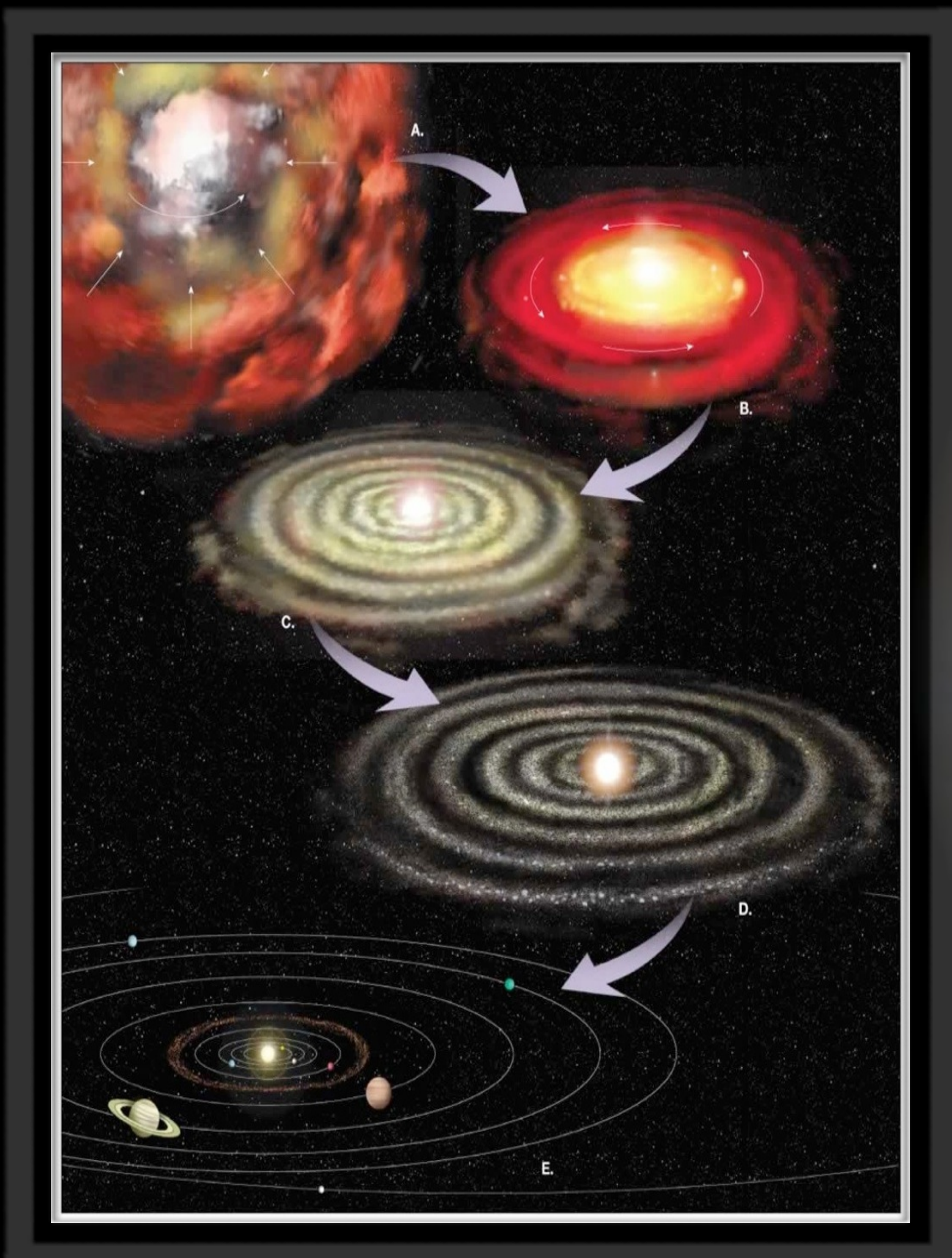
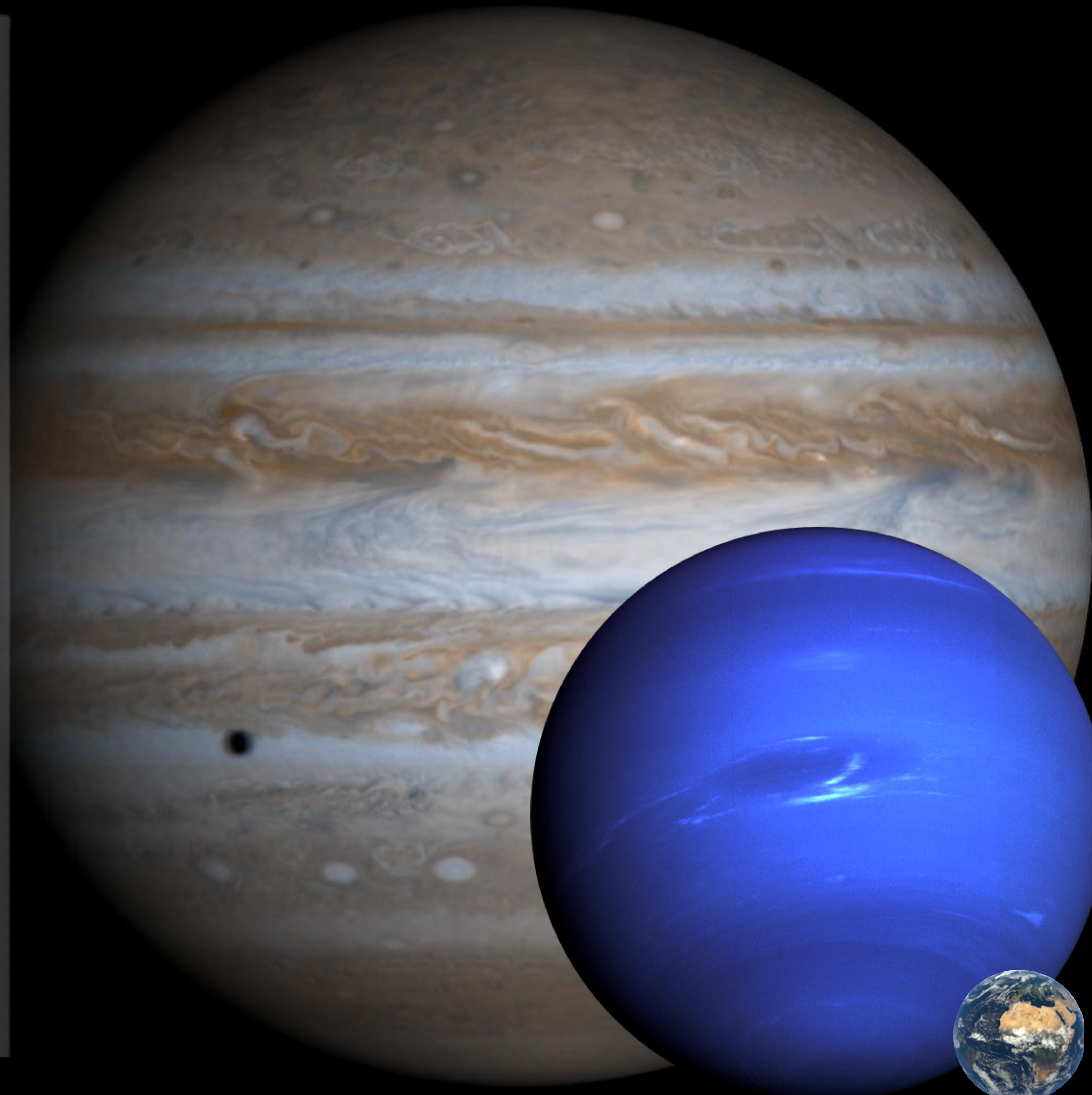
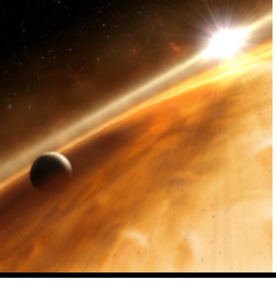
# *Hands-on Session I*



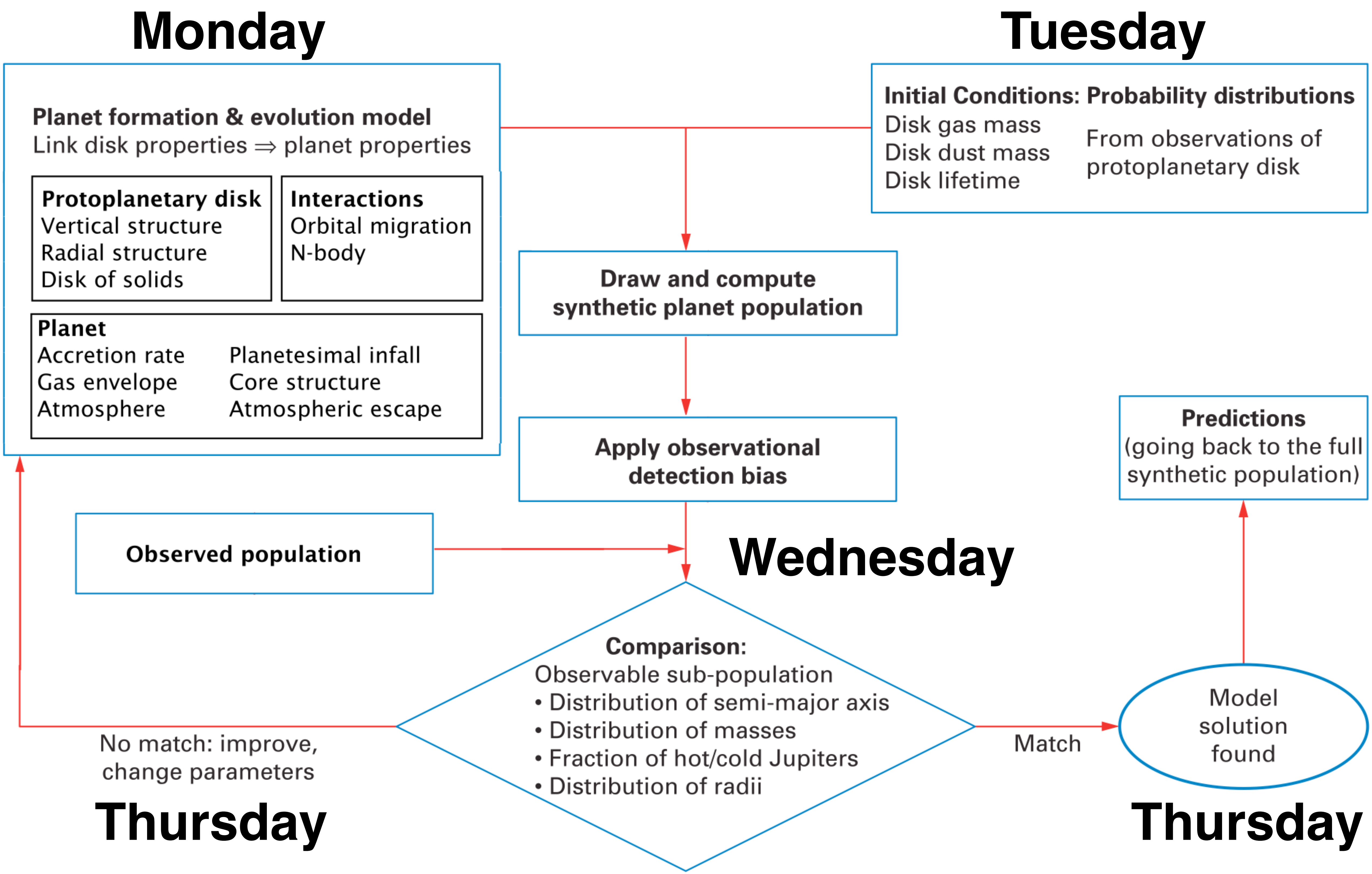



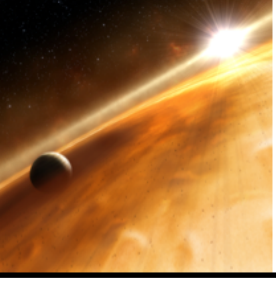
C. Mordasini & G. Bryden

Sagan Summer School 2015



# *Population synthesis*





# GlobalPFE model

## Minimum physical processes to consider

1. Structure and evolution of the protoplanetary disk
2. Accretion of solids / growth of the planetary solid core
3. Accretion of H/He / growth of the planetary gaseous envelope
4. Orbital migration
5. ~~N-body interaction among (proto)planets~~

