



# ExoPlex Mass-Radius Code

Cayman Unterborn  
School of Earth and Space Exploration  
Arizona State University

Sagan Summer Workshop  
7-17-2019

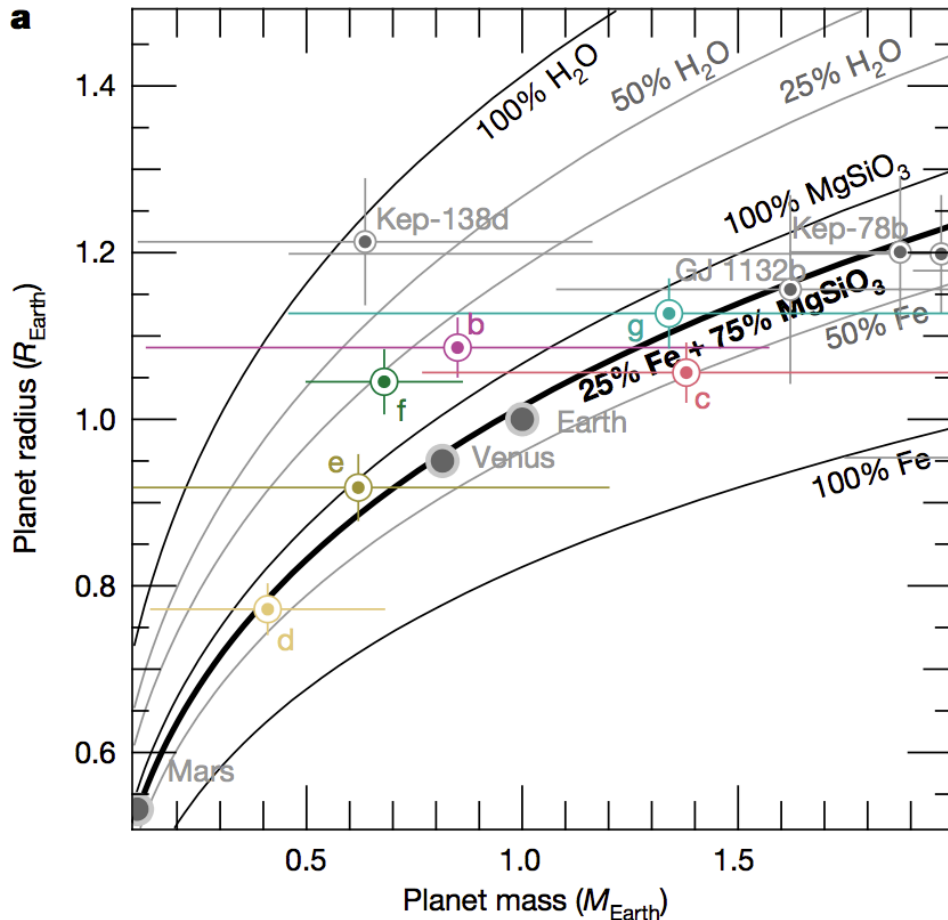
# First things first:

1. This is an actively developing code
2. This may not be the best tool for your specific job
3. It's in python, so we'll be coding!

# First things first:

1. This is an actively developing code
2. This may not be the best tool for your specific job
3. It's in python, so we'll be coding!

# How do we “measure” *composition*?



Gillon et al., 2017

Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

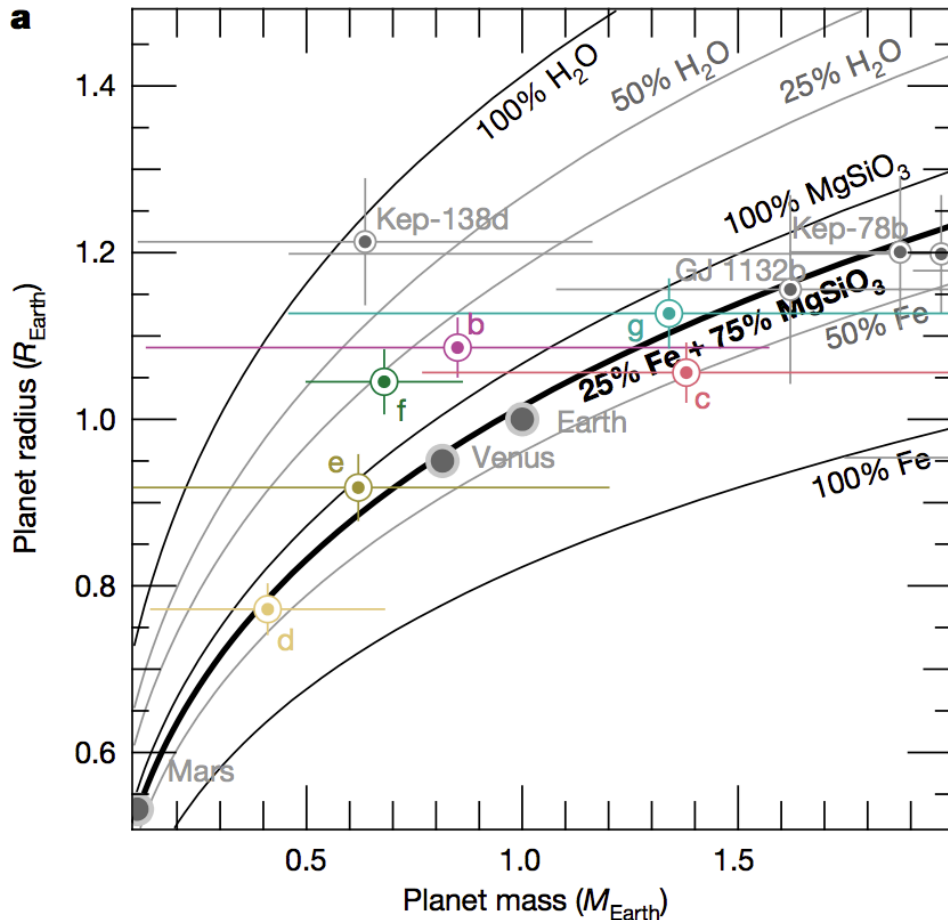
Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$

