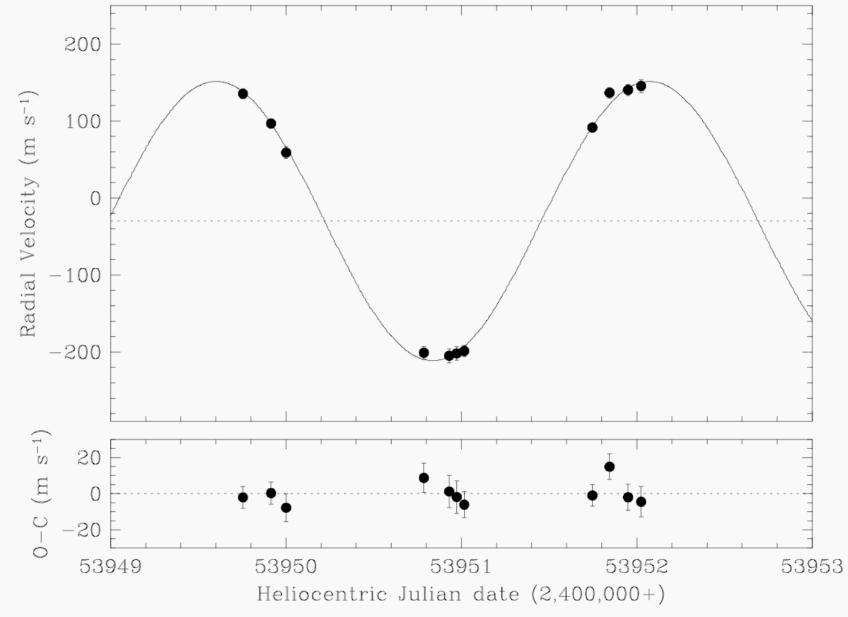
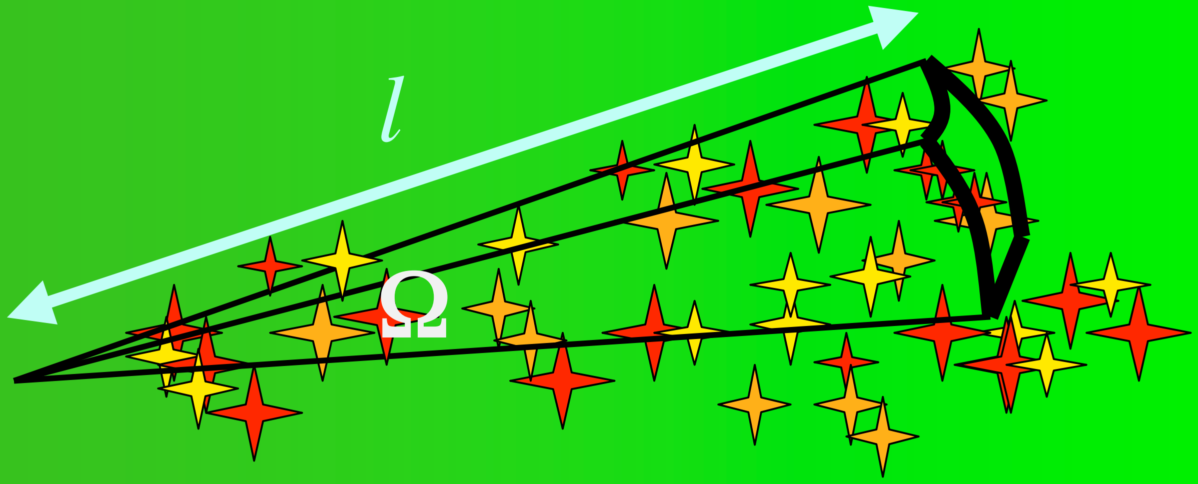


(O'Donovan et al 2006)

Lessons II: Selection Effects



$$\langle N \rangle \approx \frac{\Omega}{3} n P_{transit} l_{\max}^{3}$$

Lessons II: Selection Effects

$$\frac{S}{N} \approx N^{1/2}\frac{\delta}{\sigma}$$

(white noise)

$$N \approx \frac{R}{\pi a} N_{total}$$

$$\sigma \approx N_{photons}^{-1/2} \propto L^{-1/2} l$$

$$\delta \approx \left(\frac{r}{R}\right)^{2}$$

$$\frac{S}{N} \propto \underbrace{R^{-3/2}M^{-1/6}L^{1/2}}\underbrace{r^{2}P^{-1/3}}\underbrace{l^{-1}}$$

Lessons II: Selection Effects

$$\frac{S}{N} \propto R^{-3/2} M^{-1/6} L^{1/2} r^{2} P^{-1/3} l^{-1}$$

$$R \propto M, \quad L \propto M^{7/2}$$

$$\frac{S}{N} \propto M^{1/12} r^{2} P^{-1/3} l^{-1}$$

At fixed distance, S/N nearly independent of mass!