GlobalPFE: Toy global planet formation model for population synthesis built on the core accretion paradigm, assuming core growth via planetesimal accretion and disk driven migration (Ida & Lin 2004, Alibert et al. 2005, Mordasini et al. 2009)

## 3 modes of operation

```
./globalPFE

Mode of operation: single planet (1), systematic study (2), population synthesis (3)
```

# Input file I: *globalPFE.in*

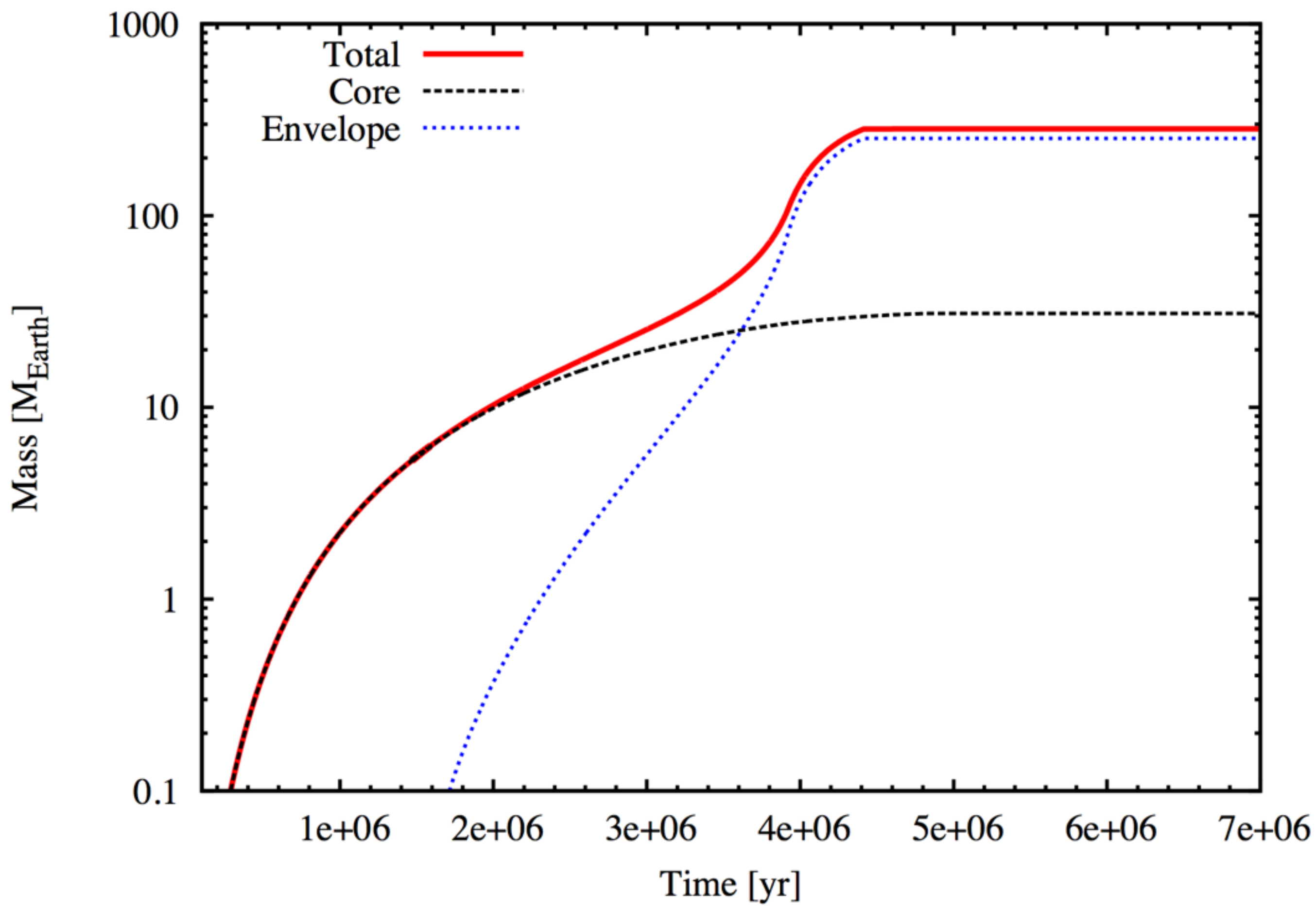
```
1.0                 !Stellar mass in Msol  (1.0)
0.03                !Inner disk radius in AU (0.03)
30.0                !Outer disk radius in AU (30)
0.0                 !Disk [Fe/H] (dex) (0)
5.0                 !Scaling factor for gas surface density (5.0)
1.2                 !disk dispersion timescale in Myr (1.2)
7.5                 !planet initial distance in AU (7.5)
```

# Input file II: *paramsPFE.in*

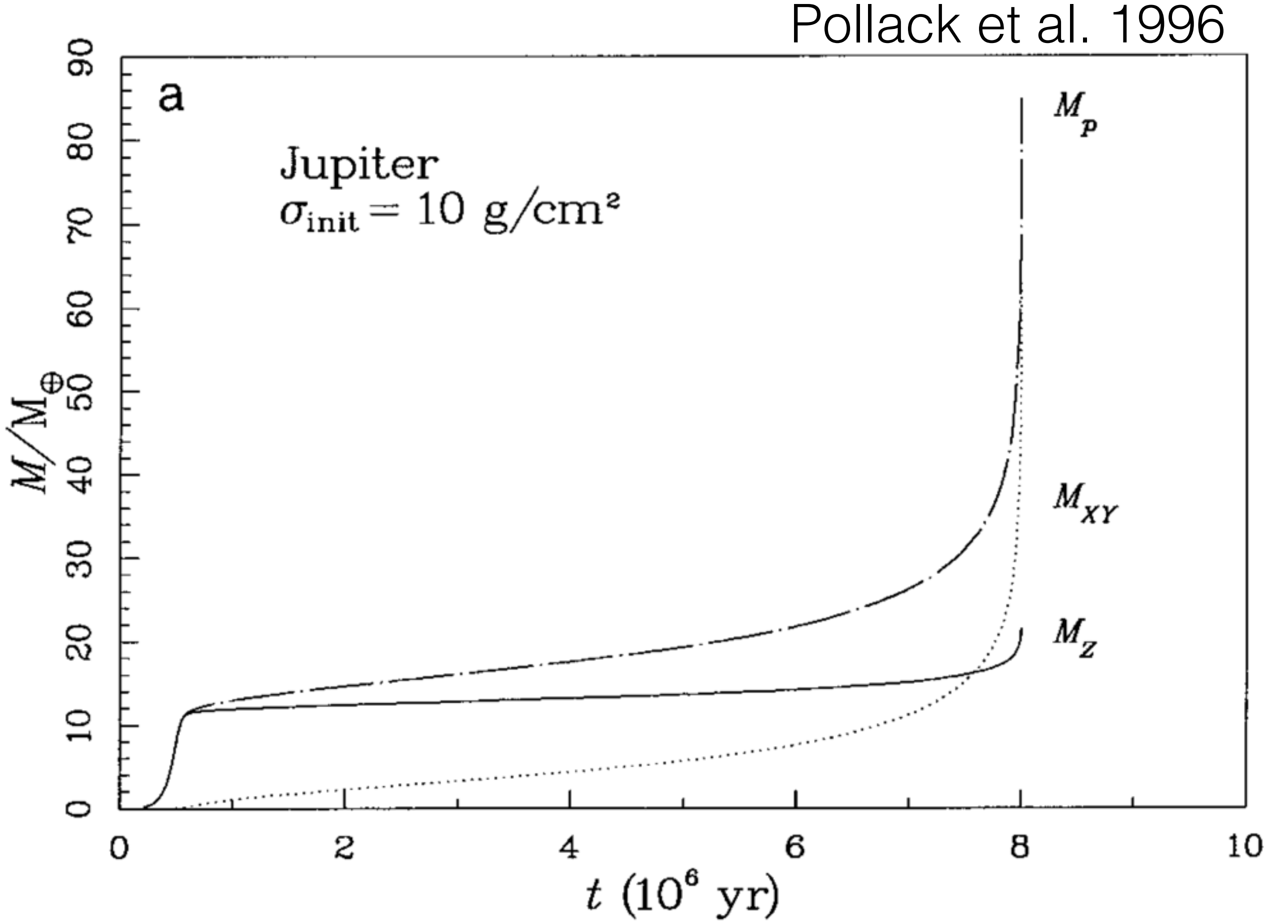
```
-0.5D0       Power law exponent for temperature in disk (-0.5D0)
1D-3         Alpha viscosity in disk (1D-3)
0.0149D0     Solar dust to gas ratio (0.0149D0)
1D-2         Opacity in protoplanetary atmosphere [cm**2/g] (1D-2)
10.4D0       Power law TKH of planet 10**pKH*M**qKH pKH (10.4)
-1.5D0       Power law TKH of planet 10**pKH*M**qKH qKH (-1.5)
5            Mode of limiting gas accretion (5)
3            Mode of type I  migration (3)
4D-3         Scaling factor for type I rate  (4D-3)
2            Mode of type II  migration(2)
2D-3         Scaling factor for type II rate (2D-3)
1D0          Scaling factor for core accretion rate (1D0)
```

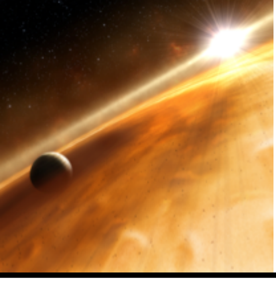
Detailed description in Sect. 7.2 of documentation