# How do we “measure” *composition*?



Gillon et al., 2017

Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$

# How do we “measure” *composition*?

Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

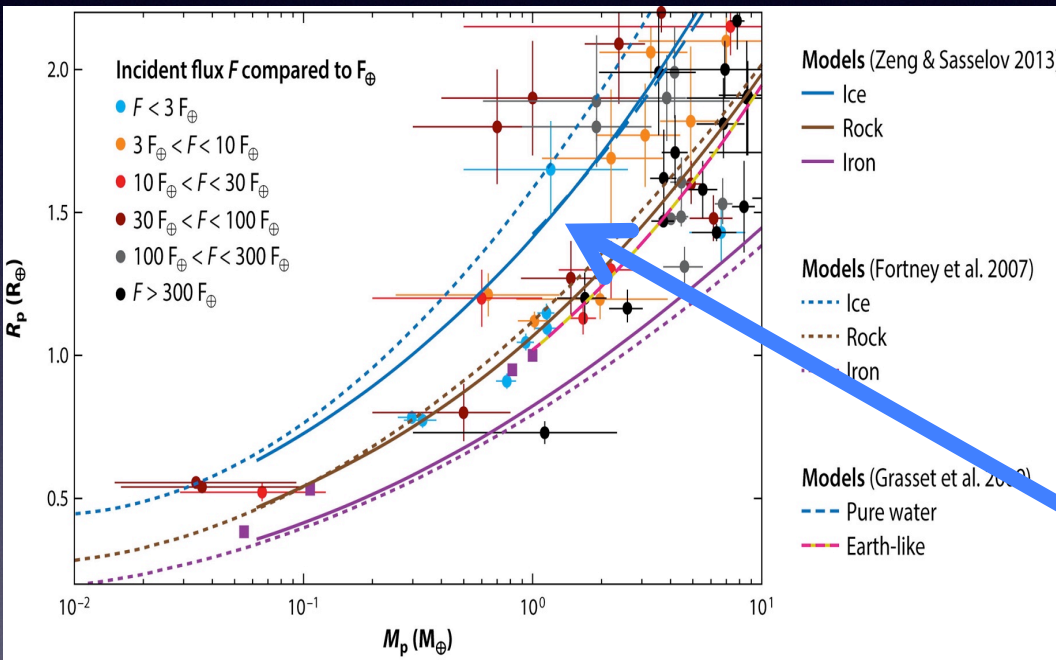
$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

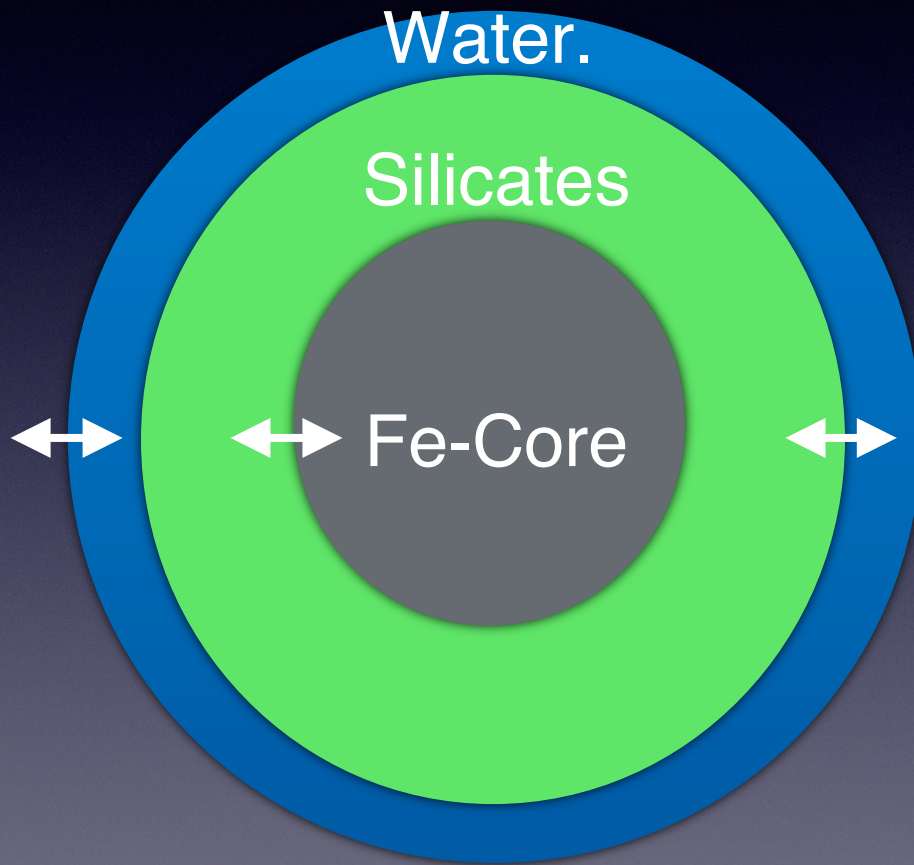
Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$



Jontof-Hutter, 2019, Ann. Rev.

# Let's Build a Planet!



Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$

# Let's Build a Planet! **M Earth Mass**

Comp.	Mass
Water	<b>M</b>
Silicate	dM
Silicate	dM
Silicate	dM
Silicate	dM
Core	dM
Core	dM
Core	<b>0</b>

Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$



# Let's Build a Planet! **M** Earth Mass

Comp.	Mass	Rad.	Pres.
Water	<b>M</b>	$dr_7$	<b>1 bar</b>
Silicate	dM	$dr_6$	dP
Silicate	dM	$dr_5$	dP <sub>2</sub>
Silicate	dM	$dr_4$	dP <sub>3</sub>
Silicate	dM	$dr_3$	dP <sub>4</sub>
Core	dM	$dr_2$	dP <sub>5</sub>
Core	dM	dr	dP <sub>6</sub>
Core	<b>0</b>	<b>0</b>	dP <sub>7</sub>

Mass in a sphere

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r),$$

Hydrostatic Equil.

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2},$$

Equation of State of material

$$\rho(r) = f(P(r), T(r), X(r))$$

Adiabatic temperature gradient

$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)},$$

# Let's Build a Planet! $M$ Earth Mass

Comp.	Mass	Rad.	Pres.	Temp.
Water	$M$	$dr_7$	$P_0 = 1 \text{ bar}$	$T_P$
Silicate	$dM$	$dr_6$	$P_0 + dP$	$T_P + dT_1$
Silicate	$dM$	$dr_5$	$P_0 + dP_1 + dP_2$	$T_P + dT_1 + dT_2$
Silicate	$dM$	$dr_4$	$\dots + dP_3$	$\dots + dT_3$
Silicate	$dM$	$dr_3$	$\dots + dP_4$	$dT_4$
Core	$dM$	$dr_2$	$\dots + dP_5$	300 K
Core	$dM$	$dr$	$\dots + dP_6$	300 K
Core	$0$	$0$	$\dots + dP_7$	300 K