Lessons II: Selection Effects

$$l_{\max} \propto M^{1/12} r^{2} P^{-1/3} \left(\frac{S}{N}\right)_{\min}^{-1}$$

$$P_{\mathrm{transit}} \approx \frac{R}{a} \propto M^{-1/3} R P^{-2/3}$$

$$\langle N \rangle \approx \frac{\Omega}{3} n P_{transit} l_{\max}^{3}$$

$$\Downarrow$$

$$\boxed{\langle N \rangle \propto P^{-5/3} r^{6} \left(\frac{S}{N}\right)_{\min}^{-3}}$$

(Gaudi et al. 2003, Gaudi 2005)

Lessons II: Selection Effects

- **S/N-limited transit surveys have strong biases**
  - **Favor short periods**
  - **# of detections strong function of S/N**
  - **Overwhelmingly favor large planets**

(Gaudi et al. 2003, Gaudi 2005 , Pont et al. 2006)

$$\boxed{\langle N \rangle \propto P^{-5/3} r^{6} \left(\frac{S}{N} \right)_{\min}^{-3}}$$

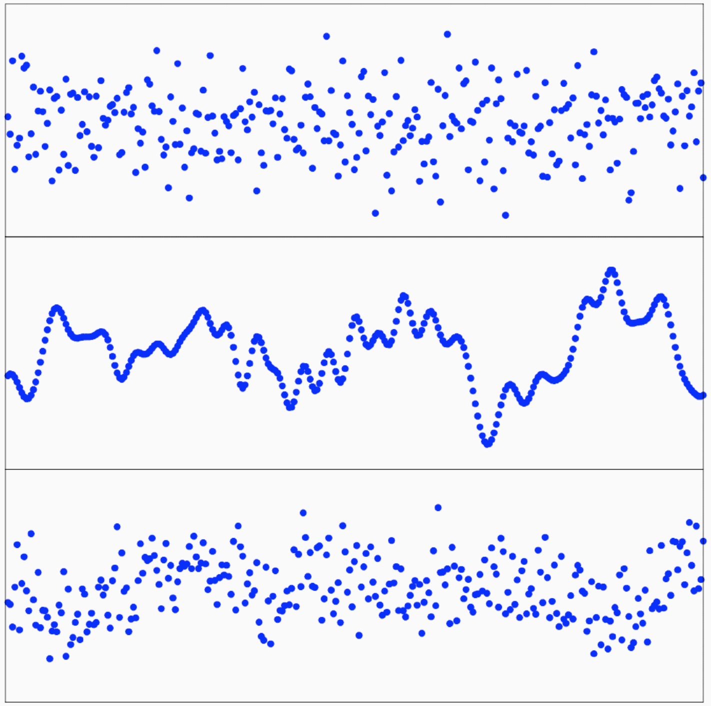
Lessons III: Correlated Noise

Most ground-based transit surveys are subject to correlated (i.e. red) noise

- **Measurements correlated on transit timescales**

- **Fundamental limit to noise**

- **Changes the statistics**



(Pont et al. 2006)

Lessons III: Correlated Noise

Relation between red and white noise

$$\left(\frac{S}{N}\right)_r \approx N_{tr}^{\;1/2}\,\frac{\delta}{\sigma_{red}}$$

$$\left(\frac{S}{N}\right)_w \approx N^{1/2}\,\frac{\delta}{\sigma}$$

$$\left(\frac{S}{N}\right)_r = \left(\frac{S}{N}\right)_w \left(\frac{\sigma}{\sigma_r}\right)\left(\frac{N}{N_{tr}}\right)^{-1/2} \approx \left(\frac{S}{N}\right)_w n_k^{-1/2}$$

Effective S/N considerably lower with correlated noise

(Pont et al. 2006)

Lessons III: Correlated Noise

$$\frac{S}{N} \approx N_{tr}^{\;1/2} \frac{\delta}{\sigma_{red}}$$

(red noise)

