## **High-Contrast Instruments (Theory)**

N. Jeremy Kasdin Princeton University

Friday, July 25, 14

# Imaging a Star–A Simple Ray Optics Description



## Imaging a Star–A Simple Ray Optics Description



### Star and Planet



## However, we need to include diffraction



$$E_0(x,y) = \frac{1}{j\lambda} \int \int_{\Sigma} E_1(\xi,\eta) \frac{\exp(jkr_{01})}{r_{01}} \cos\theta d\xi d\eta$$
$$r_{01} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$$
The Huygens-Fresnel Principle

See Goodman, Introduction to Fourier Optics

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#### The Huygens-Fresnel Principle

# This is too complicated so we approximate in different regimes.

See Goodman, Introduction to Fourier Optics

## Important approximations ...

### S-Huygens (very near field)

 $+ x^{2} + y$ 

### Fresnel Number

$$E(x,y) = \frac{e^{jkS}}{j\lambda z} \int \int_{-\infty}^{\infty} E(\xi,\eta) e^{j\frac{k}{2S} \left[ (x-\xi)^2 + (y-\eta)^2 \right]} d\xi d\eta$$

$$\frac{R^2}{\lambda S} \sim \mathcal{O}(1)$$

Fresnel (near field)

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$$\frac{R^2}{\lambda z} \sim \mathcal{O}(1) \quad x, y \ll z$$

### Fraunhoffer (far field)

$$E(x,y) = \frac{e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z} \int \int_{-\infty}^{\infty} E(\xi,\eta)e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi d\eta \qquad \qquad \frac{R^2}{\lambda z} \ll 1$$

#### Fourier Transform

Or a focusing lens

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Or a focusing lens

## **Star & Planet with Diffraction**





## **Star & Planet with Diffraction**





## **Star & Planet with Diffraction**







## Resolution

### Two Meter Telescope

#### Ten Meter Telescope



Even a two-meter telescope can resolve a planet at 1 AU about closest stars.

## **The Problem is Contrast**



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Traub & Jucks

## **High-Contrast Imaging**

To image the planet with a ground or space telescope there are five important metrics:

- **Contrast:** The ratio of the peak of the stellar point spread function to the halo at the planet location.
- Inner Working Angle: The smallest angle on the sky at which the needed contrast is achieved and the planet is reduced by no more than 50% relative to other angles.
- Throughput: The ratio of the open telescope area remaining after high-contrast is achieved.
- **Bandwidth**: The wavelengths at which high contrast is achieved.
- Sensitivity: The degree to which contrast is degraded in the presence of aberations.

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- Throughput: The ratio of the fraction of light in the central core of the PSF to the same fraction in an Airy function.
- **Bandwidth**: The wavelengths at which high contrast is achieved.
- Sensitivity: The degree to which contrast is degraded in the presence of aberations.

In the remainder of this talk I will describe how we achieve highcontrast using a coronagraph.

Later, Aki Roberge will describe how it is done using a Starshade.

Tomorrow, Laurent Pueyo will describe how the planet is extracted from the image.

Coronagraphy

Modify the optical path of the telescope to reduce the stellar halo in the planet "discovery zone" (increase contrast) while allowing sufficient planet light to transmit through.

## The "Lyot Coronagraph"



## The "Lyot Coronagraph"



## **Bandlimited Lyot**

#### Classical Lyot (Gaussian)



#### **Bandlimited Lyot**

Kuchner & Traub (2002)

## **Bandlimited Lyot**

#### Classical Lyot (Gaussian)



The Final Field L(u) (M(u) \* A(u))

Aperture

A(u)

M(u)

M(u) \* A(u)

Lyot Stop

L(u)

#### Kuchner & Traub (2002)

**Bandlimited Lyot** 



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## **A General Picture**

Coronagraph is a linear operator



## **A General Picture**

Coronagraph is a linear operator



## **A General Picture**

Coronagraph is a linear operator



## Instrument Contrast (on-axis behavior)



The Instrument Contrast Ratio (at a specific wavelength)

$$C_{i} = \frac{\int_{\Delta\Omega} P_{c}(\omega)d\omega}{\Delta\Omega P_{o}(0)} = \frac{\int_{S} |\mathcal{A}_{c}(x)|^{2}dx}{\Delta\Omega A_{o}^{2}} \left[1 - \frac{\int_{\Delta C} P_{c}(\omega)d\omega}{\int_{-\infty}^{\infty} P_{c}(\omega)d\omega}\right]$$