# Let's Build a Planet! $M$ Earth Mass

Comp.	Mass	Rad.	Pres.	Temp.	Grav.
Water	$M$	$dr_7$	$P_0 = 1 \text{ bar}$	$T_P$	$\dots + dg_7$
Silicate	$dM$	$dr_6$	$P_0 + dP$	$T_P + dT_1$	$\dots + dg_6$
Silicate	$dM$	$dr_5$	$P_0 + dP_1 + dP_2$	$T_P + dT_1 + dT_2$	$\dots + dg_5$
Silicate	$dM$	$dr_4$	$\dots + dP_3$	$\dots + dT_3$	$\dots + dg_4$
Silicate	$dM$	$dr_3$	$\dots + dP_4$	$dT_4$	$\dots + dg_3$
Core	$dM$	$dr_2$	$\dots + dP_5$	300 K	$dg + dg_2$
Core	$dM$	$dr$	$\dots + dP_6$	300 K	$dg$
Core	$0$	$0$	$\dots + dP_7$	300 K	$0$

# Let's Build a Planet! $M$ Earth Mass

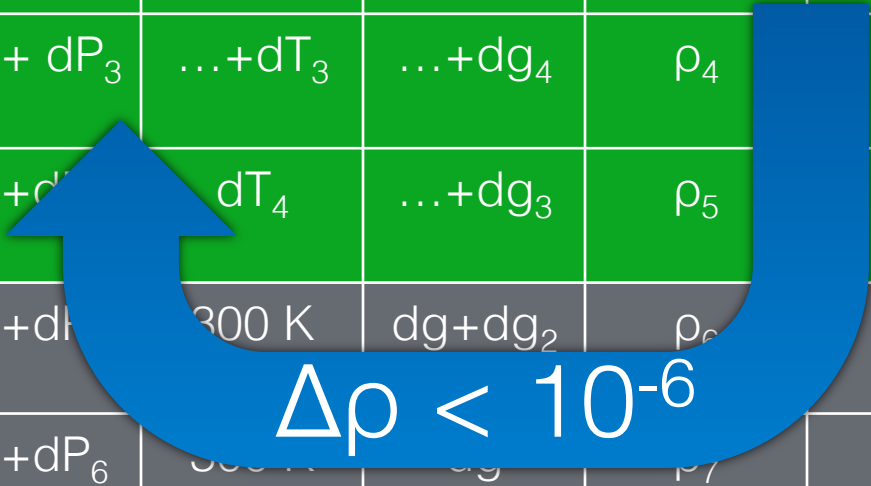
Comp.	Mass	Rad.	Pres.	Temp.	Grav.	Density
Water	$M$	$dr_7$	$P_0 = 1$ bar	$T_P$	$\dots + dg_7$	$\rho_1$
Silicate	$dM$	$dr_6$	$P_0 + dP$	$T_P + dT_1$	$\dots + dg_6$	$\rho_2$
Silicate	$dM$	$dr_5$	$P_0 + dP_1$ $+ dP_2$	$T_P + dT_1$ $+ dT_2$	$\dots + dg_5$	$\rho_3$
Silicate	$dM$	$dr_4$	$\dots + dP_3$	$\dots + dT_3$	$\dots + dg_4$	$\rho_4$
Silicate	$dM$	$dr_3$	$\dots + dP_4$	$dT_4$	$\dots + dg_3$	$\rho_5$
Core	$dM$	$dr_2$	$\dots + dP_5$	300 K	$dg + dg_2$	$\rho_6$
Core	$dM$	$dr$	$\dots + dP_6$	300 K	$dg$	$\rho_7$
Core	$0$	$0$	$\dots + dP_7$	300 K	$0$	$\rho_8$

# Let's Build a Planet! $M$ Earth Mass

Comp.	Mass	Rad.	Pres.	Temp.	Grav.	Density	Rad.
Water	$M$	$dr_7$	$P_0 = 1$ bar	$T_P$	$\dots + dg_7$	$\rho_1$	$dr_7$
Silicate	$dM$	$dr_6$	$P_0 + dP$	$T_P + dT_1$	$\dots + dg_6$	$\rho_2$	$dr_6$
Silicate	$dM$	$dr_5$	$P_0 + dP_1$ $+ dP_2$	$T_P + dT_1$ $+ dT_2$	$\dots + dg_5$	$\rho_3$	$dr_5$
Silicate	$dM$	$dr_4$	$\dots + dP_3$	$\dots + dT_3$	$\dots + dg_4$	$\rho_4$	$dr_4$
Silicate	$dM$	$dr_3$	$\dots + dP_4$	$dT_4$	$\dots + dg_3$	$\rho_5$	$dr_3$
Core	$dM$	$dr_2$	$\dots + dP_5$	300 K	$dg + dg_2$	$\rho_6$	$dr_2$
Core	$dM$	$dr$	$\dots + dP_6$	300 K	$dg$	$\rho_7$	$dr$
Core	$dM$	$0$	$\dots + dP_7$	300 K	$0$	$\rho_8$	$0$

# Let's Build a Planet! **M Earth Mass**

Comp.	Mass	Rad.	Pres.	Temp.	Grav.	Density	Rad.
Water	<b>M</b>	$dr_7$	$P_0 = 1$ bar	$T_P$	$\dots + dg_7$	$\rho_1$	$dr_7$
Silicate	$dM$	$dr_6$	$P_0 + dP$	$T_P + dT_1$	$\dots + dg_6$	$\rho_2$	$dr_6$
Silicate	$dM$	$dr_5$	$P_0 + dP_1$ $+ dP_2$	$T_P + dT_1$ $+ dT_2$	$\dots + dg_5$	$\rho_3$	$dr_5$
Silicate	$dM$	$dr_4$	$\dots + dP_3$	$\dots + dT_3$	$\dots + dg_4$	$\rho_4$	$dr_4$
Silicate	$dM$	$dr_3$	$\dots + dP_4$	$\dots + dT_4$	$\dots + dg_3$	$\rho_5$	$dr_3$
Core	$dM$	$dr_2$	$\dots + dP_5$	300 K	$dg + dg_2$	$\rho_6$	$dr_2$
Core	$dM$	$dr$	$\dots + dP_6$	300 K	$dg$	$\rho_7$	$dr$
Core	$dM$	<b>0</b>	$\dots + dP_7$	300 K	<b>0</b>	$\rho_8$	<b>0</b>