$$N_{tr} \overset{?}{\approx} \frac{T}{P} f$$

$$\sigma_{red} \approx \text{constant}$$

$$\delta \approx \left(\frac{r}{R}\right)^2$$

$$\frac{S}{N} \propto \underbrace{R^{-2}}_{}\,\underbrace{r^2 P^{-1/2}}_{}$$

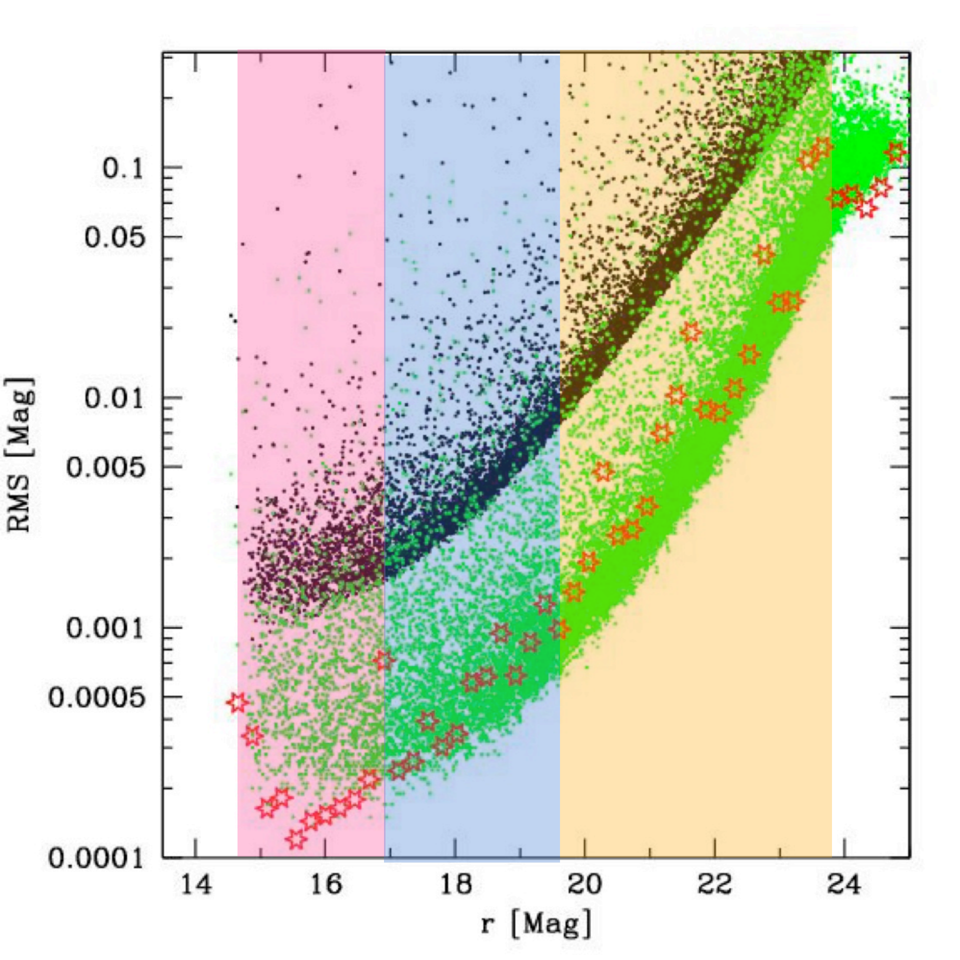
Lessons III: Correlated Noise

When correlated noise dominates:

- Effective S/N considerably lower
- Detectability doesn’t depend on stellar brightness!
- Strong (inverse) dependence on stellar mass
- No volume effect - Threshold statistics
    - Stronger aliasing effects
- Changes the optimal observing strategy

Lessons IV: All Regimes

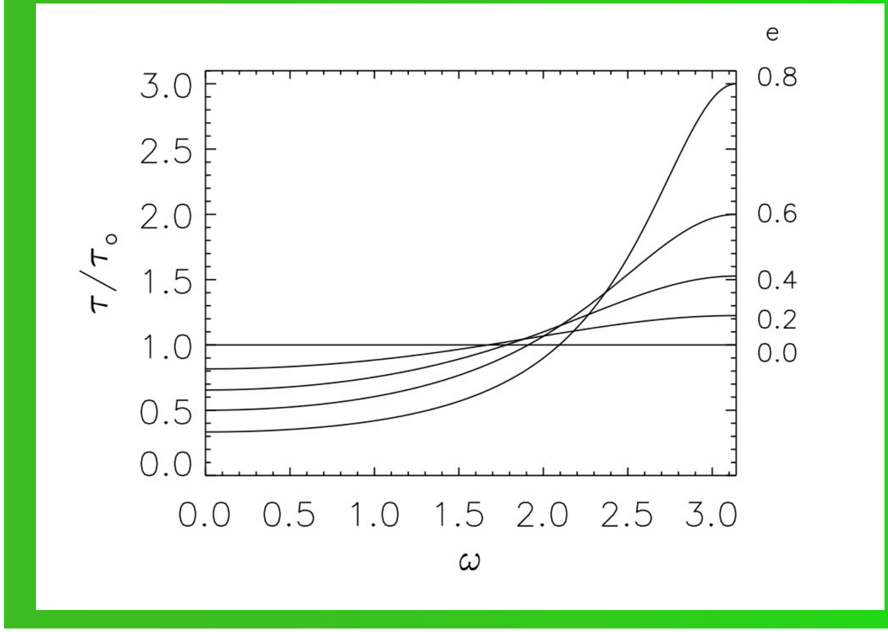
- **Correlated noise**
- **Source noise**
- **Sky noise**
  - **often provides the fundamental limit**
- **Other noise sources**
  - **scintillation**

