Fourier Optics Theory and Fundemantals

Iva Laginja

Sagan Summer Workshop 2024

22 July 2024, Caltech, Pasadena

What is light and how do we describe it?

Light as a wave and E-field, Fourier optics and optical systems

What are the goals?

Understand and describe…

Physically manipulate…

Numerically imitate…

…the **propagation** of **light** from point A to point B.

What are the goals?

Understand and describe…

Physically manipulate…

Numerically imitate…

…the **propagation** of **light** from point A to point B.

Image credit: Getty images

What are the goals?

Understand and describe…
Physically manipulate…
Numerically imitate…

…the **propagation** of **light** from point A to point B.

What are the goals? → Optics in HCI

Understand and describe…
Physically manipulate…
Numerically imitate…

…the **propagation** of **light** from point A to point B…

…through a **high-contrast imaging (HCI)** instrument.

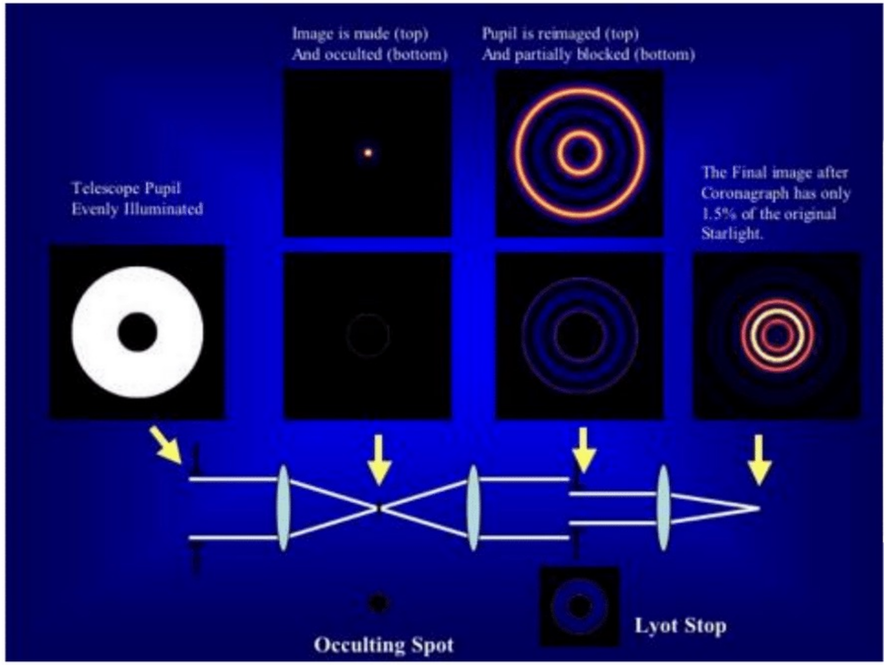
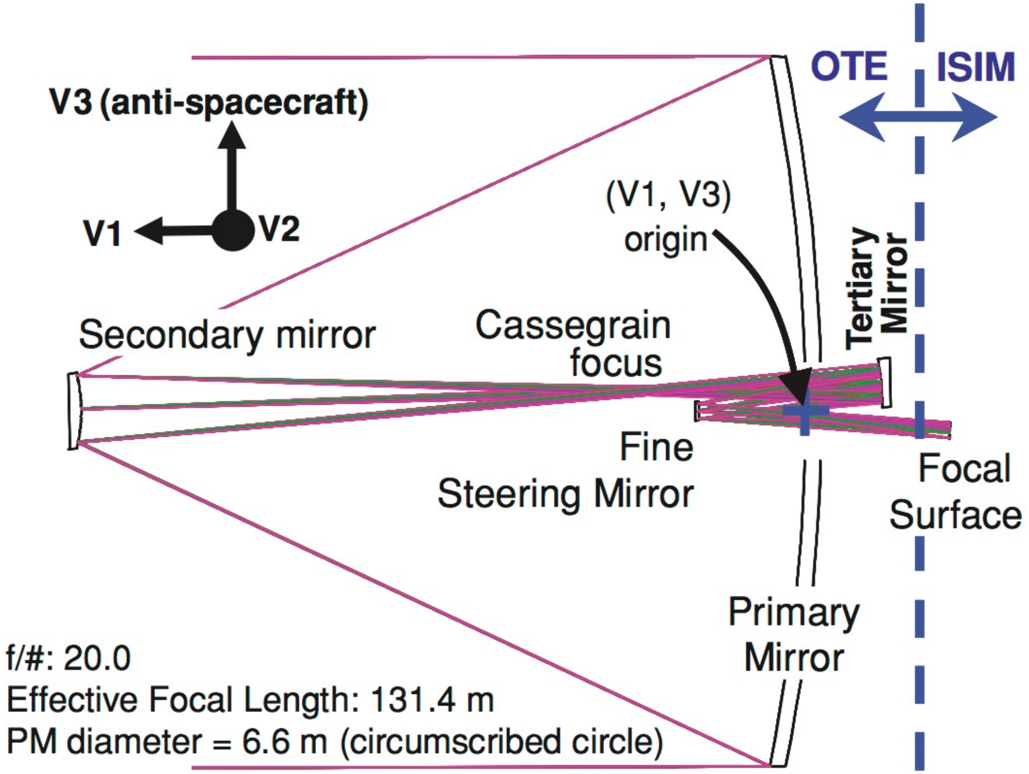