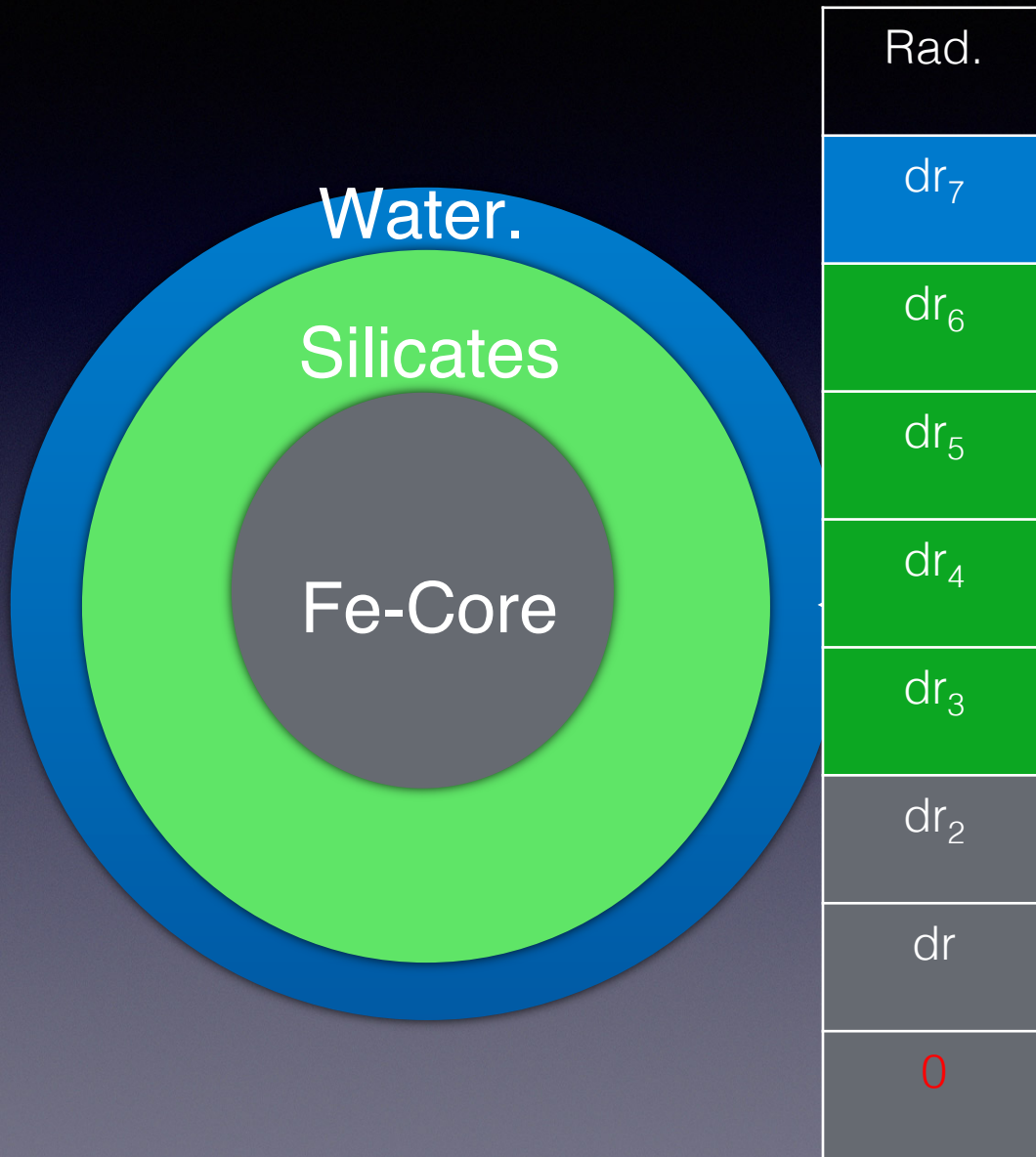
# Equations of State

$$\rho(r) = f(P(r), T(r), X(r))$$
$$\frac{dT(r)}{dr} = \frac{\alpha(r)g(r)}{C_P(r)}$$