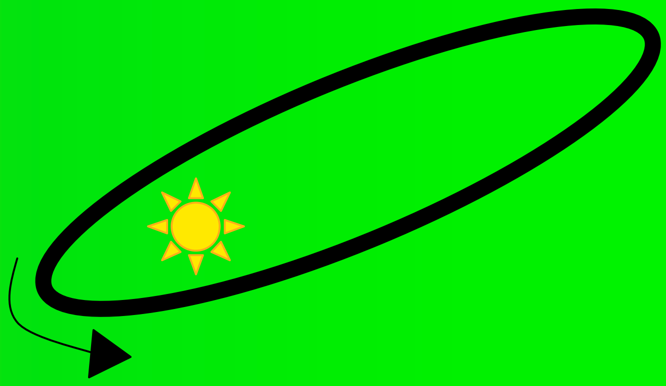


(Hartman et al. 2006)

Future Considerations: Eccentricity

- **Work by Chris Burke**
- **Eccentric orbits change detectability**
  - Changes transit duration
    - Shorter near periastron, longer near apastron
  - Changes transit probability
    - Higher transit probability near periastron





Results

Upper Limits:

- 47 Tuc
- Open Clusters
- Field

OGLE

- Detailed analysis
  - HJ Frequency = 1/310
  - VHJ Frequency = 1/690
- Comparison with RV
  - metallicity bias

Gilliland et al. 2000
Weldrake et al. 2005

Results

Upper Limits:

- 47 Tuc
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  - HJ Frequency = 1/310
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- Comparison with RV
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PISCES - Mochejska et al. (2005, 2006)

STEPSS - Burke et al. (2006)

Results

Upper Limits:

- 47 Tuc
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OGLE

- Detailed analysis
  - HJ Frequency = 1/310
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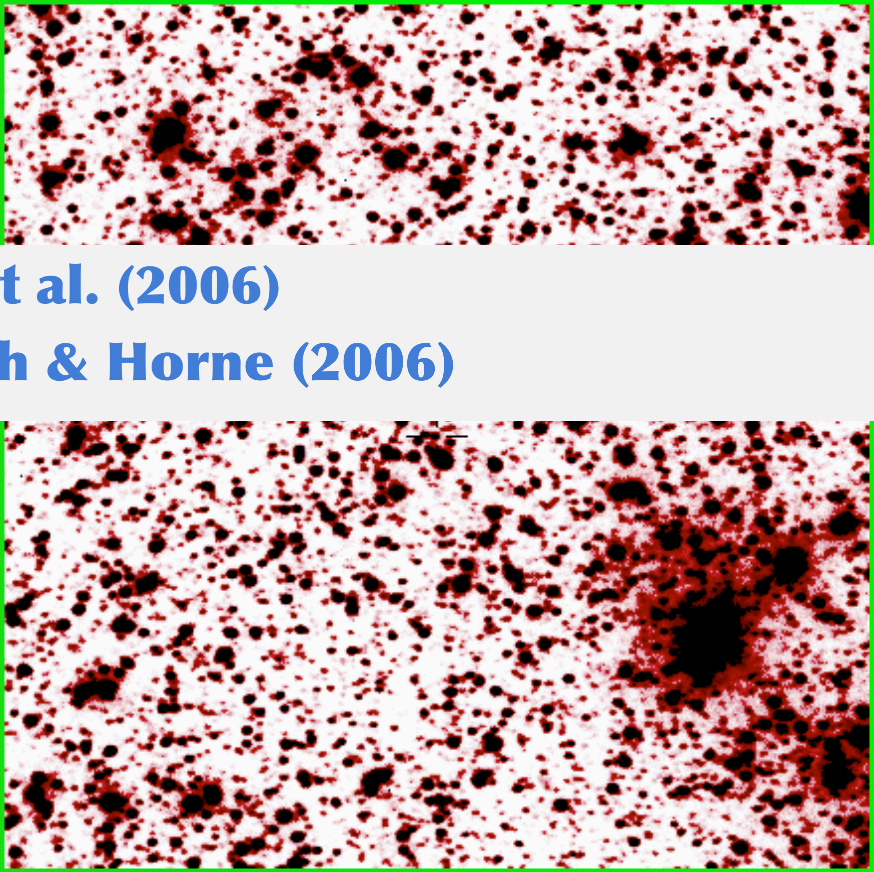


Hood et al. (2006)

Bramich & Horne (2006)

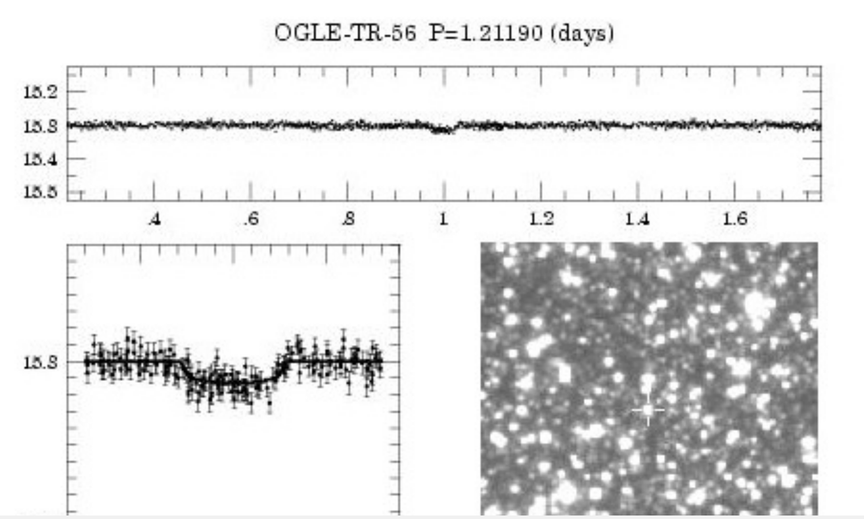
Results

Upper Limits:

- 47 Tuc
- Open Clusters
- Field

OGLE (seasons 1+2)

- Detailed analysis
  - Detailed forward model
  - Selection effects
- Comparison with RV
  - metallicity bias



Gould et al. (2006), Fressin et al. (2007)

OGLE vs RV

1-3 days

$$f_{VHJ} = \left(\frac{1}{690}\right) \times 1^{+1.10}_{-0.54}$$

$$f_{VHJ} \approx \frac{0}{1330} \leq \frac{1}{440}$$

3-5 days

$$f_{HJ} = \left(\frac{1}{310}\right) \times 1^{+1.39}_{-0.59}$$

$$f_{HJ} \approx \frac{8}{1330} \approx \frac{1}{170}$$

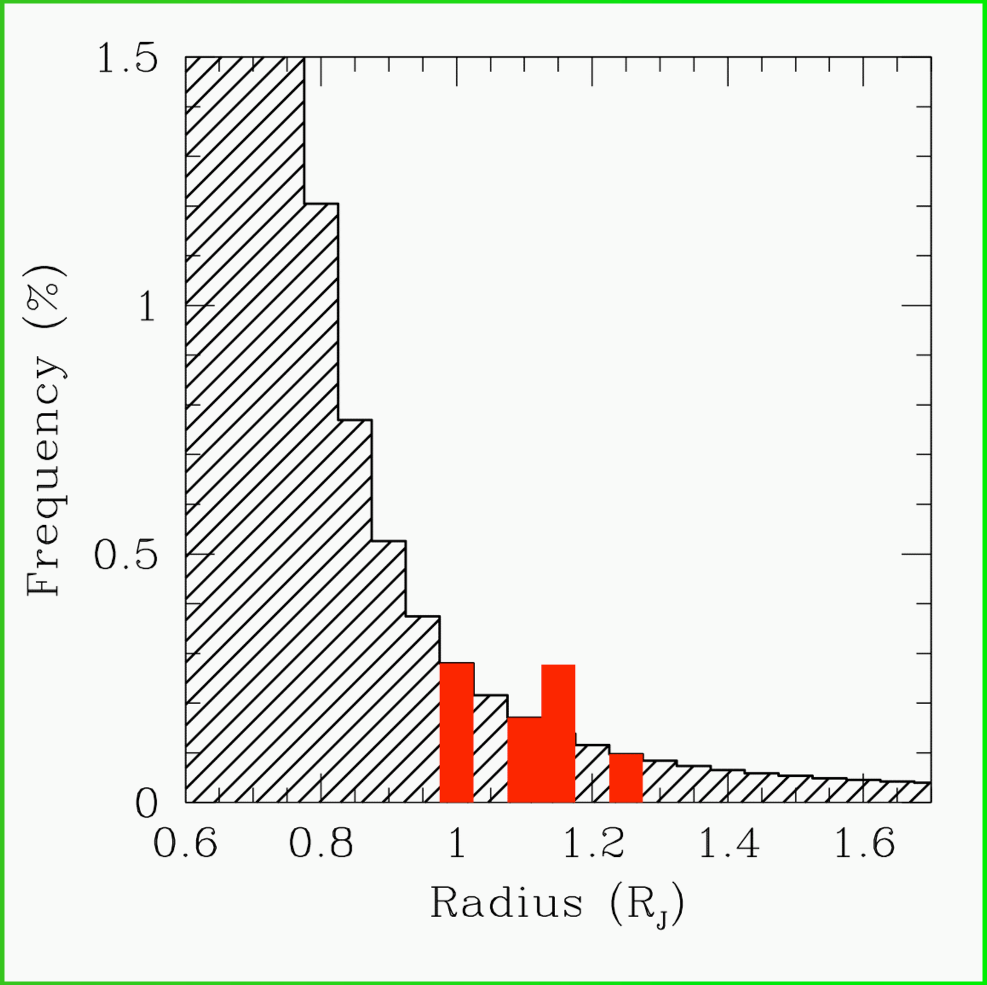
Gould et al. (2006)