# Formation of a single planet




Pollack et al. 1996

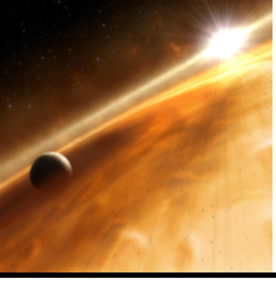




# *Output file:* **tracks_XXXXX.dat**

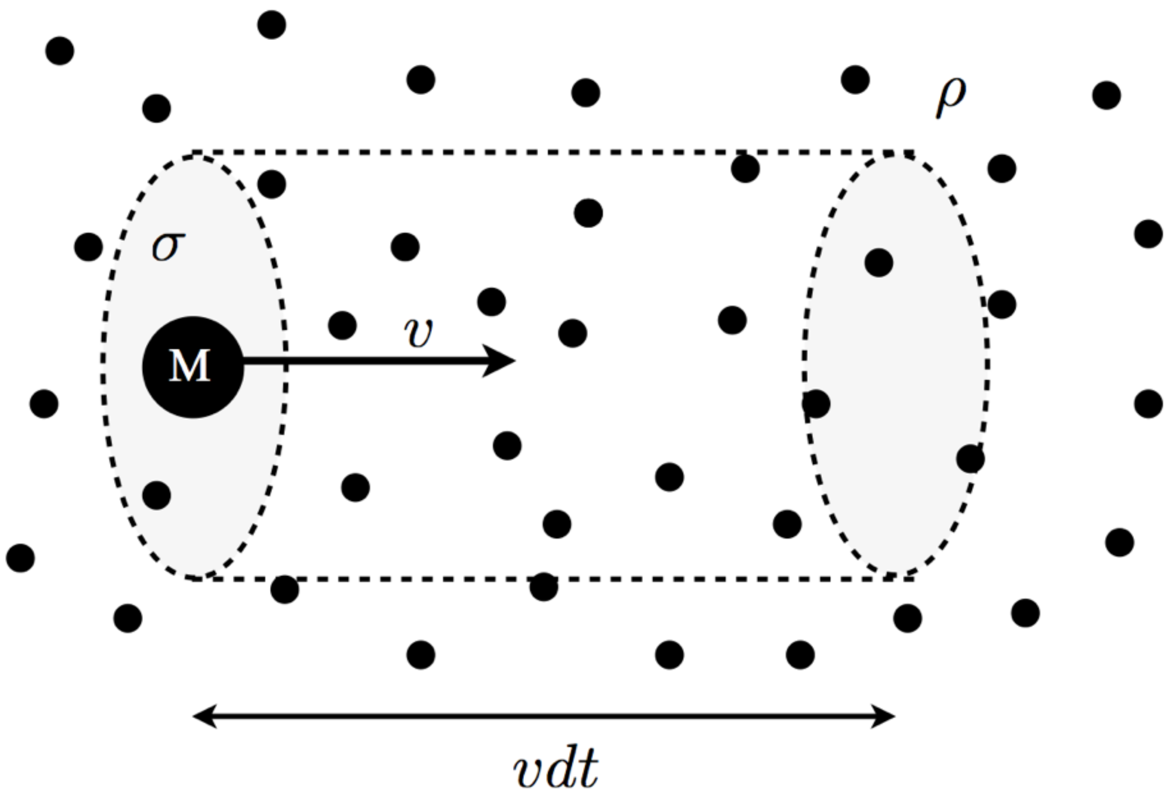
1. itime: Timestep number
2. time/an: Time in years
3. Mc/Mearth: Core mass in Earth masses
4. Me/Mearth: Envelope mass in Earth masses
5. (Mc+Me)/Mearth: Total mass in Earth masses
6. Mgdisk/Msol: Disk gas mass in solar masses
7. Mddisk/Mearth: Disk solid mass in Earth masses
8. Mdotc/Mearth*an: Core accretion rate (Mearth/yr)
9. Mdote/Mearth*an: Gas accretion rate (Mearth/yr)
10. ap/AU: Planet semi-major axis/AU
11. dt/an: Timestep duration/yrs
12. Sigmag(ip): Disk gas surface density at planet position [$g/cm^2$]
13. Sigmad(ip): Disk planetesimal surface density at planet position [$g/cm^2$]
14. Hdisk(ip)/AU: Disk vertical scale height
15. Rhills/AU: Hill sphere
16. dlsigmagdlr: Local slope of gas surface density
17. adot/AU*an: Migration rate (AU/yr)

Detailed description in Sect. 8 of documentation



# Accretion of planetesimals I

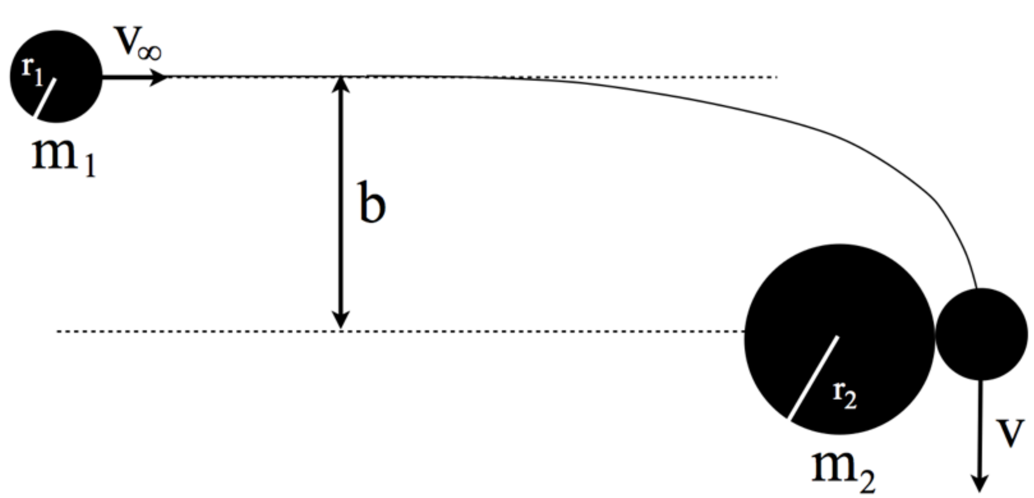
Growth by collisional accretion of background planetesimals



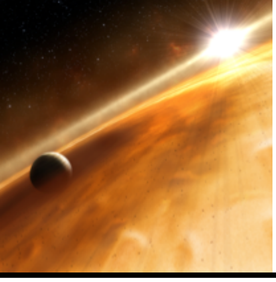

$$dm_p = \rho_s \sigma v_\infty dt$$

$$\frac{dm_p}{dt} \simeq \rho_s v \sigma = \rho_s v_\infty \left( \pi r^2 \left( 1 + \frac{v_{esc}^2}{v_\infty^2} \right) \right)$$

$$v_\infty \approx \sqrt{e^2 + i^2}\ v_{\rm Kep}$$




Safronov equation



# Accretion of planetesimals II

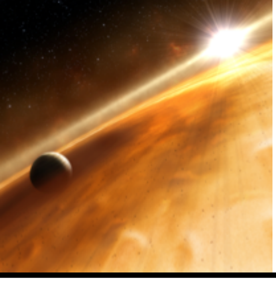
Oligarchic growth during presence of gas disk

$$\tau_{\rm c,gas} = 1.2 \times 10^5 \ {\rm yr} \left( \frac{\Sigma_{\rm d}}{10 \ {\rm g \ cm^{-2}}} \right)^{-1} \left( \frac{a_{\rm p}}{1 \ {\rm AU}} \right)^{1/2} \left( \frac{M_c}{M_\oplus} \right)^{1/3} \left( \frac{M_\star}{M_\odot} \right)^{-1/6} \times \left[ \left( \frac{\Sigma_{\rm g}}{2400 \ {\rm g \ cm^{-2}}} \right)^{-2/5} \left( \frac{a_{\rm p}}{1 \ {\rm AU}} \right)^{2/20} \left( \frac{m}{10^{18} \ {\rm g}} \right)^{2/15} \right]$$

Orderly growth after dissipation of gas disk

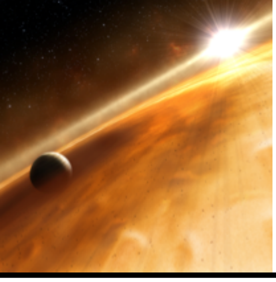
$$\tau_{\rm c,nogas} = 2 \times 10^7 \ {\rm yr} \left( \frac{\Sigma_{\rm d}}{10 \ {\rm g \ cm^{-2}}} \right)^{-1} \left( \frac{a_{\rm p}}{1 \ {\rm AU}} \right)^{3/2} \left( \frac{M_c}{M_\oplus} \right)^{1/3} \left( \frac{M_\star}{M_\odot} \right)^{-1/2} \left( \frac{\rho_p}{1 \ {\rm g \ cm^{-3}}} \right)^{2/3}$$

$$\dot{M}_{\rm c} = \frac{M_{\rm c}}{\tau_{\rm c}}$$



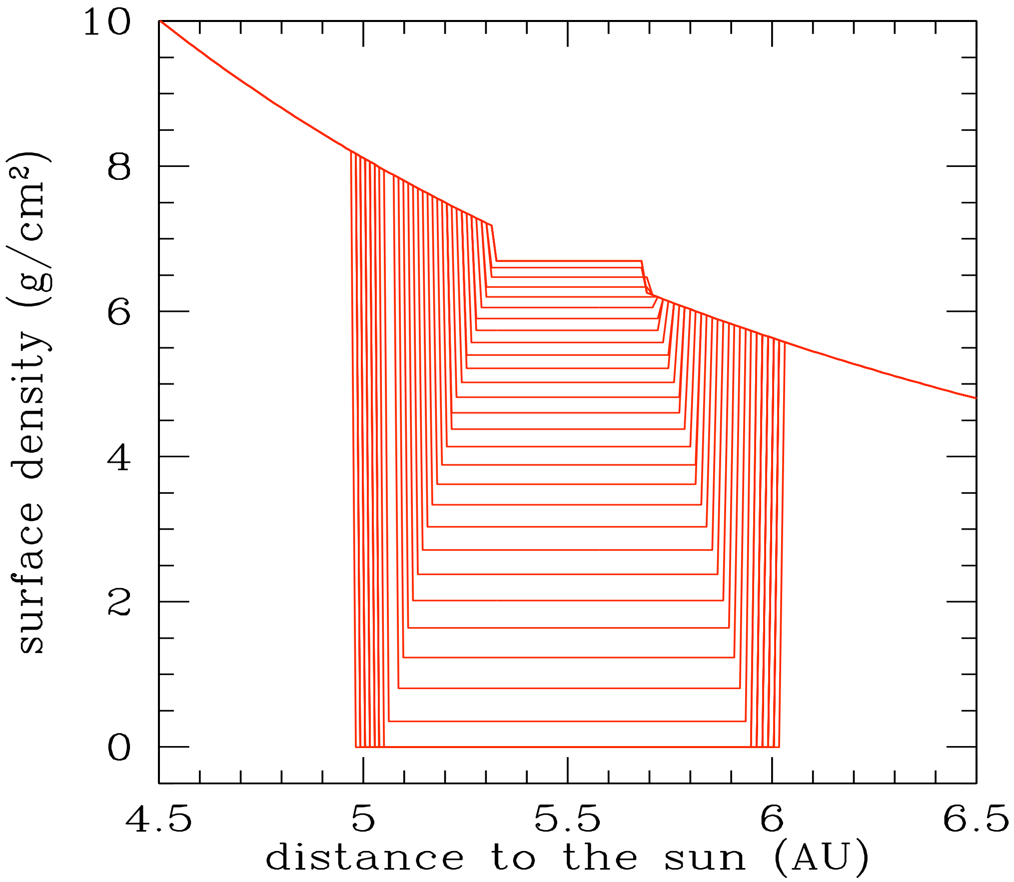
# Accretion of planetesimals III

```
SUBROUTINE Mdotcore(time,dt,Mstar,ndisk,Rdisk,Sigmag,Mc,Me,ap,Mdotc,Sigmadmean,gasdisk,Mfeed,CMdotc)

!-------------------------------------------------------------------------------
!calculate the accretion rate during the presence of the gas disk with IL04 Eq. 6
!-------------------------------------------------------------------------------
tauc=1.2D5*an*(Sigmadmean/10D0)**(-1D0)*(ap/AU)**0.5*((Mc+Me)/Mearth)**(1D0/3D0)*(Mstar/Msol)**(-1D0/6D0)*&
     ((Sigmag(ip)/2.4D3)**(-1D0/5D0)*(ap/AU)**(1D0/20D0)*(mpla/1D18)**(1D0/15D0))**2D0

!-------------------------------------------------------------------------------
!calculate the accretion rate after the dissipation of the gas disk with IL04 Eq. 9
!-------------------------------------------------------------------------------
taue=2D7*an*(Sigmadmean/10D0)**(-1D0)*((Mc+Me)/Mearth)**(1D0/3D0)*(rhoe/1D0)**(2D0/3D0)*&
     (Mstar/Msol)**(-0.5D0)*(ap/AU)**1.5D0

!-------------------
!core accretion rate
!-------------------
IF(gasdisk)THEN
   mdotc=CMdotc*Mc/tauc
ELSE
   mdotc=CMdotc*Mc/taue
END IF

!----------------------
!increment the core mass
!----------------------
!cannot accrete more than mass in the feeding zone
Mc=Mc+MIN(dt*mdotc,Mfeed)
```