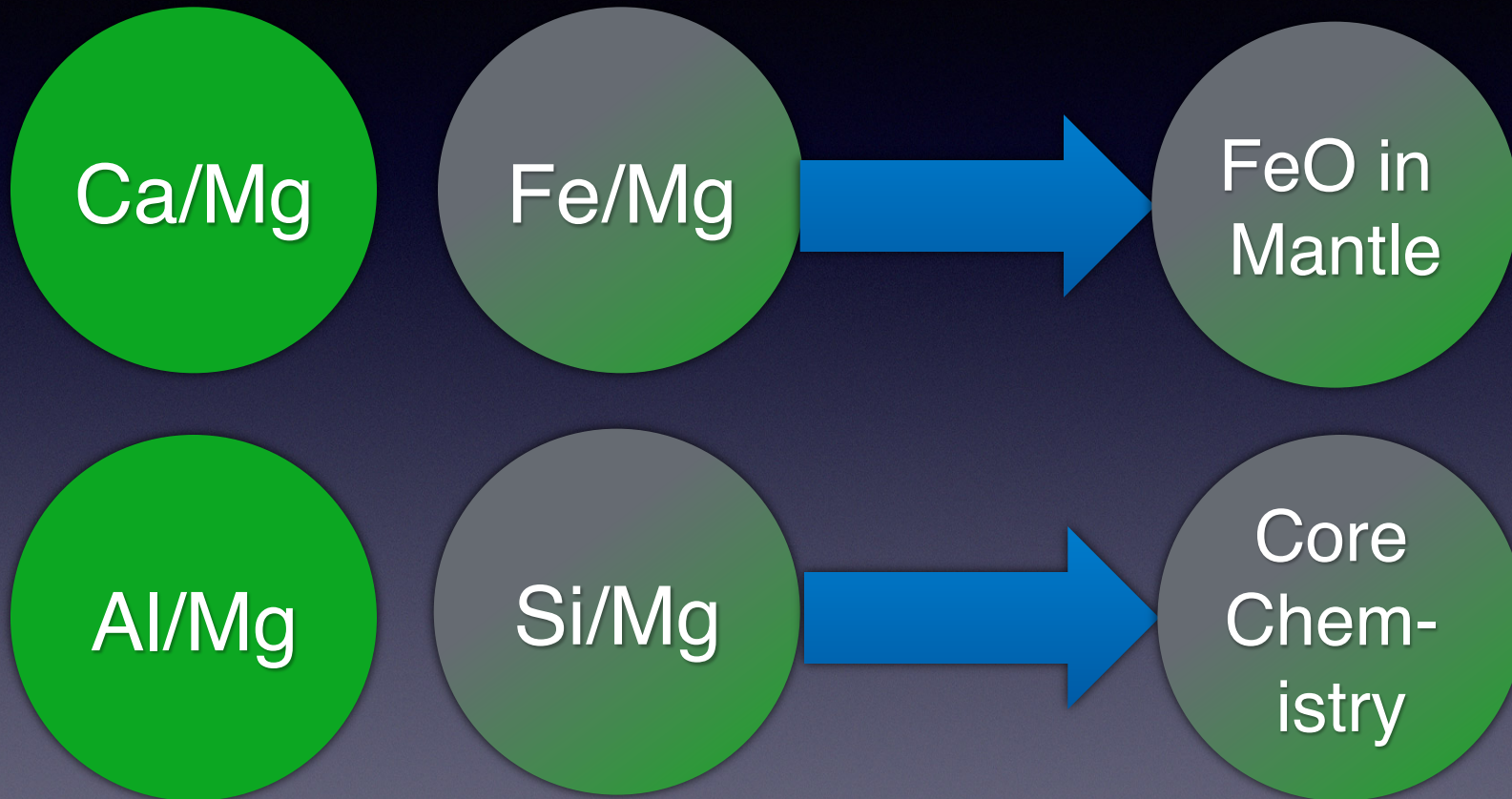
IAPWS

PerPlex or Grids

EoS from Shock experiments

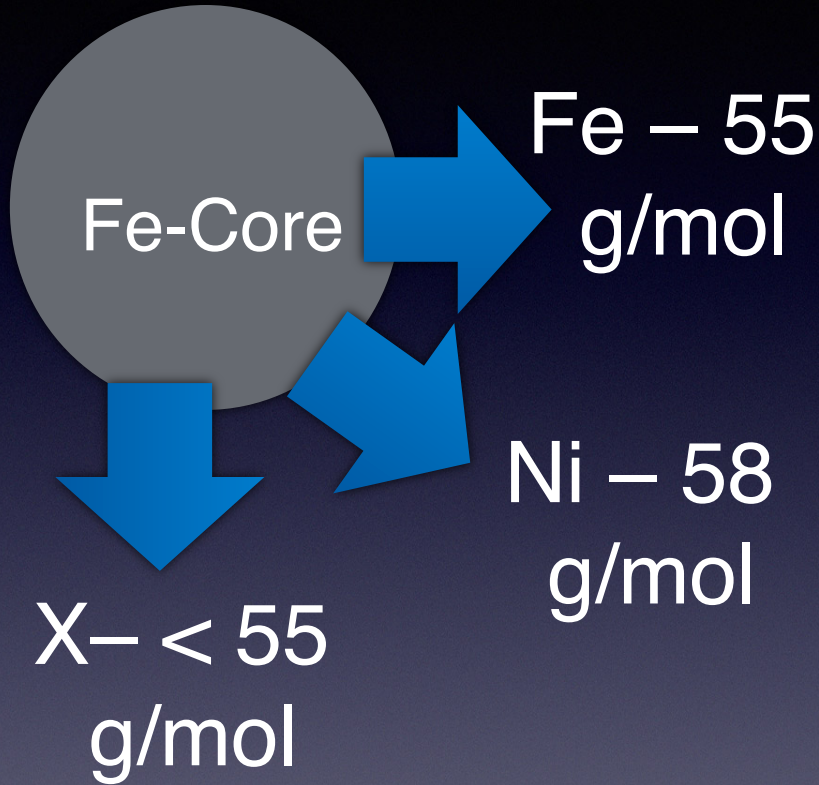


# Calculating Composition