OGLE & RV consistent

Metallicity bias

⇒ generically expect fewer transiting planets

Planet radii from OGLE

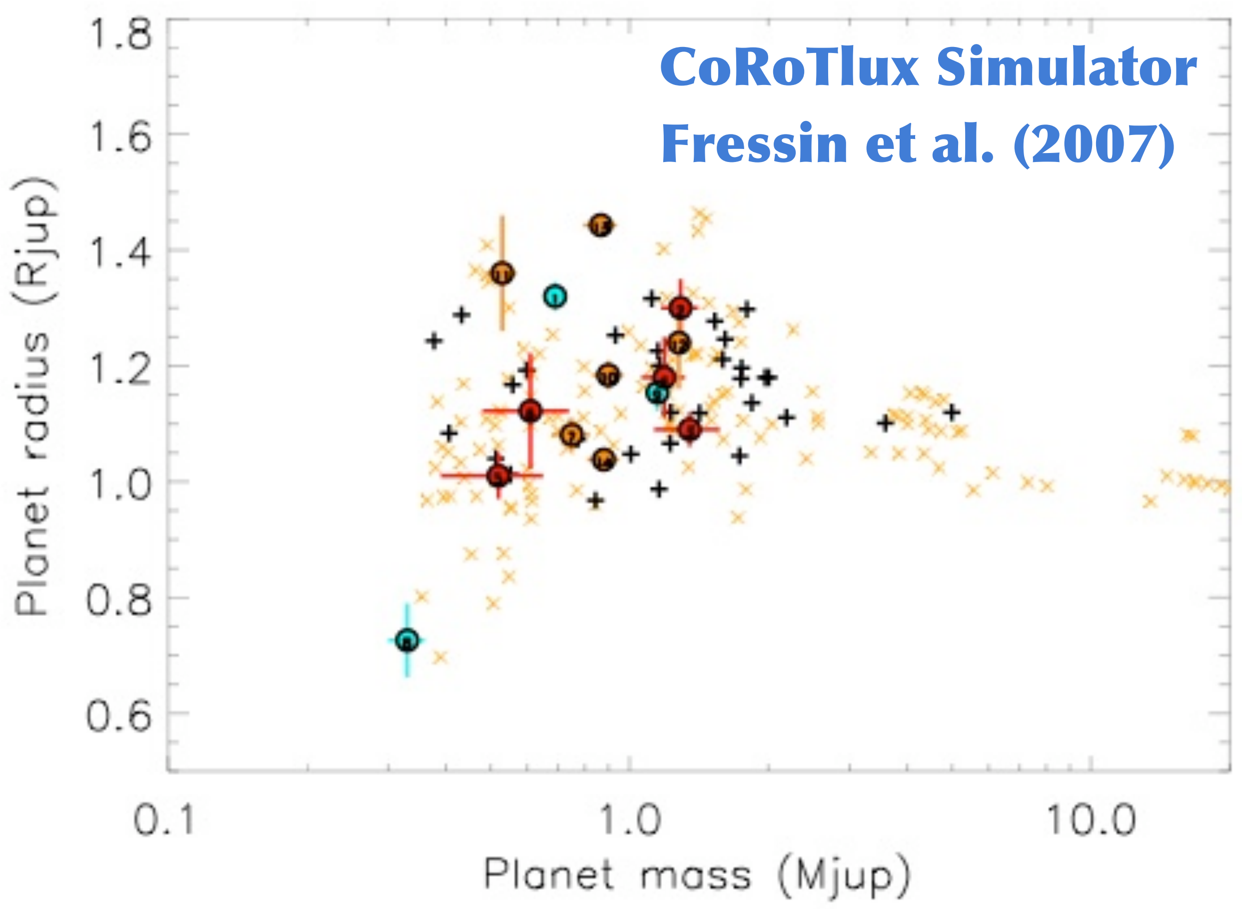


Gould et al. (2006)

Bloated planets are rare.

Weak constraints on sub-Jupiter sized planets.