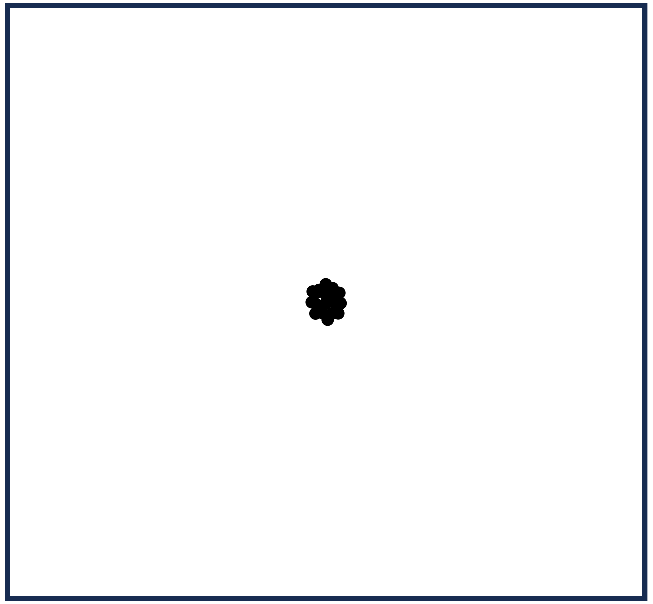
Figure courtesy of A. Sivaramakrishnan

Geometric optics vs. wave optics

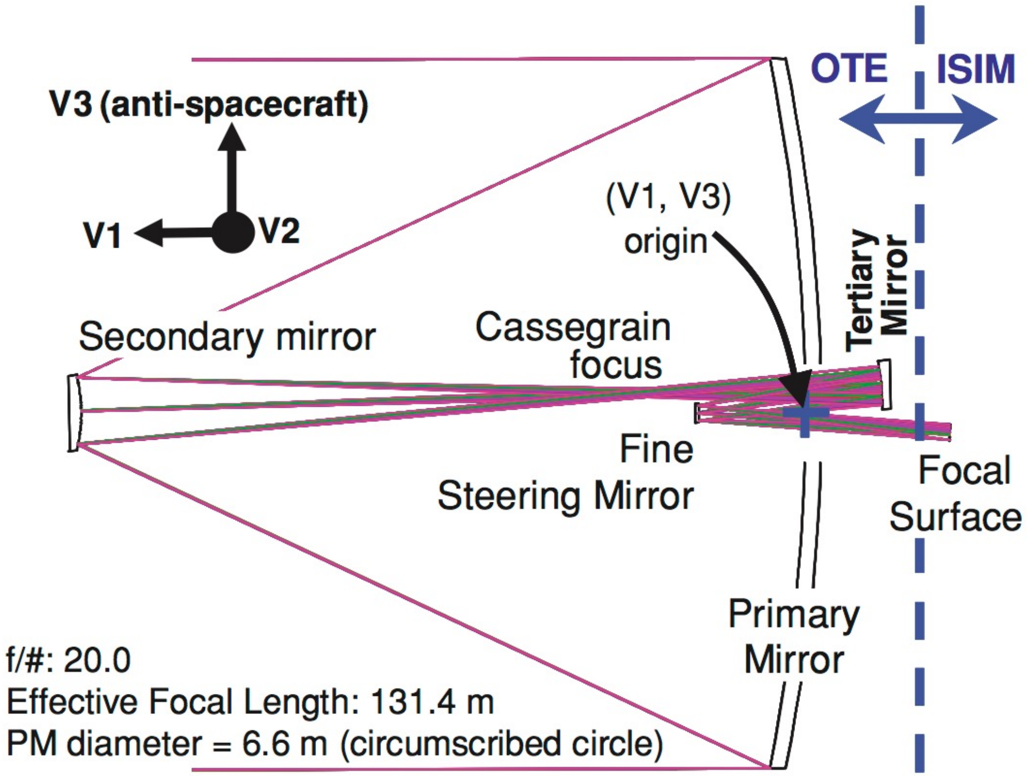


JWST optical design. Credit: Gardner et al., 2006.

Spot diagram



Geometric optics vs. wave optics



JWST optical design. Credit: Gardner et al., 2006.