# Accretion of planetesimals IV

Accretion from a feeding zone with spatially constant planetesimal surface density $\Sigma_P$




$$\dot{\Sigma}_{\rm d} = -\frac{(3M_*)^{1/3}}{6\pi a_{\rm p}^2 B_L M_{\rm p}^{1/3}} \dot{M}_{\rm c}$$

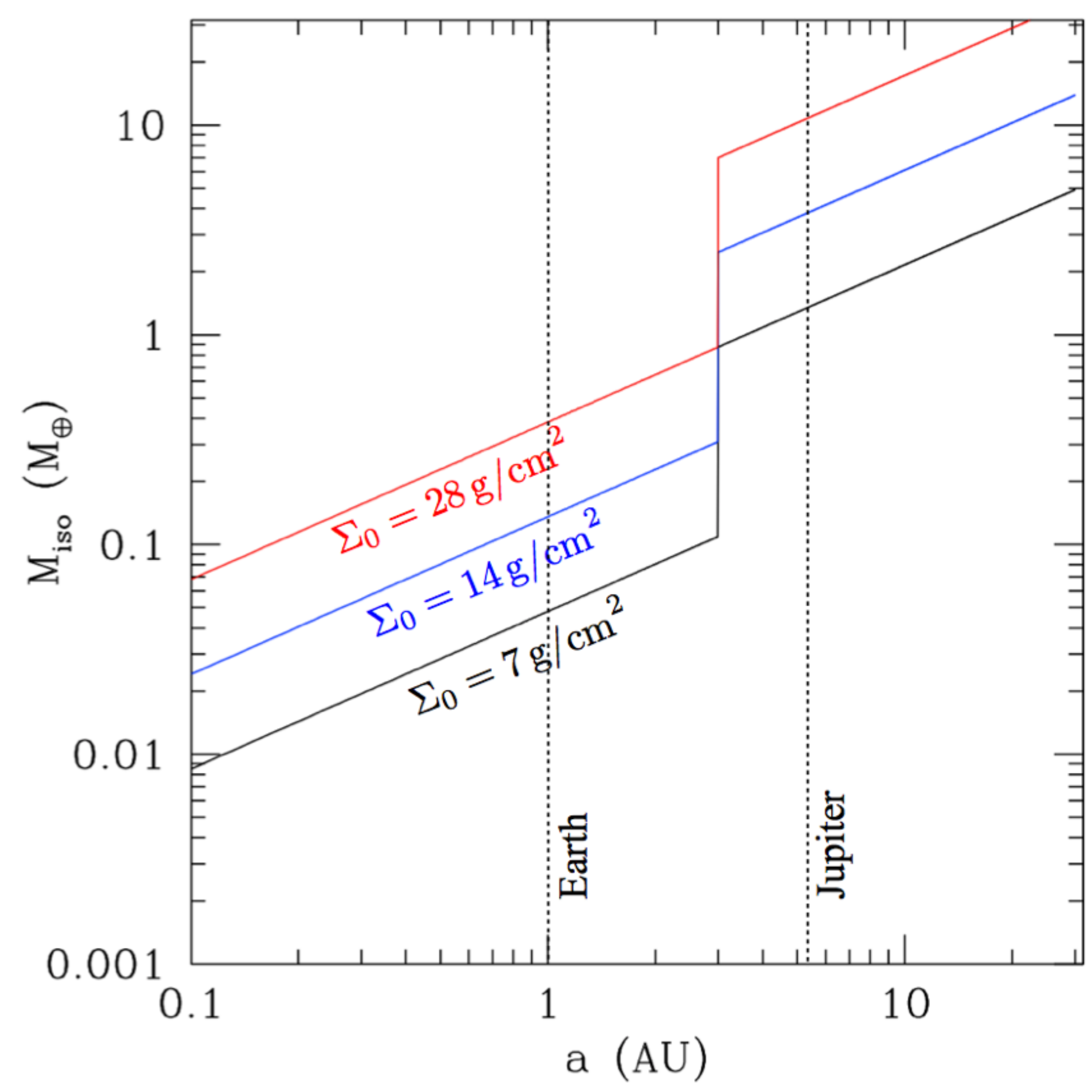
$$W_{feed} = B_L R_H = B_L \left(\frac{M}{3M_\star}\right)^{1/3} a$$

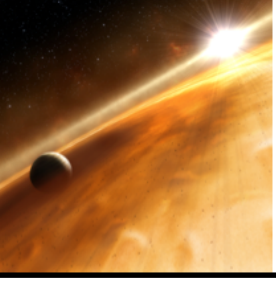
Without migration and planetesimal drift:

Growth to the isolation mass

$$m_{isolation} \approx \frac{(4\pi r^2 \Sigma)^{\frac{3}{2}}}{(3M_\star)^{\frac{1}{2}}}$$

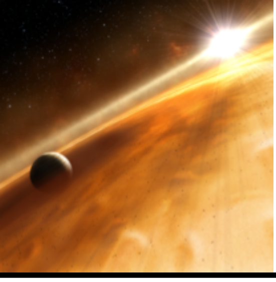
$$\approx 0.07 \left(\frac{a}{1\mathrm{AU}}\right)^3 \left(\frac{\Sigma}{10\mathrm{gcm}^{-2}}\right)^{3/2} M_\oplus$$





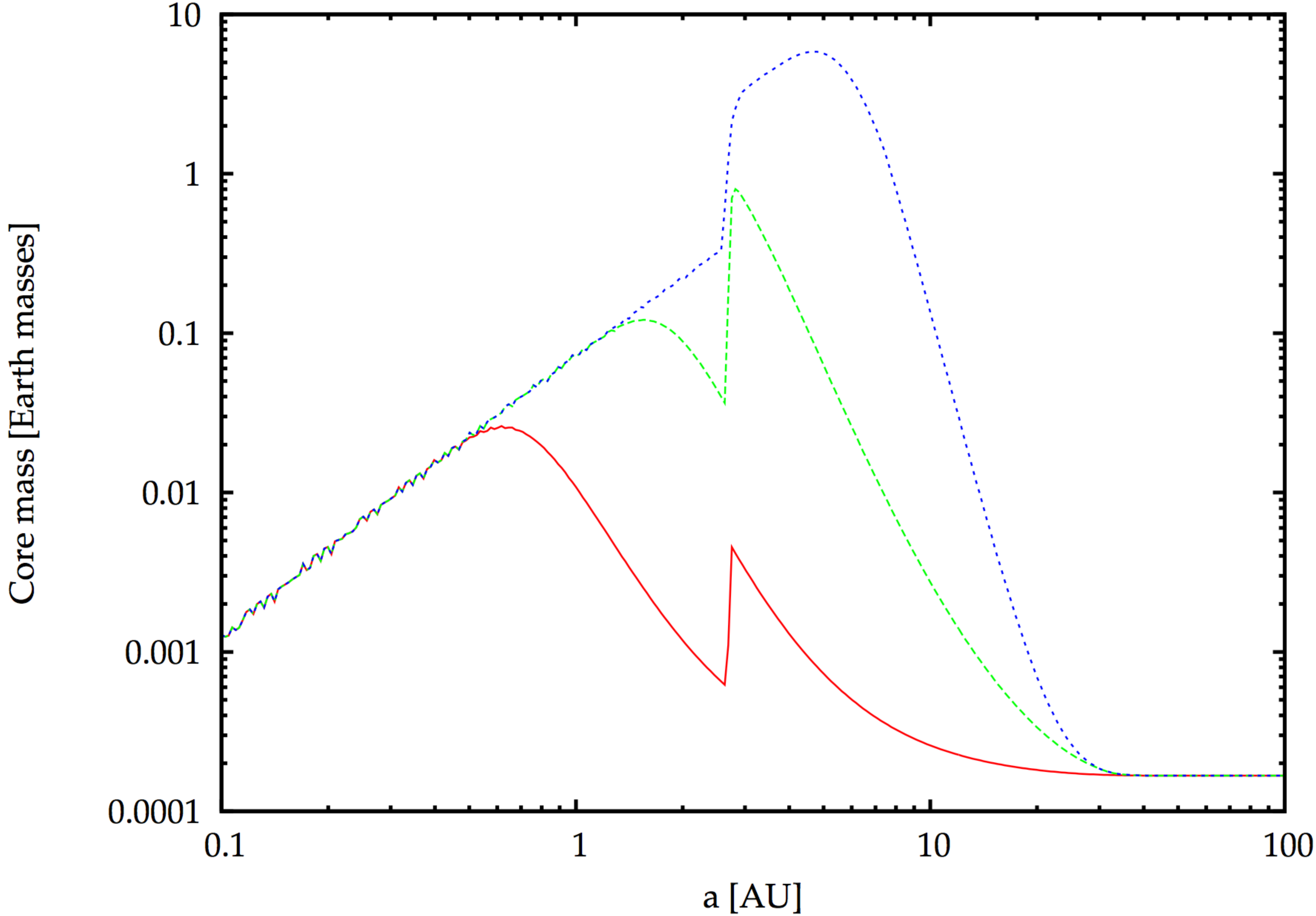
# Accretion of planetesimals V

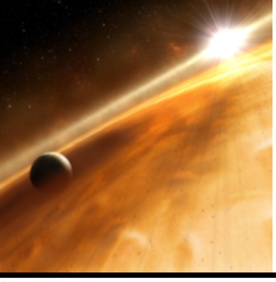
```
SUBROUTINE evodiskd(time,dt,Mstar,ndisk,Rdisk,Sigmad,Mddisk,Mdotc,Mc,Me,ap,Sigmadmean,gasdisk,Mfeed)


  !------------------------
  !Mass in the feeding zone
  !------------------------
  Mfeed=0D0
  Do i=imin,imax-1
     Mfeed=Mfeed+PI*(Rdisk(i+1)**2D0-Rdisk(i)**2D0)*0.5D0*(Sigmad(i+1)+Sigmad(i))
  END DO

  IF(DEBUG)WRITE(*,*)Mfeed/Mearth,-dt*Mdotc/Mearth

  !------------------------------------
  !subtract mass accreted by the planet
  !------------------------------------
  Mfeed=Mfeed-dt*Mdotc
  Mfeed=max(Mfeed,0D0)

  !------------------------------------------------
  !uniform mean surface density in the feeding zone
  !------------------------------------------------
  Sigmadmean=Mfeed/(PI*(Rdisk(imax)**2D0-Rdisk(imin)**2D0))
  IF(Sigmadmean<1D-20)Sigmadmean=0D0

  Do i=imin,imax
     Sigmad(i)=Sigmadmean
  END DO
```



# *Accretion of planetesimals VI*

Core mass as a function of semi-major axis at 0.1 Myr (red), 1 Myr (green) and 10 Myr.