Increasing sophistication



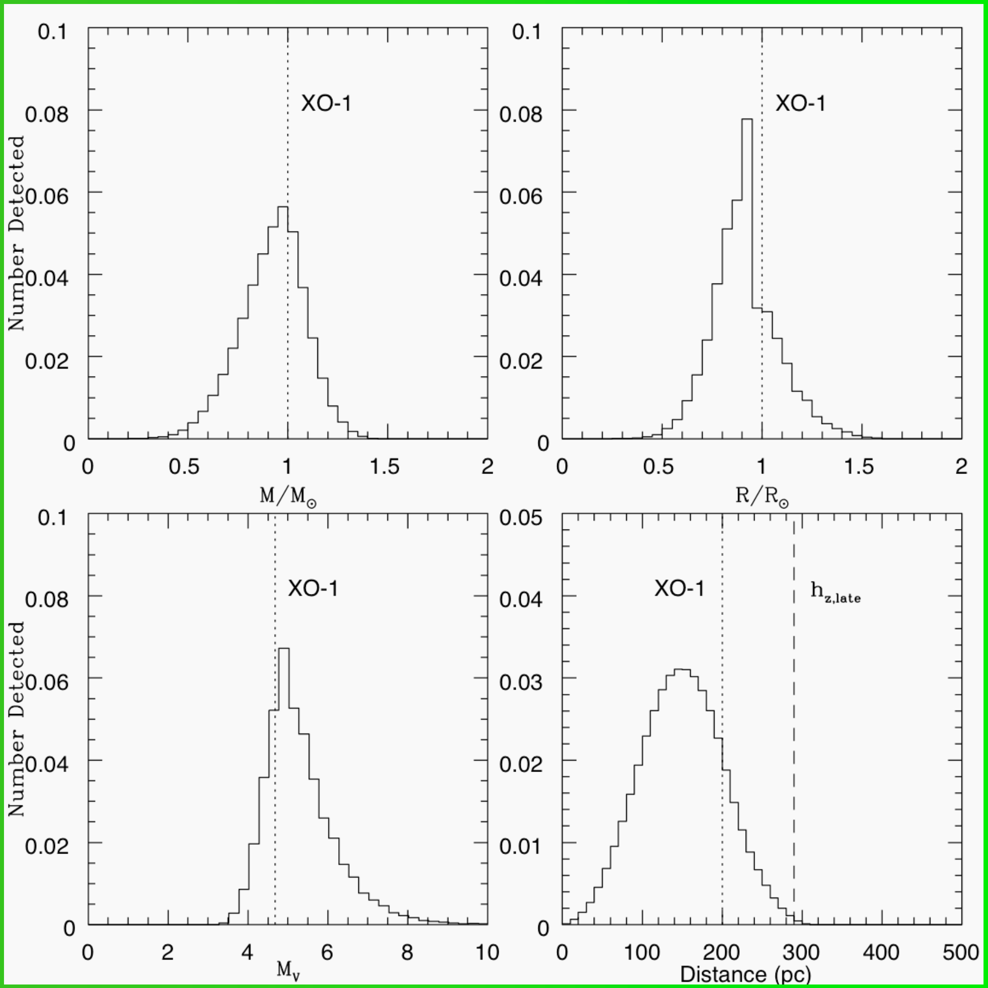
For OGLE; consistent with Gould et al.

General agreement with RV + planet models

Predictions

- **Horne (2003), Brown (2003), Pepper & Gaudi (2005), Gillon et al (2005), Fressin et al (2007)**
- **Extended model (w/ Thomas Beatty, CfA)**
  - **Galactic structure**
  - **Signal-to-noise ratio detection criteria**
  - **Noise sources (source, sky, scintillation, saturation, red & white noise)**
  - **Magnitude limit(s)**
  - **Mass, radius, effective temperature distribution**
  - **Arbitrary bandpasses**
  - **Visibility/transit requirements**
  - × **False Positives**
  - × **Blending**
  - × **Binaries**

Predictions: XO

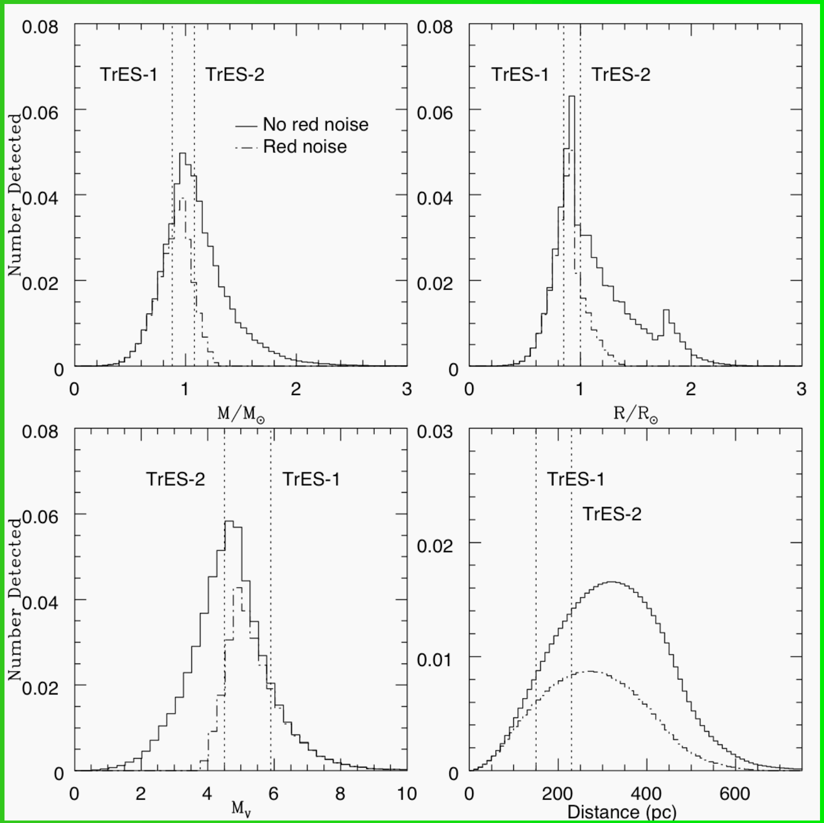


(Beatty & Gaudi, in prep)