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#### Reduce the exit amplitude

## Instrument Contrast (on-axis behavior)



The Instrument Contrast Ratio (at a specific wavelength)

$$C_{i} = \frac{\int_{\Delta\Omega} P_{c}(\omega) d\omega}{\Delta\Omega P_{o}(0)} = \frac{\int_{S} |\mathcal{A}_{c}(x)|^{2} dx}{\Delta\Omega A_{o}^{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{\int_{\Delta C} P_c(\omega) d\omega}{\int_{-\infty}^{\infty} P_c(\omega) d\omega}$$

#### Reduce the exit amplitude

Shift the energy (uncertainty principal)

## **Coronagraph Families**

- Lyot & Bandlimited Lyot (Gemini, Keck, Hubble, Subaru, Palomar, VLT, JWST NICI, AFTA)
- 4 Quadrant Phase Mask (JWST MIRI, VLT, LBT)
- Optical Vortex (Palomar, VLT, LBT)
- AIC, VNC and other nullers

- Apodized pupils (VLT)
- Shaped pupils (SPICA, Subaru, AFTA)
- Pupil remappers (PIAA) (Subaru)
- Apodized phase plate (MMT, Magellan, VLT)



APLC (GPI, VLT/SPHERE, Palomar)



## **Coronagraphs That Change Amplitude**

Focal Plane Amplitude Mask: Lyot & Bandlimited Lyot, AIC

#### Focal Plane Phase Mask: 4QPM, Vector Vortex



## **Coronagraphs That Reshape PSF**

Pupil Plane Amplitude Mask: Shaped Pupils, PIAA

Pupil Plane Phase Mask: APP

Slepian, D., "Analytic Solution of Two Apodization Problems", September, 1965

#### **Pupil Apodization**



**Point Spread Function** 

The "optimal" apodization that maximally concentrates light is the Prolate Spheroidal Wavefunction, based on finite uncertainty principle.



#### Shaped pupil contrast independent of wavelength.



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## Shaped Pupil Zoo (1D)



#### Pupils designed via optimization under certain constraints
#### **Direct 2 D Optimization of SPs**





<u>JWST</u> 45% Throughput 5 to 15 lambda/D 10<sup>-5</sup> Contrast

#### First Lab Test of 2D SP at Princeton



A. Carlotti, E. Young, G. Che

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#### Manufacturing 2D-Optimized Shaped Pupils

- Recent breakthrough: Reflective SPs (RSPs)
  - Silicon wafers with absorptive (black silicon) and reflective regions





Testing 1<sup>st</sup> black Si masks now in HCIL.







#### Shaped Pupil Coronagraph for WFIRST-AFTA



#### Intensity in First Focal Plane



#### Contrast in Final Image (10-8)



# **Pupil Mapping (PIAA)**



Nearly 100% throughput 100% search area small (<2 lambda/ d) Inner Working Angle

Guyon (2003), Vanderbei & Traub (2003, 2005)

Pupil Mapping for Apodization

#### Apodizing Phase Plates (Codona, Kenworthy)





Installed in 2010 for NaCo L' band imaging

One sided discovery zone



#### 2 sided: with 0/π masks (Carlotti) ... or with quarter wave plates & Wollaston prism (Snik)







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#### How is Phase used to Change Amplitude?