# Accretion of gas I

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

$$\frac{dl}{dr} = 4\pi r^2 \rho \left(\epsilon - T\frac{\partial S}{\partial t}\right) \qquad \frac{dT}{dr} = \frac{T}{P}\frac{dP}{dr}\nabla$$

$$\nabla = \frac{d\ln T}{d\ln P} = \min(\nabla_{\rm ad}, \nabla_{\rm rad}) \qquad \nabla_{\rm rad} = \frac{3}{64\pi\sigma G}\frac{\kappa l P}{T^4 m}$$

1D structure equations of planetary H/He envelope

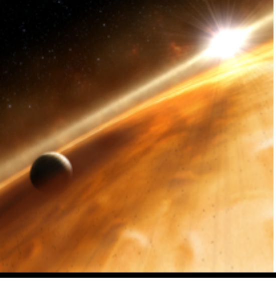
Parameterization via KH timescale

$$M_{\rm c,crit} = 10 M_\oplus \left(\frac{\dot{M}_{\rm c}}{10^{-6} M_\oplus \ {\rm yr}^{-1}}\right)^{1/4} \left(\frac{\kappa}{1 \ {\rm g \ cm}^{-2}}\right)^{1/4}$$

$$\tau_{\rm KH} = 10^{p_{\rm KH}} \ {\rm yr} \left(\frac{M_{\rm p}}{M_\oplus}\right)^{q_{\rm KH}} \left(\frac{\kappa}{1 \ {\rm g \ cm}^{-2}}\right)$$

$$\dot{M}_{\rm e,KH} = \frac{M_{\rm p}}{\tau_{\rm KH}}$$

$p_{\rm KH} = 10.4$ and $q_{\rm KH} = -1.5$, and $\varkappa = 10^{-2}$ g/cm$^2$



# Accretion of gas II

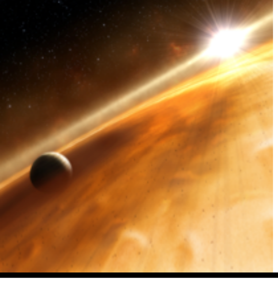
Limits to gas accretion rate:

Bondi rate

$$\dot{M}_{\rm e,Bondi} \approx \frac{\Sigma_{\rm g}}{H}\left(\frac{R_{\rm H}}{3}\right)^3 \Omega$$

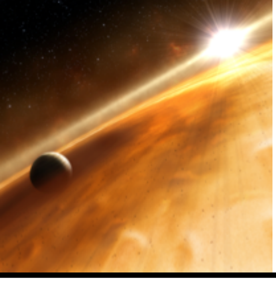
Gas accretion rate in the disk

$$\dot{M}_{\rm e,visc} = 3\pi\nu\Sigma_{\rm g} \qquad \nu = \alpha H^2 \Omega$$



# Accretion of gas III

```
SUBROUTINE Mdotgas(time,dt,Mstar,ndisk,Rdisk,Sigmag,Hdisk,Mc,Me,ap,Mdotc,alpha,kappa,pKH,qKH,ilimMe,Mdote,Mgdisk)


!calculate the critical core mass
Mcrit=10D0*Mearth*(Mdotc/(1e-6*Mearth/an))**0.25*(kappa/1D0)**0.25

!calculate the Kelvin Helmholtz timescale
IF(Mc>Mcrit)THEN
   tauKH=10D0**pKH*an*(Mp/Mearth)**qKH*(kappa/1D0)
   mdote=Mp/tauKH
ELSE
   tauKH=1D10*an
   mdote=0D0
END IF

!cannot accrete more than at the Bondi rate
mdotbondi=sigmag(ip)/Hdisk(ip)*SQRT(G*Mstar/ap**3)*(Rhills(Mp,Mstar,ap)/3D0)**3D0
mdote=MIN(mdote,mdotbondi)

!calculate the disk accretion rate and limit gas accretion to that
nudisk=alpha*Hdisk(ip)**2D0*SQRT(G*Mstar/ap**3D0)
Mdotdisk=flub*3D0*PI*nudisk*Sigmag(ip)
mdote=MIN(mdote,mdotdisk)
```



# Truncation of gas accretion I

1) At the gas isolation mass

$$M_{\mathrm{iso,e}} = \sqrt{\frac{(4\pi 2 a_{\mathrm{p}}^2 \Sigma_{\mathrm{g}})^3}{3M_\star}}$$

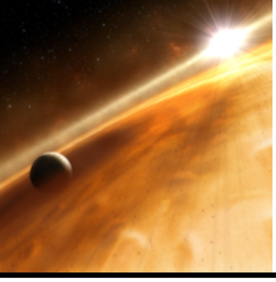
2) Hard limit at gap formation

$$R_{\mathrm{H}} > H$$

3) Decrease of rate due to gap formation

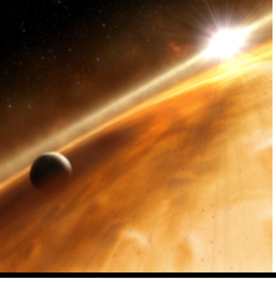
$$f_{\mathrm{va04}} = 1.668 \left(\frac{M_{\mathrm{p}}}{M_{\mathrm{Jup}}}\right)^{1/3} \exp\left(-\frac{M_{\mathrm{p}}}{1.5 M_{\mathrm{Jup}}}\right) + 0.04$$

$$\dot{M}_{\mathrm{e,va04}} = f_{\mathrm{va04}} 3\pi\nu\Sigma_{\mathrm{g}}.$$



# Truncation of gas accretion II

```
!-----------------------------------------
!various ways to terminate the accretion
!-----------------------------------------

!1) by the gas isolation mass
IF(ilimMe==1)THEN

   !calculate the gas isolation mass. Assume width of feeding zone is 2 RH as in IL04
   Meiso=(4D0*PI*2D0*ap**2D0*Sigmag(ip))**1.5D0/(3D0*Mstar)**0.5D0

   !limit Mdot at the isolation mass
   IF(Me>=Meiso)THEN
      mdote=0d0
      IF(DEBUG)WRITE(*,*)'Gas isolation mass reached',Meiso/Mearth
   END IF

!2) if the Hills sphere is larger than H (gap opening)
ELSE IF(ilimMe==2)THEN

   !limit Mdot at the gap opening mas
   IF(Hdisk(ip)<Rhills(Mp,Mstar,ap))THEN
      mdote=0d0
      IF(DEBUG)WRITE(*,*)'Gap opened',Mp/Mearth,Rhills(Mp,Mstar,ap)/AU,Hdisk(ip)/AU
   END IF

...
```



# Gas driven orbital migration I

1) Low mass planets: type I migration

$$\tau_{\mathrm{typeI}} = \frac{1}{2.728 + 1.082 p_{\mathrm{g}}} \left( \frac{c_s}{a_{\mathrm{p}} \Omega} \right)^2 \frac{M_*}{M_{\mathrm{p}}} \frac{M_*}{a_{\mathrm{p}}^2 \Sigma_{\mathrm{g}}} \Omega^{-1}$$

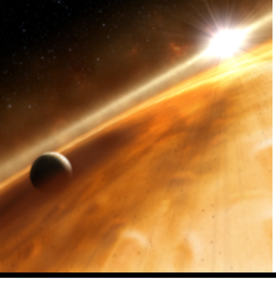
$$\dot{a}_{\mathrm{p}} = -\frac{a_{\mathrm{p}}}{\tau_{\mathrm{typeI}}}$$

2) High mass planets: type II migration if $R_{\mathrm{H}} > H$

$$\dot{a}_{\mathrm{p}} = \mathrm{sign}(a_{\mathrm{p}} - R_{\mathrm{m}}) \; u_{\mathrm{r}} \; \min \left( 1, \frac{2 \Sigma_{\mathrm{g}} a_{\mathrm{p}}^2}{M_{\mathrm{p}}} \right)$$

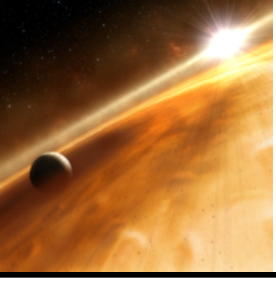
$$u_{\mathrm{r}} \; = \; 3\nu / (2 a_{\mathrm{p}})$$

$$R_{\mathrm{m}} = 10 \; \mathrm{AU} \exp \left( \frac{2t}{5 \tau_{\mathrm{disk}}} \right)$$