# Core Chemistry



$$\rho(r) = f(P(r), T(r), X(r))$$

P = Mass/  
Volume

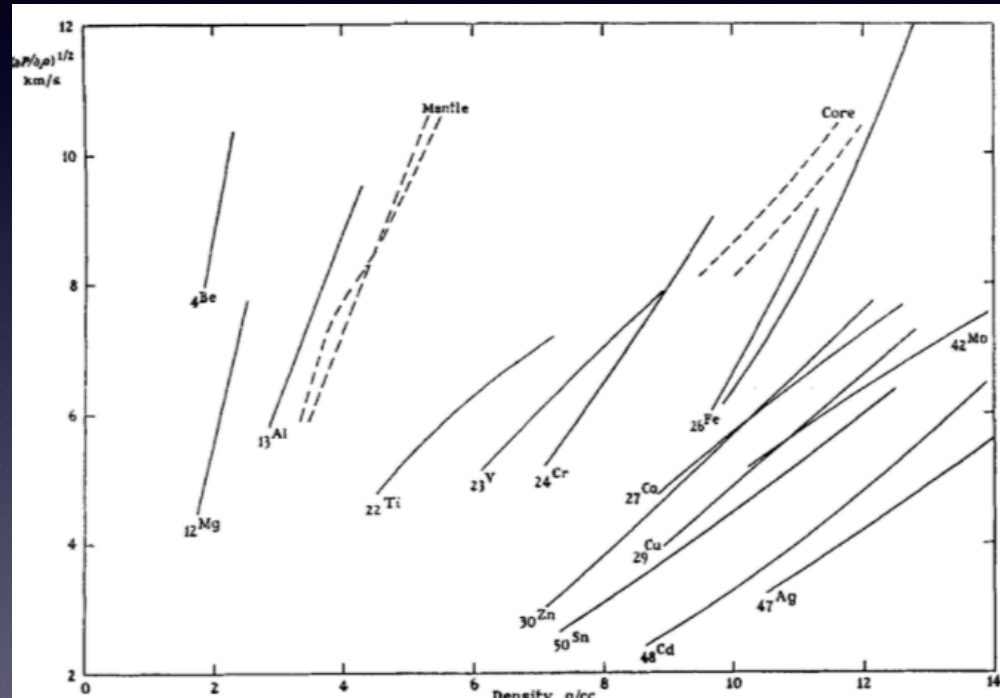
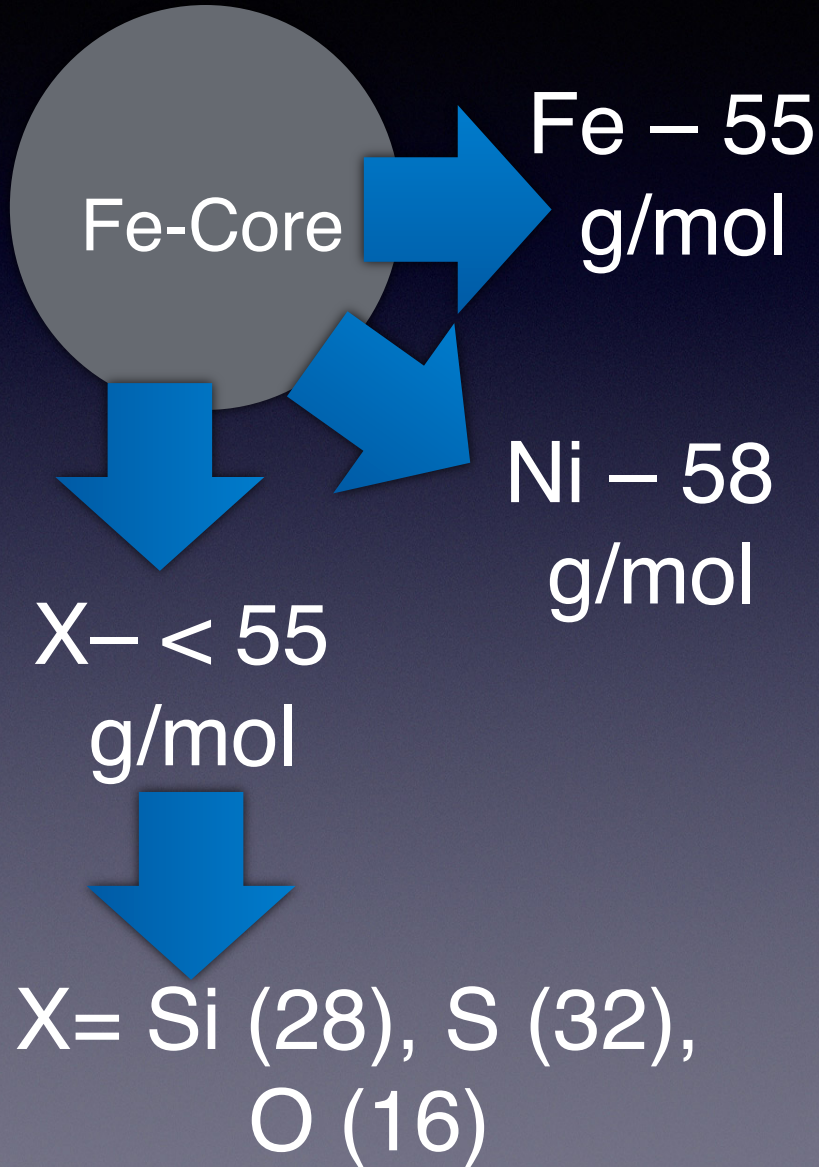