Predicted XO Detections for V<12, S/N > 20, r=R$_J$

00 hrs	04 hrs	08 hrs	12 hrs	16 hrs	20 hrs	Total
0.45	0.47	0.45	0.37	0.42	0.51	2.67

Predictions: TrES



(Beatty & Gaudi, in prep)

Predicted TrES Detections for R<13, S/N > 20, r=R$_J$

	And0	Cyg1	Cas0	Per1	UMa0	CrB0	Lyr1	And1	And2	Tau0	UMa1	Total	Total Red $(S/N)_r>8$
VHJs	0.30	0.33	0.29	0.27	0.14	0.15	0.28	0.22	0.18	0.21	0.13	2.53	1.46
HJs	0.31	0.31	0.29	0.28	0.14	0.17	0.27	0.19	0.11	0.16	0.14	2.35	1.00
Both	0.61	0.64	0.58	0.55	0.28	0.32	0.55	0.41	0.29	0.37	0.27	4.88	2.46

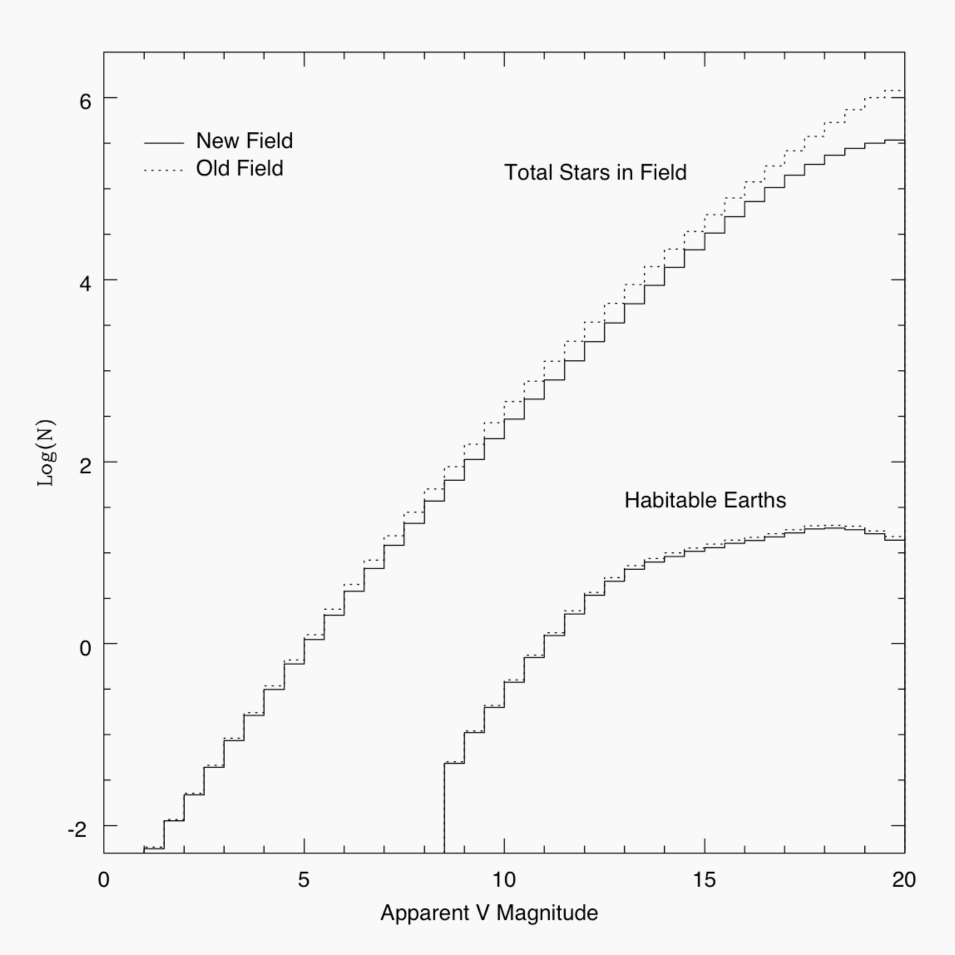
Predictions: S/N limit

	VHJ	HJ	Both
S/N>10	3.96	4.47	8.43
S/N>15	3.23	3.30	6.53
S/N>20	2.53	2.35	4.88
S/N>25	1.94	1.66	3.60
S/N>30	1.49	1.19	2.68

Predicted TrES Detections for $R<13$, $r=R_J$

Predictions: Space Surveys

- **Kepler**
  - HJ+VHJ ~ 54
  - Total # J ~ 80
  - Most fairly faint
  - TrES-2 likely only one with V<12
  - 50 Habitable Earths
- **CoRoT**
  - May find as many or more Jupiters



(Beatty & Gaudi, in prep)

Summary

- Interpretation and predictions require accurate simulations
- Naïve estimates fail.
- Strong selection effects.
- Correlated noise important
- OGLE surveys consistent with RV
- Current surveys require careful modelling
- Must include all selection cuts consistently
- XO & TrES predicted to have handfuls of detections