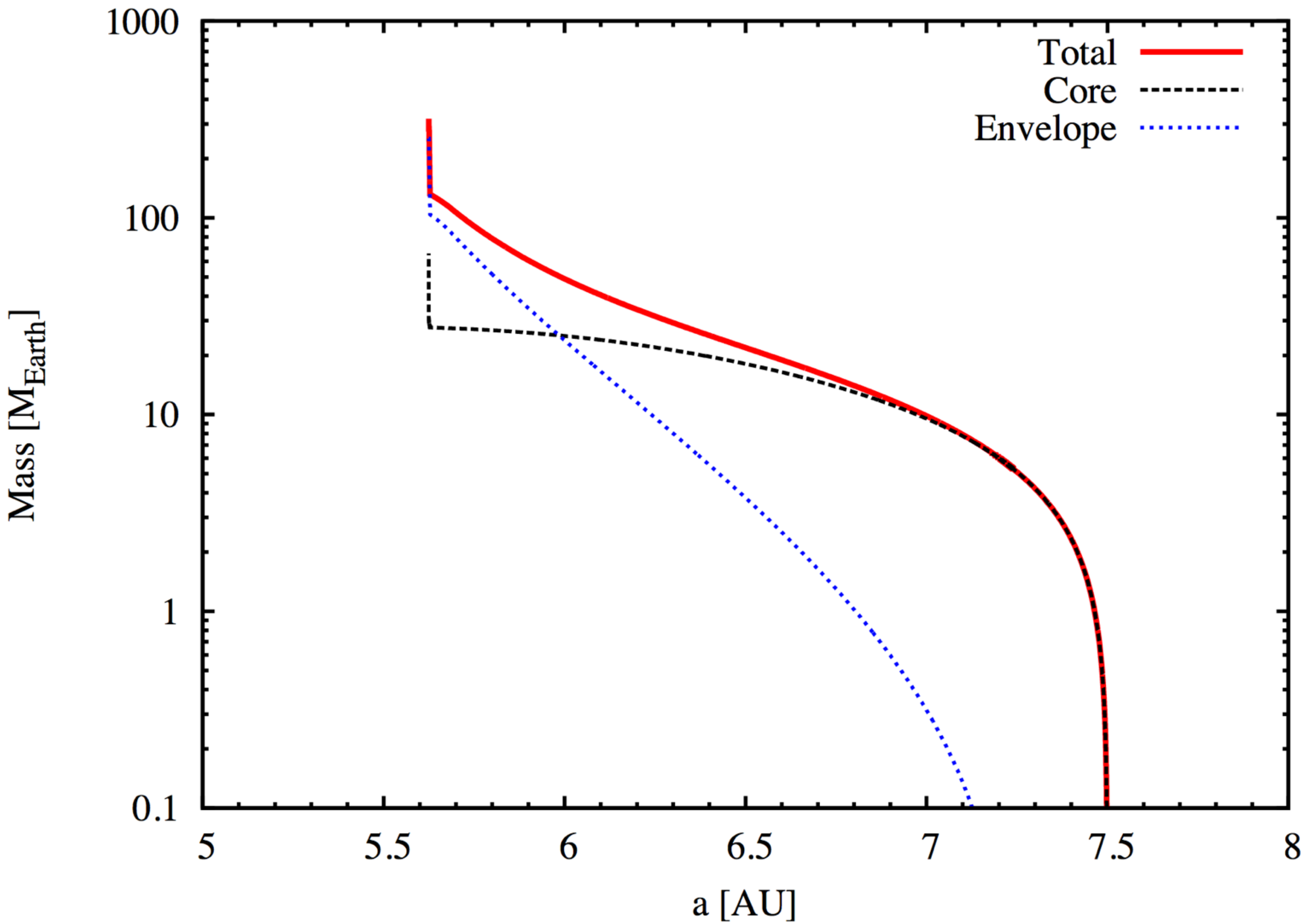


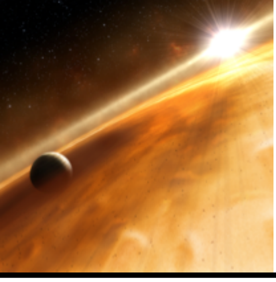
# Gas driven orbital migration II

```
SUBROUTINE Migration(time,dt,Mstar,ndisk,Rdisk,Sigmag,Hdisk,Mc,Me,taudisk,ap,Rindisk,Routdisk,alpha,&
     itypeI,C1,itypeII,C2,pT,dlsigmagdlr,adot)


!--------------------------------------------------------------
!Type I and Type II migration according to different authors
!--------------------------------------------------------------

IF(Hdisk(ip)>Rhills(Mp,Mstar,ap))THEN  !Type I migration

   !get local power law exponent of Sigmag
   IF(Sigmag(ip)>0D0)THEN
      call drivequadradisk(Ndisk,Rdisk,Sigmag,ap,yloc,yderi)
      dlsigmagdlr=yderi*ap/Sigmag(ip)
   ELSE
      yloc=0D0
      yderi=0D0
   END IF

   IF(itypeI==1)THEN !IL08 Eq.12
      IF(Sigmag(ip)>0D0)THEN
         tauI=1D0/(2.728+1.082*dlsigmagdlr)*(Hdisk(ip)/ap)**2D0*(Mstar/Mp)*(Mstar/(ap**2D0*Sigmag(ip)))*(1D0/OmegaKp)
         adot=-C1*ap/tauI
      ELSE
         adot=0D0
      END If



     ...
```



# Gas driven orbital migration III





# Structure gas disk

Initial profile

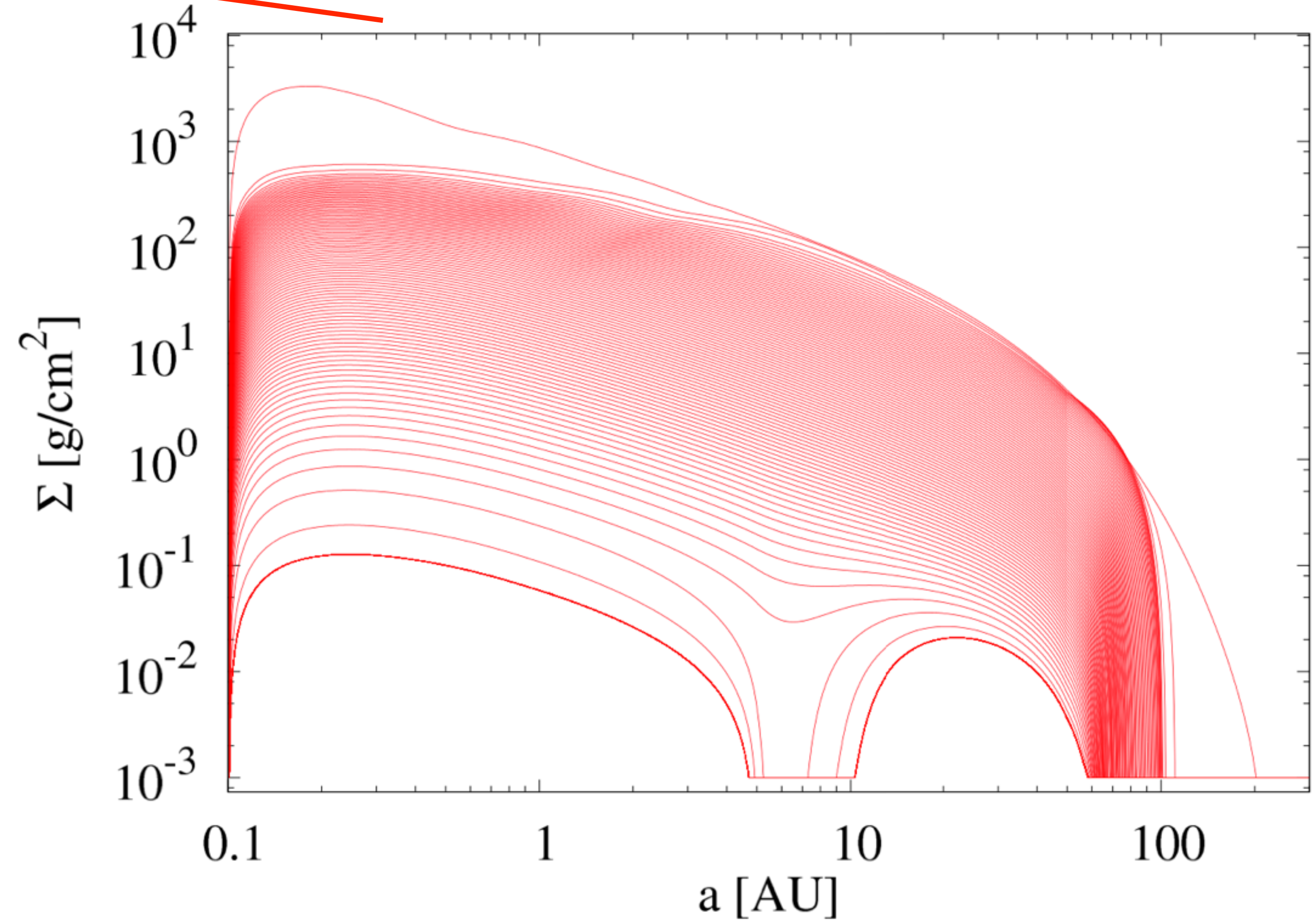
$$\Sigma_{\rm g}(t=0,r) = \Sigma_{\rm g,0} f_{\rm g} \left(\frac{r}{1\rm AU}\right)^{p_{\rm g,0}} \exp\left[-\left(\frac{r}{R_{\rm out}}\right)^{2+p_{\rm g,0}}\right]\left(1-\sqrt{\frac{r}{R_{\rm in}}}\right)$$

$$p_{\rm g} = -p_T - 3/2$$

$$\Sigma_{\rm g} \propto r^{-1}$$

Evolution

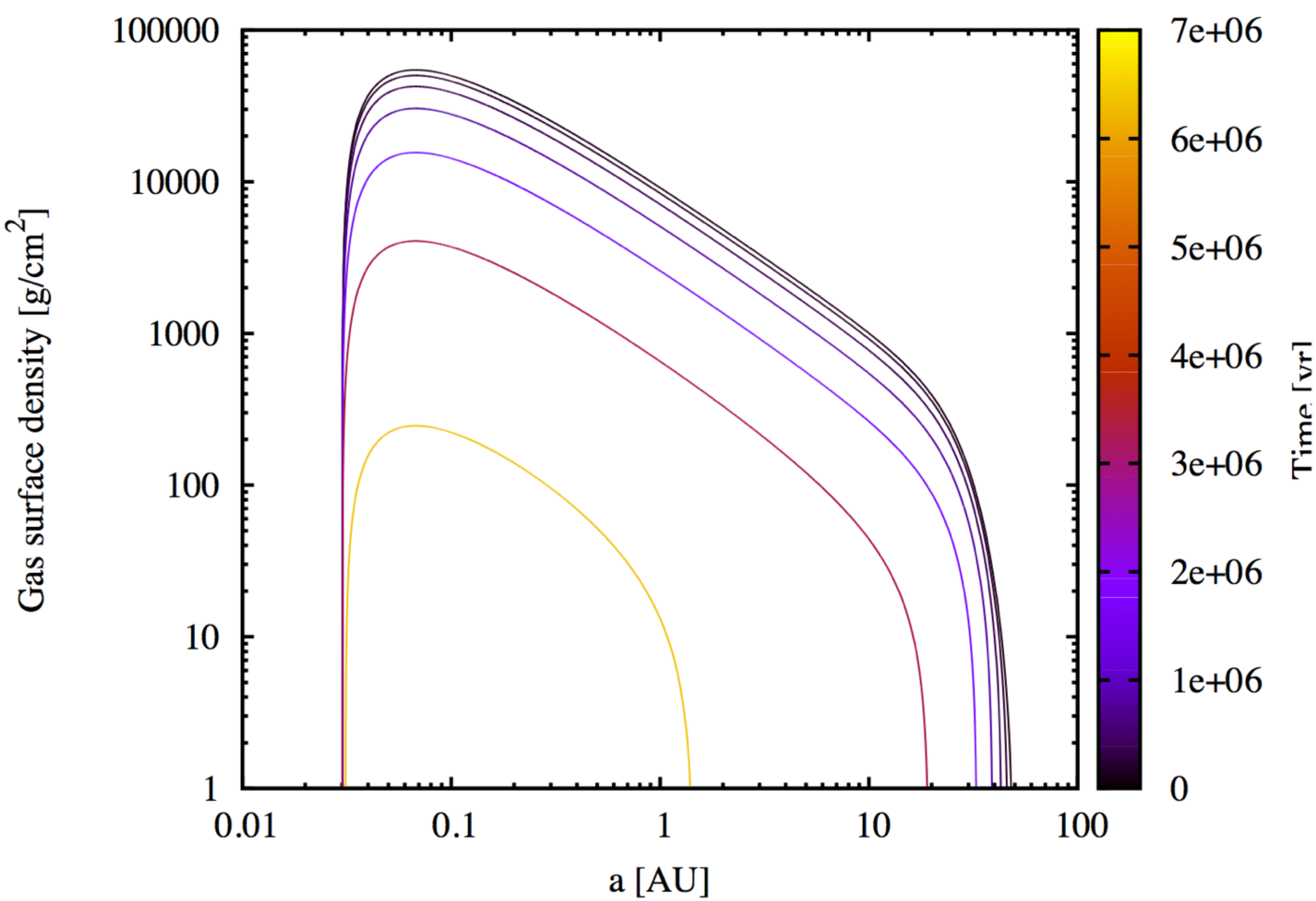
$$\dot{\Sigma}_{\rm g} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\tilde{\nu}\Sigma r^{1/2}\right] + \dot{\Sigma}_w(r) + \dot{Q}_{\rm planet}(r)$$