# **Coronagraphs That Combine Both**

# Combine apodized pupil with focal plane mask and Lyot stop: APLC, SPLC, ACAD

### **Apodized Pupil Lyot Coronagraph**

#### Soummer et al. 2005, 2009, 2011





GPI design: contrast > 1e7 at 5  $\lambda$ /D with central obstruction and 20% bandpass

### **Apodized Pupil Lyot Coronagraph**

- Generalized prolate spheroidal apodizers exist for any aperture geometry and focal mask diameter
- Quasi-Achromatic Solutions exist for large enough mask (e.g. with GPI with 5.6 lambda/D mask *diameter*)





Soummer et al. 2011

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### **Shaped Pupil Lyot Coronagraph**



Simultaneously optimize pupil and Lyot plane

Gains smaller iwa and more throughput

from Neil Zimmerman



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# Phase mask coronagraphs with on-axis telescopes (Carlotti, Mawet, Pueyo); here w/ 4QPM; ask D.Mawet for Vortex.

obscuration & spiders limit high-contrast ; apodizer can retrieve it



#### Shaped Pupil Lyot Coronagraph for WFIRST-AFTA



Intensity in First Focal Plane

#### Intensity in Lyot Plane

#### Contrast in Final Image (10-8)



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### Instrument Performance (off-axis behavior)



 $P_o(\omega) = \left| \mathcal{F} \left\{ \mathcal{A}_o \right\} \right|^2$ 

**Off-Axis Point Spread Function** 

#### <u>Metrics</u>

- Throughput (and iwa)
- Sharpness

• 
$$\tilde{Q} = Q \sum_{\Delta S} \bar{P}_{ij}$$

Q is the ratio of the planet flux at the center of the PSF to the background flux there.

$$Q = \frac{C}{C_i + C_{eq}}$$

Detection Time (Kasdin et al. 2006)

$$t_d = \frac{1}{\beta} \frac{\left(K - \gamma \sqrt{1 + \frac{\tilde{Q} \Xi_{\Delta S}}{\Psi_{\Delta S}}}\right)^2}{T_R \tilde{Q} \Psi_{\Delta S}}$$

$$\beta = \epsilon \eta^2 \Delta \lambda I_p A(T_A)_{airy}$$

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**Off-Axis Point Spread Function** 

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$$\tilde{Q} = Q \sum_{\Delta S} \bar{P}_{ij}$$

Q is the ratio of the planet flux at the center of the PSF to the background flux there.

$$Q = \frac{C}{C_i + C_{eq}}$$

Detection Time (Kasdin et al. 2006)

$$t_s = \frac{(SNR)^2 \left(\Xi \tilde{Q} + \Psi_{\Delta S}\right)}{\beta Q \Psi_{\Delta S}}$$

$$\beta = \epsilon \eta^2 \Delta \lambda I_p A(T_A)_{airy}$$

# Throughput

There are four possible measures of throughput often quoted: Total Throughput:

$$T = \frac{\int \int_{-\infty}^{\infty} P_o(u, v) du dv}{\int \int_{-\infty}^{\infty} P(u, v) du dv} = \frac{\int \int_{-\infty}^{\infty} |\mathcal{A}_o(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |\mathcal{A}(x, y)|^2 dx dy} = \frac{\tilde{A}_o}{A} \quad \text{For binary pupils} = \frac{A_o}{A}$$

#### <u>Airy Throughput:</u>

$$T_A = T \frac{\int \int_{\Delta S} P_o(u, v) du dv}{\int \int_{-\infty}^{\infty} P_o(u, v) du dv} = \frac{\int \int_{\Delta S} P_o(u, v) du dv}{\int \int_{-\infty}^{\infty} P(u, v) du dv} = \frac{\int \int_{\Delta S} P_o(u, v) du dv}{A}$$

#### Useful Throughput (Guyon, et al. 2006):

Maximum fraction of planet light that can be separated from starlight.

#### Effective Throughput:

$$T_R = \frac{\int \int_{\Delta S} P_o(u, v) du dv}{\int \int_{\Delta S} P(u, v) du dv} = \frac{T_A}{(T_A)_{airy}}$$

Note:  $P_o$  is a function of angle in image plane.

# Inner Working Angle

Where the effective throughput drops by 50%

(Maybe where  $t_d$  doubles to allow for sharpness change?)



# Sharpness

$$\Psi_{\Delta S} = \frac{\sum_{ij} \bar{P}_{ij}^2}{(\sum_{ij} \bar{P}_{ij})^2}$$

Note that sharpness is a strong function of the PSF sampling.