FIG. 5.—Hydrodynamical sound velocity as function of density for metals and for the Earth's mantle and core.

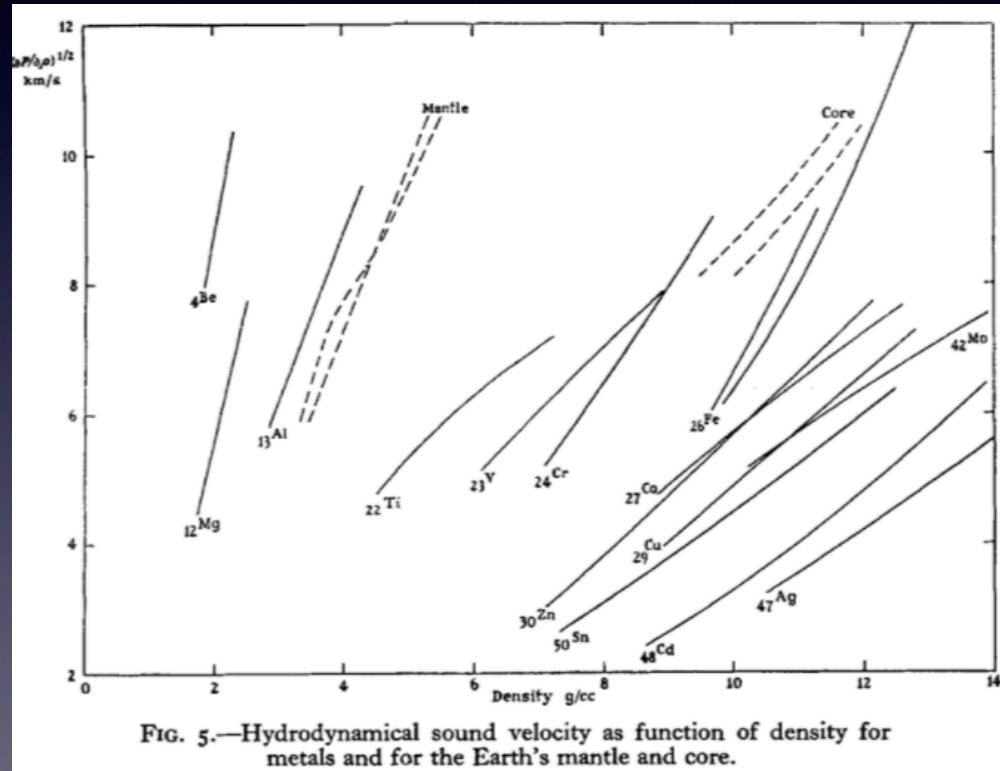
Birch, 1964 JGR

# Core Chemistry



$$\rho(r) = f(P(r), T(r), X(r))$$

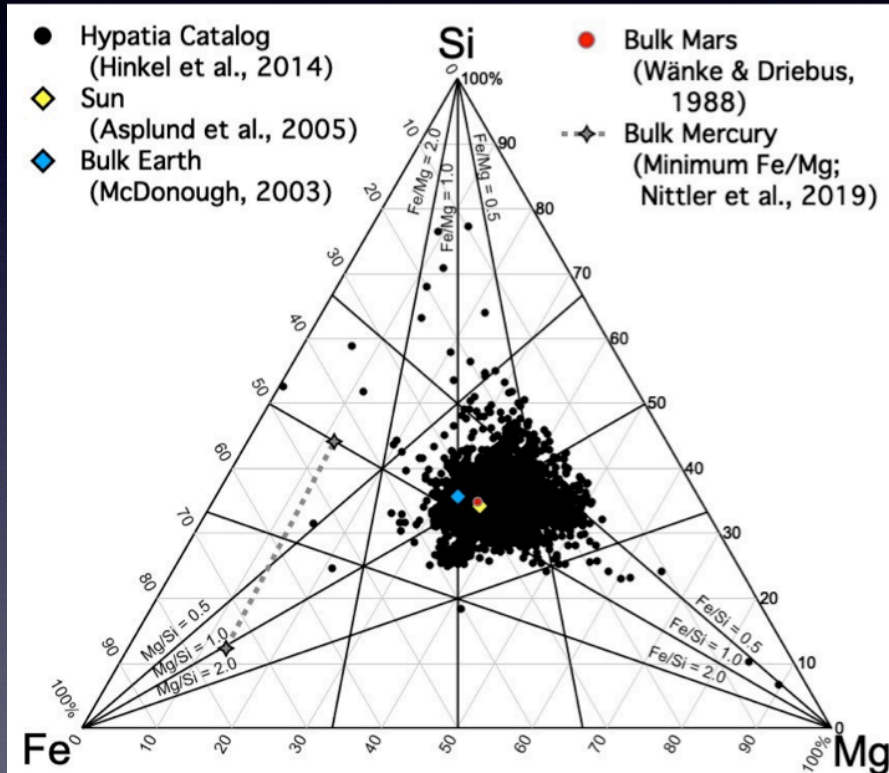
P = Mass/  
Volume



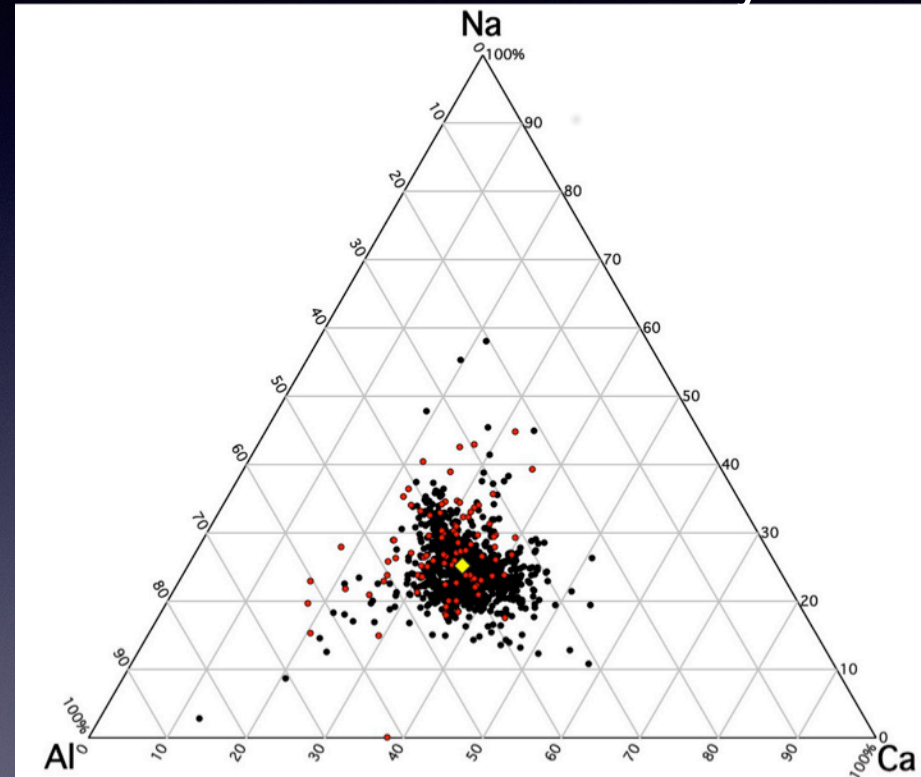
Birch, 1964 JGR

# Stellar diversity ~ Planet diversity

Refractory: 1-to-1-ish



Moderately Volatile  
Ultra-refractory

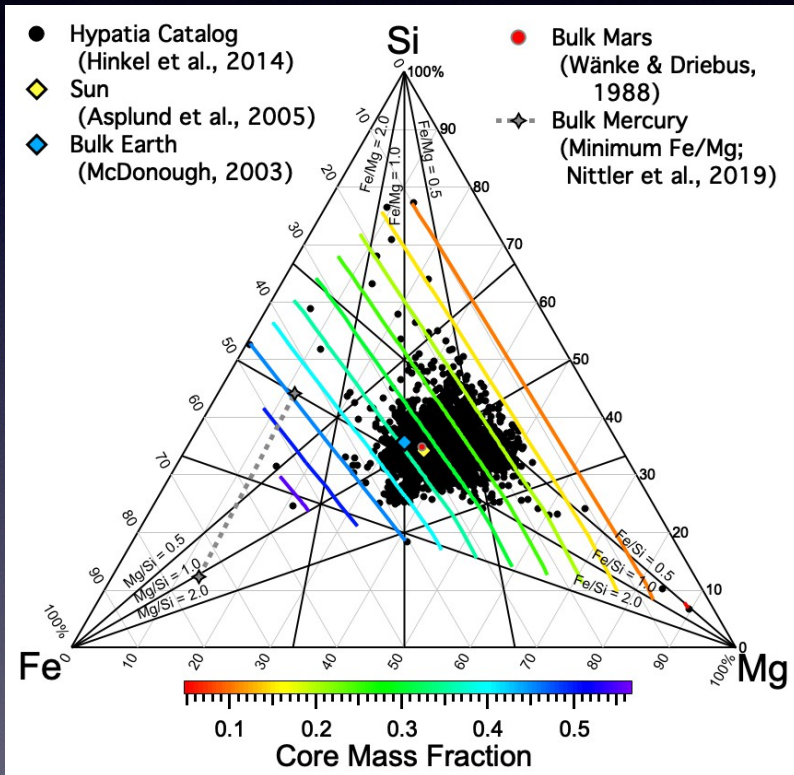


Earth: 40%  $\uparrow$  + 50%  $\circ$   
+ 10% everything else

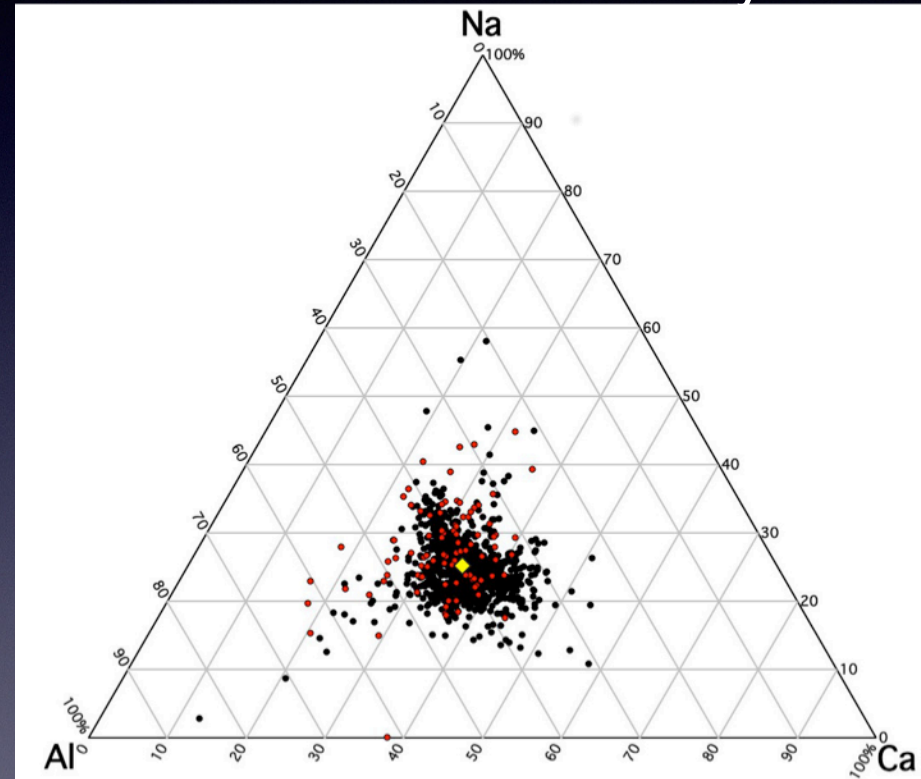