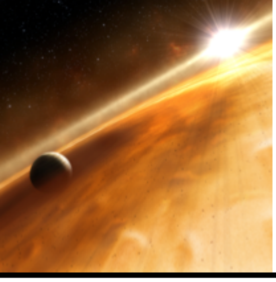


# Evolution of the gas disk

Exponential decrease and photoevaporation

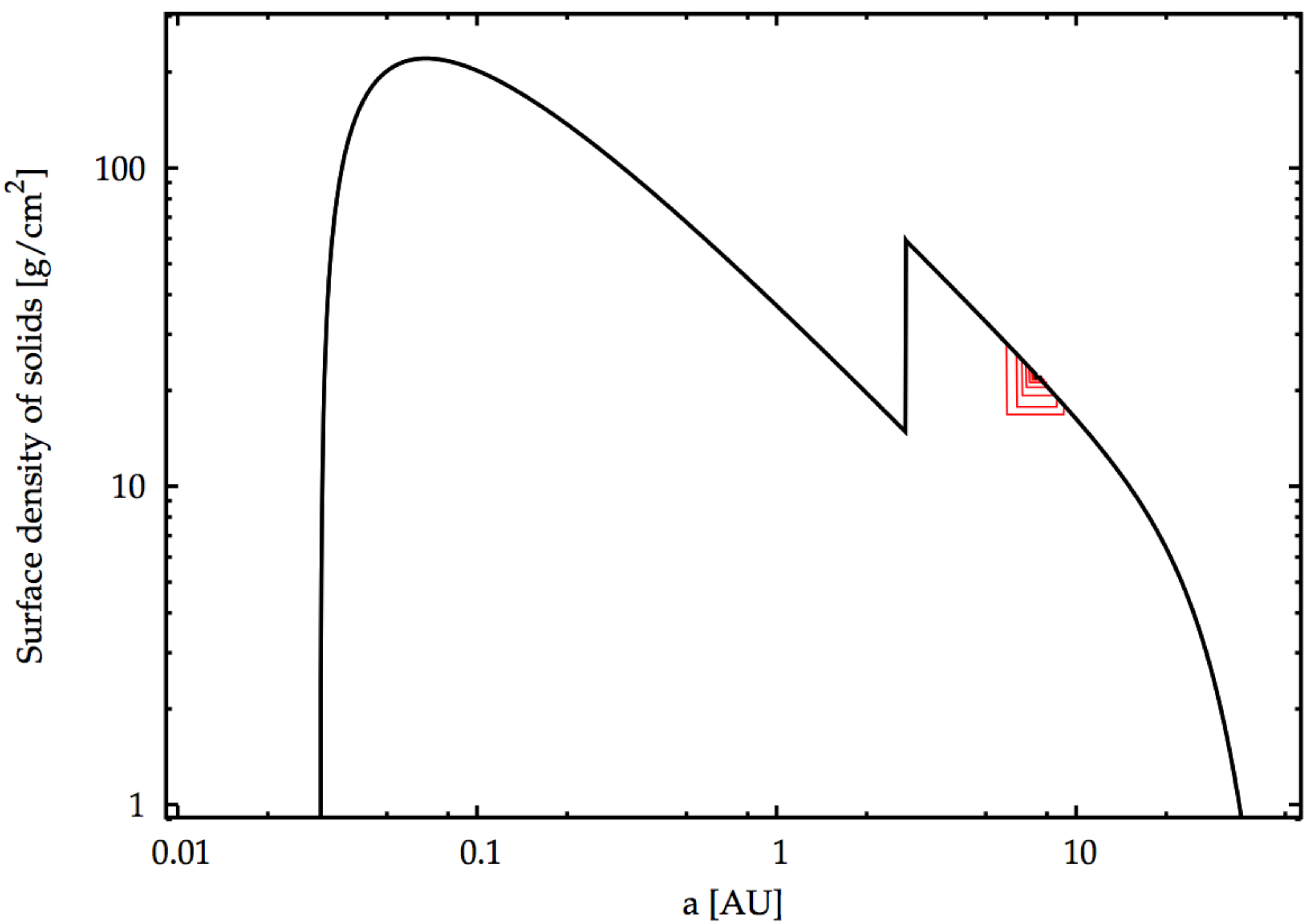
$$\dot{\Sigma}_{\rm g}(r) = -\frac{\Sigma_{\rm g}(r)}{\tau_{\rm disk}} + \dot{\Sigma}_{\rm wind}$$



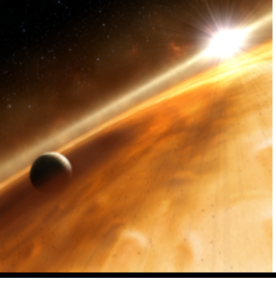


# Structure disk of solids

$$\Sigma_d(t=0,r) = f_{\mathrm{D/G},\odot} 10^{[\mathrm{Fe/H}]} \eta_{\mathrm{ice}} \Sigma_{\mathrm{g}}(t=0,r)$$



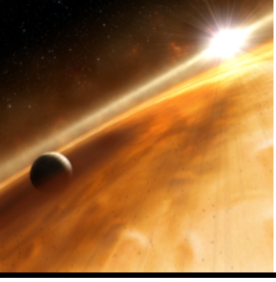

Evolution only via accretion onto the core



# *Output file:* diskevo.dat

1. time/an: time/yr
2. rdisk(i)/AU: radius/AU
3. Sigmag(i): gas surface density
4. Sigmad(i): solid surface density
5. yderi: power law exponenent of the surface density
6. Tdisk(i): temperature
7. Hdisk(i)/AU: vertical scale height

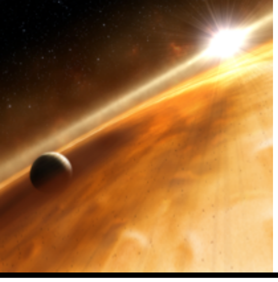
Detailed description in Sect. 8 of documentation