Critically sampled Sharpness

•Airy = 0.12 •Prolate = 0.08 •Lyot = 0.06



### Wavefront Aberrations

Atmospheric distortions and imperfect optics degrade contrast



Aberrations significantly degrade contrast: 10<sup>10</sup> ~10<sup>5</sup>

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### **Wavefront Aberrations**

Atmospheric distortions and imperfect optics degrade contrast



Remember: Different coronagraphs have different sensitivities to various orders of aberrations.



Aberrations significantly degrade contrast: 10<sup>10</sup> ~10<sup>5</sup>

#### Typical Ground Adaptive Optics Phase Conjugation



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#### Typical Ground Adaptive Optics Phase Conjugation



Planet imaging requires "Extreme Adaptive Optics" with Shigh format DMs to correct mid-spatial frequencies.



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# **Deformable Mirrors**



Xinetics Electrorestrictive MEMS Deformable Mirror (BMC)

Continuous facesheet FOV determined by number of actuators Model surface as linear sum of basis functions Usually influence function as basis function Measured response from a single poked actuator Approximately Gaussian shape

# Direct Images of HR8799 with AO



Marois, Macintosh, et al. (2008)



#### Hale/Palomar (vortex)

Serabyn, et al. (2008)

Gemini/NICI (Lyot)

The planets were later "discovered" in older HST images without AO.

# Next Generation of Extreme AO on ground GPI, SPHERE, SCExAO+CHARIS



Gemini Planet Imager

APLC coronagraph with 4000 actuator MEMS DM.

# Next Generation of Extreme AO on ground GPI, SPHERE, SCExAO+CHARIS



# **Beyond Extreme AO**

On ground, aberrations are predominantly phase.

For very high contrast in space, need to worry about noncommon path error and amplitude errors. Limit contrast to 1e-5 to 1e-7.

Solution: Focal Plane Wavefront Sensing and Control with two Deformable Mirrors

# **Beyond Extreme AO**

On ground, aberrations are predominantly phase.

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Solution: Focal Plane Wavefront Sensing and Control with two Deformable Mirrors

Today, "coronagraph" refers to both the optical design and the wavefront control system!

# **Focal Plane Wavefront Sensing & Control**



Need to estimate complex field from only intensity

#### **Control Algorithms:**

Speckle Nulling (Brown & Burrows) Energy Minimization (Malbet & Shao) Electric Field Conjugation (Giveon) Stroke Minimization (Pueyo) Estimation Algorithms: DM Diversity (Borde & Traub, Belikov) Gerchberg-Saxton (Kay) Kalman Filtering (Groff)

# **Single DM Control**

Because controlling amplitude, only single-sided dark hole.



Shaped Pupil •  $4-10 \lambda/D$ 

- 10% bandpass
- 2.4 x 10<sup>-9</sup> contrast [Belikov et al. 2007]



Band-Limited Lyot •  $4-10 \lambda/D$ 

- 10% bandpass
- 6.4 x 10<sup>-10</sup> contrast [Moody et al. 2008]



PIAA
2-3.4 λ/D
monochromatic
1.9 x 10<sup>-8</sup> contrast

[Belikov et al. 2011]

Because using phase to amplitude conversion, controller is chromatic and bandwidths limited. OWA determined by # of actuators.

#### **Dual DM Control** First test at JPL HCIT in August, 2013 (monochromatic).



IWA =  $5 \lambda/D$ OWA =  $9 \lambda/D$  $3.6 \times 10^{-9}$  contrast

Riggs, et al. (2013)

#### High-Contrast Imaging Laboratory, Princeton University

#### Future? Hybridizing coronagraph with DMs to generate contrast



#### **DM** Setting

**Shaped Pupil** 

#### **One-Sided Dark Hole**

- Contrast: 5x10<sup>-9</sup>
- Transmission: 61%
- Stroke: 0.91 \lambda
- IWA: 4 \lambda/D
- OWA: 22 \lambda/D

Riggs, et al. (2014)

# Hybrid Lyot Coronagraph



#### **Baseline design for WFIRST/AFTA**

From John Trauger, JPL

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