Unterborn et al., 2019

# Stellar diversity ~ Planet diversity

Refractory: 1-to-1-ish



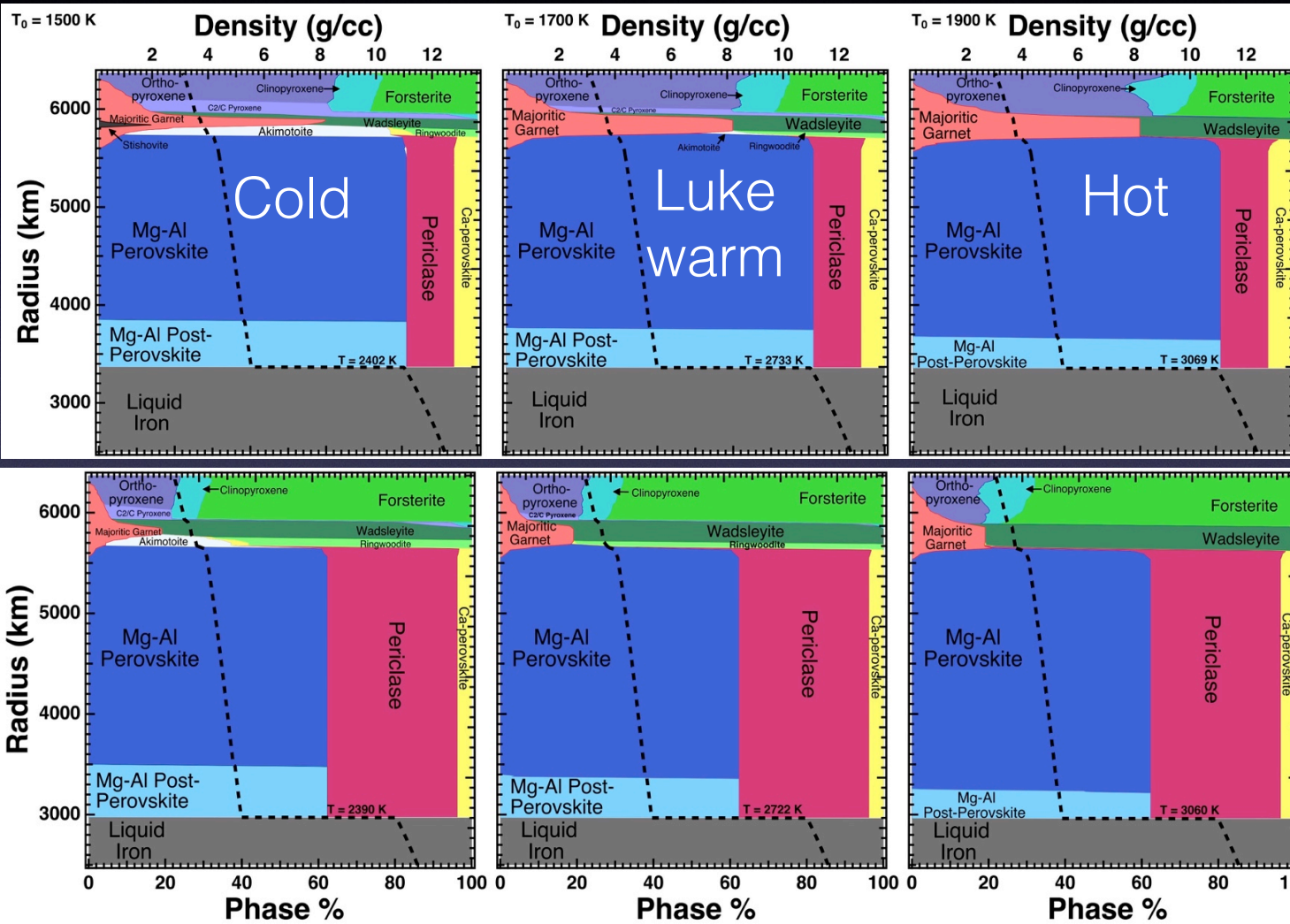
Moderately Volatile  
Ultra-refractory



Earth: 40%  $\uparrow$  + 50% O  
+ 10% everything else

Unterborn et al., 2019

# ExoPlex

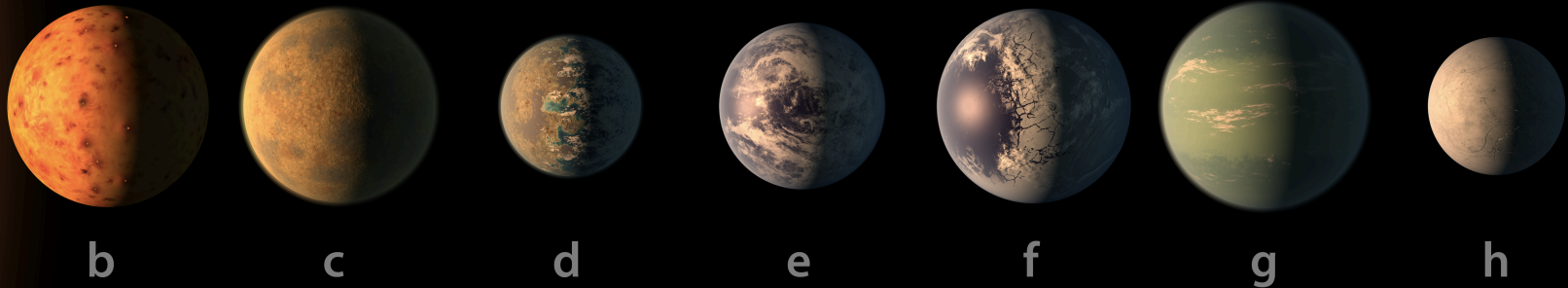


Earth  
 $\text{Mg/Si} = 1.1$   
 $\text{Fe/Mg} = 0.9$   
 $\text{Al/Mg} = 0.1$   
 $\text{Ca/Mg} = 0.07$

Random Star  
 $\text{Mg/Si} = 1.5$   
 $\text{Fe/Mg} = 0.5$   
 $\text{Al/Mg} = 0.04$   
 $\text{Ca/Mg} = 0.04$

# Hands-on Session: Characterizing TRAPPIST-1

## TRAPPIST-1 System



Illustration