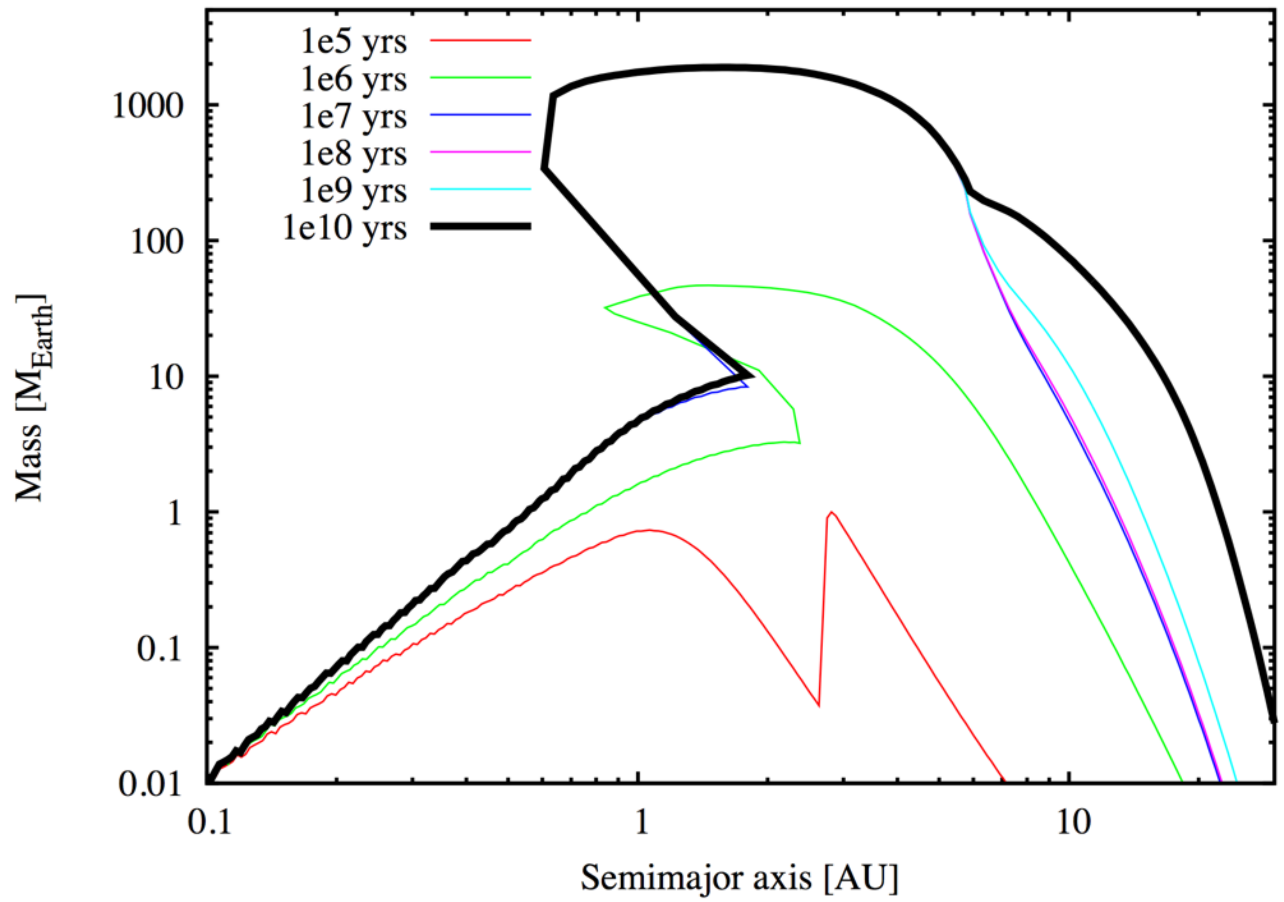
# *Formation of a planetary system*

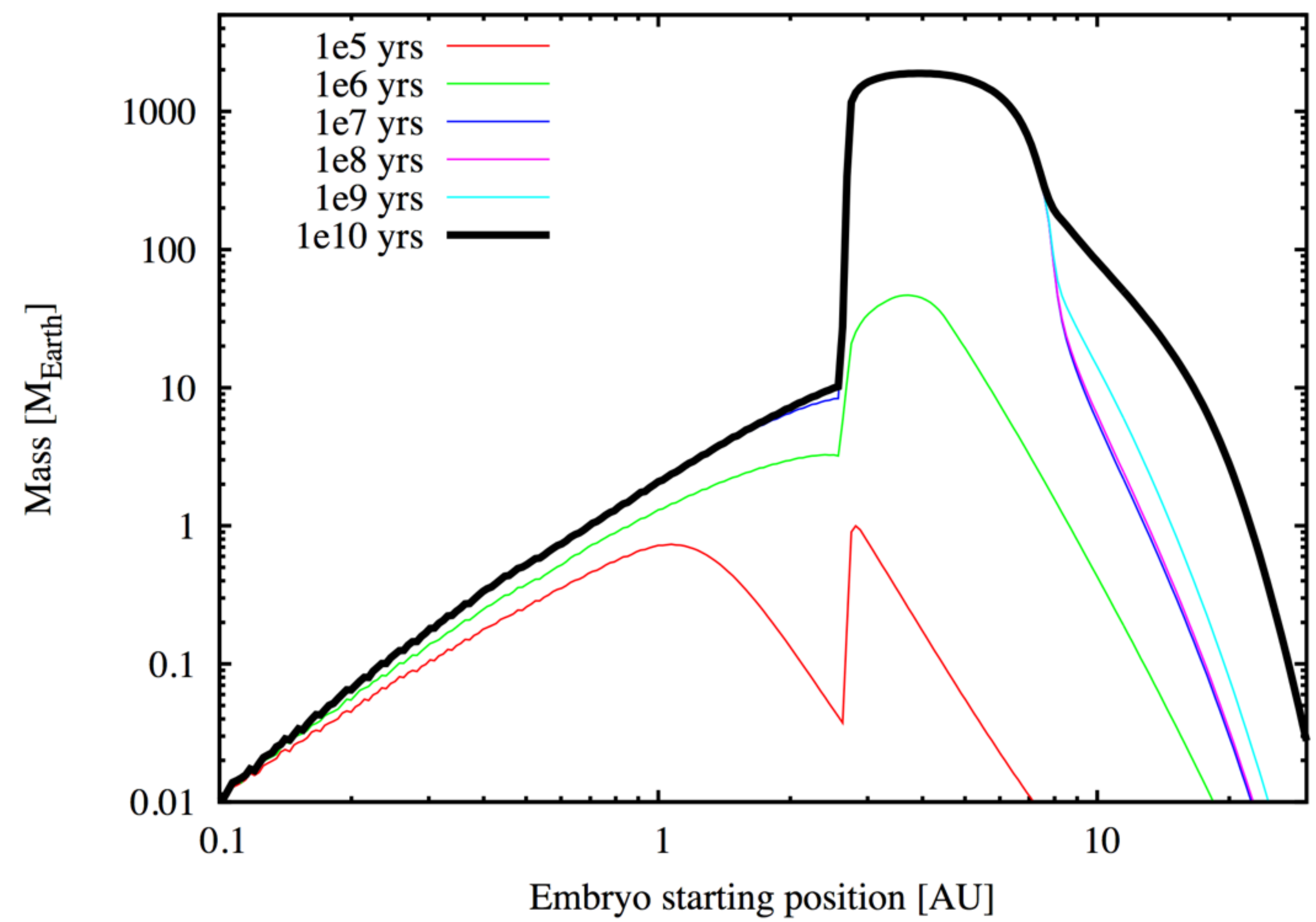
All initial conditions are kept constant, except for the semi-major axis which is systematically varied between 0.1 and 100 AU, distributed uniformly in log(a) (301 values)

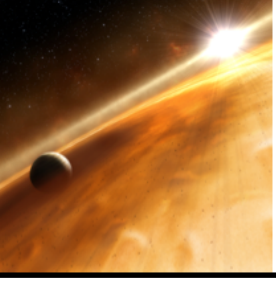
```
./globalPFE
Mode of operation: single planet (1), systematic study (2), population synthesis (3)
2
```



# Formation of a planetary system



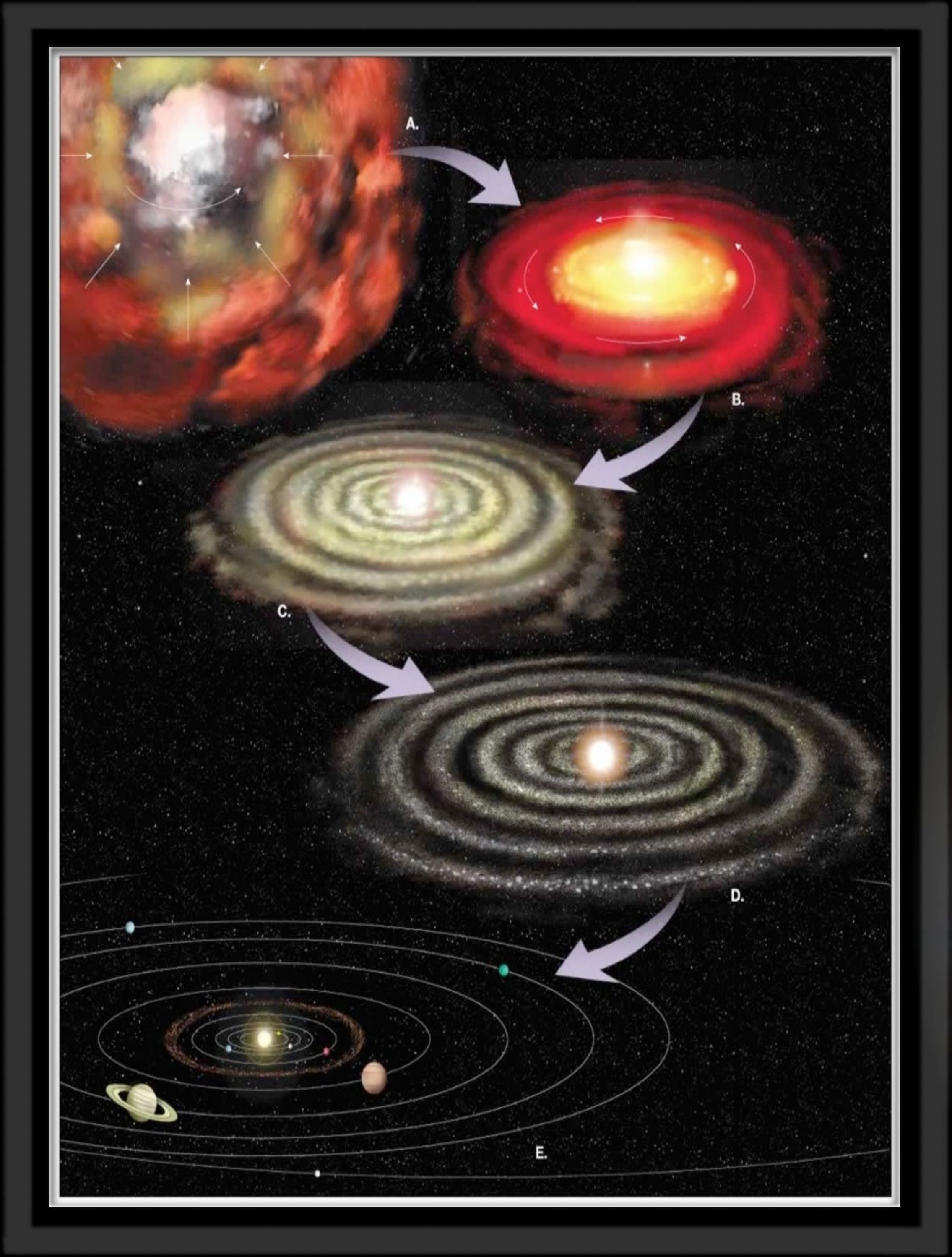



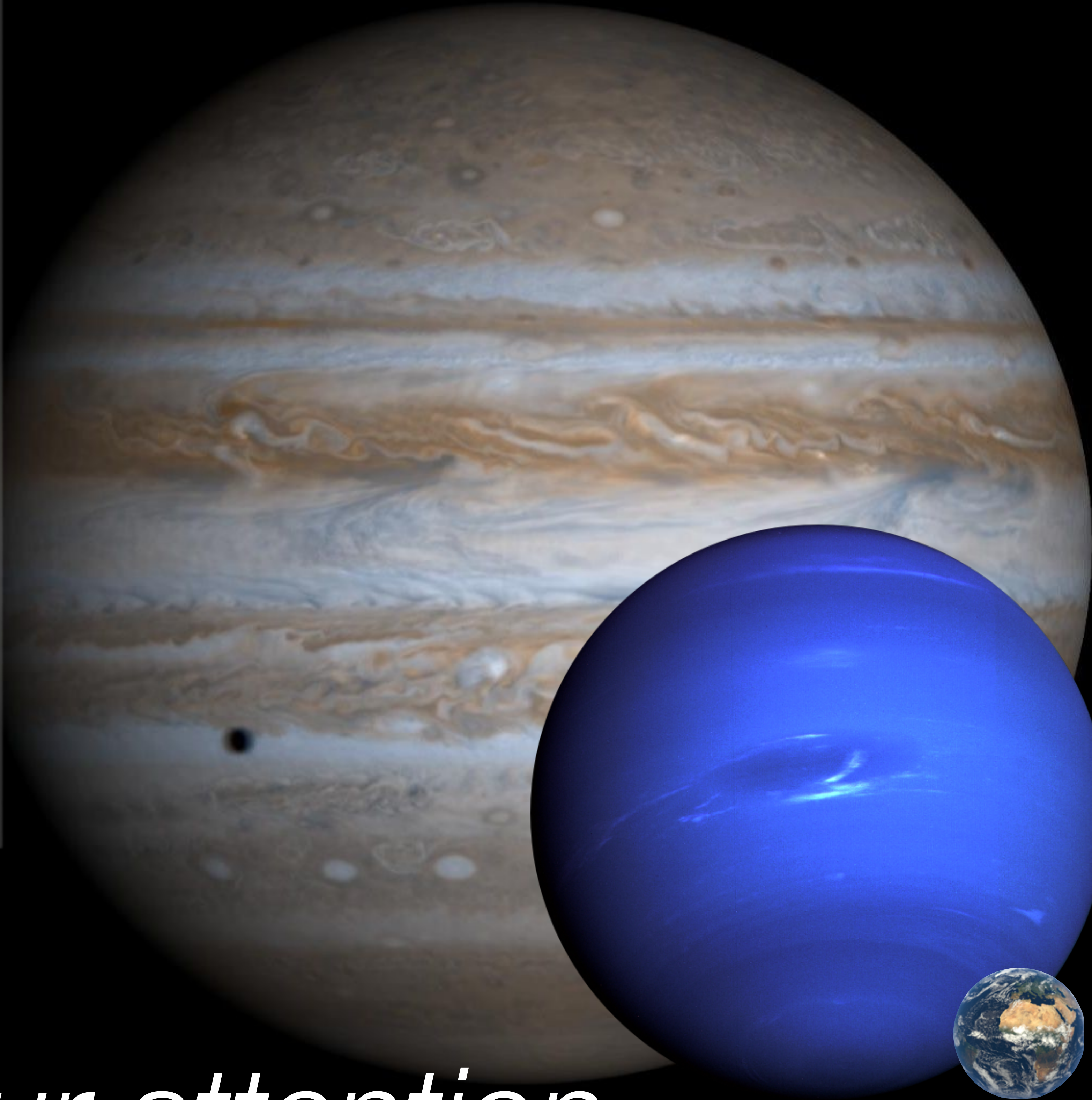


# Model limitations

As a toy model, GlobalPFE has many limitations. The most important are:

1. One embryo per disk: no dynamics, no competition for gas and solids, no eccentricity excitation, no capture in mean motion resonances …
2. Gas disk driven migration only (no scattering, no Kozai, no planetesimal driven migration). Only inward migration (loc. isothermal type I).
3. Core growth by accretion of planetesimals only, no pebble accretion
4. Simplistic disk model (fixed temperature profile, no viscous evolution, no real photoevaporation model)
5. Simplistic gas accretion model (no calculation of the envelope structure)
6. No evolution of the disk of solid: no dust/planetesimal drift, growth, fragmentation, no eccentricity/inclination evolution
7. No planetary internal structure and evolution: the planetary radius and luminosity are not calculated. The effect of atmospheric escape/ envelope evaporation is also neglected.



Thanks for your attention