Spot diagram

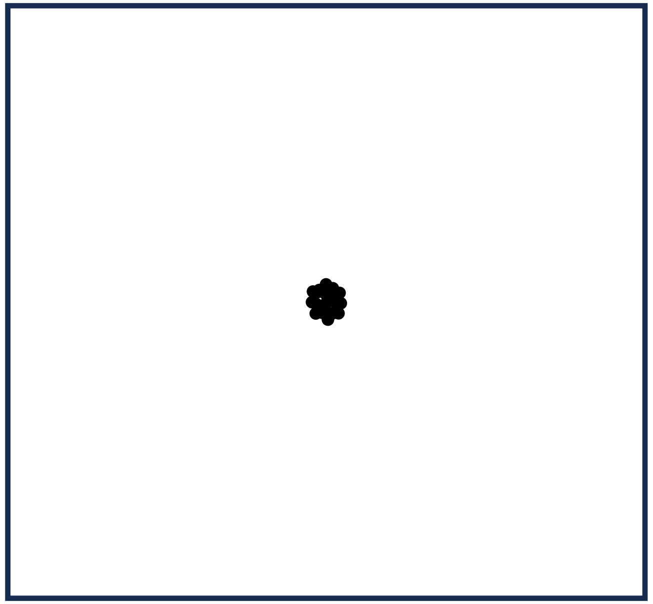
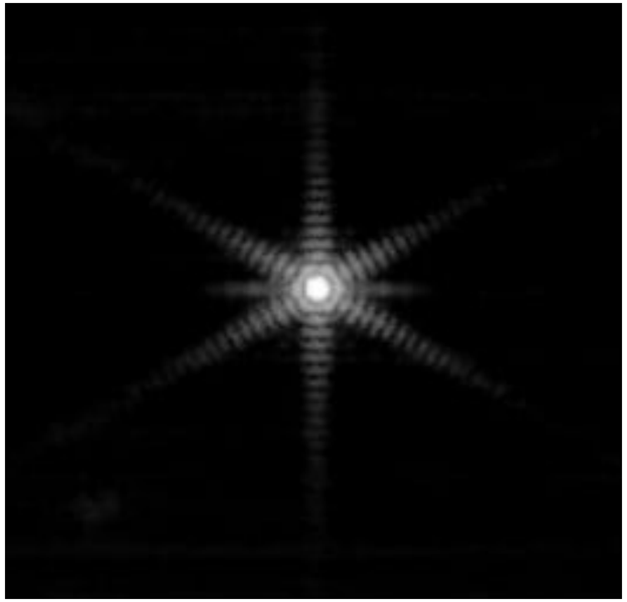
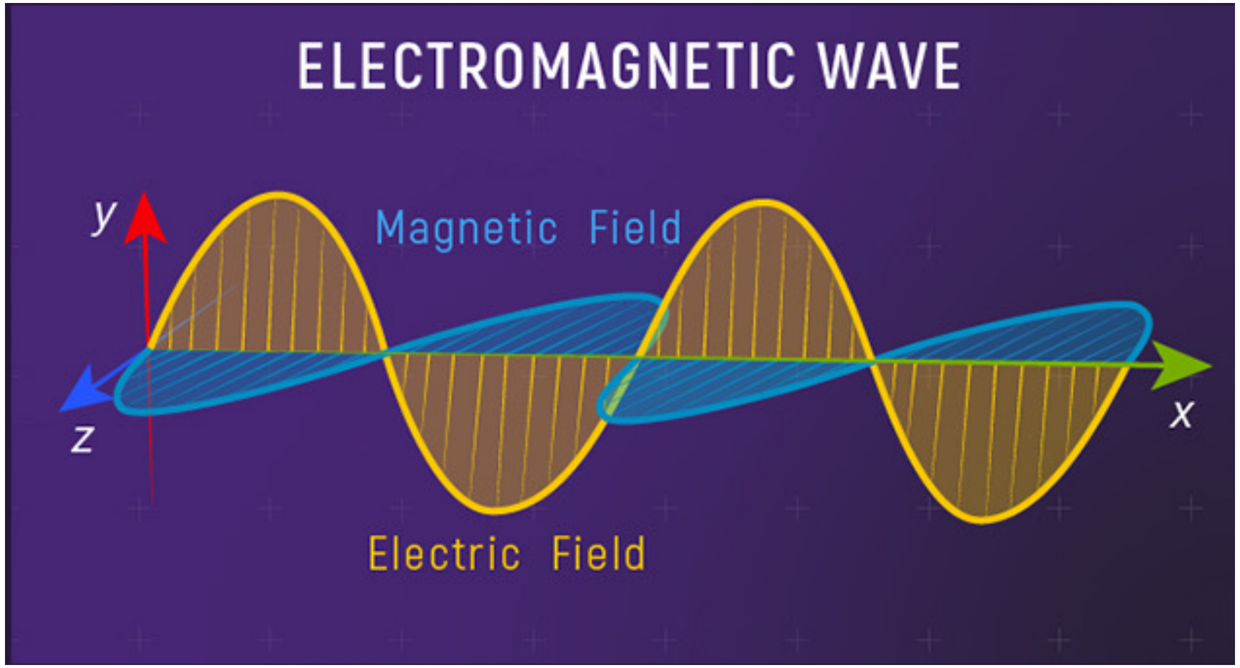


Image (JWST)



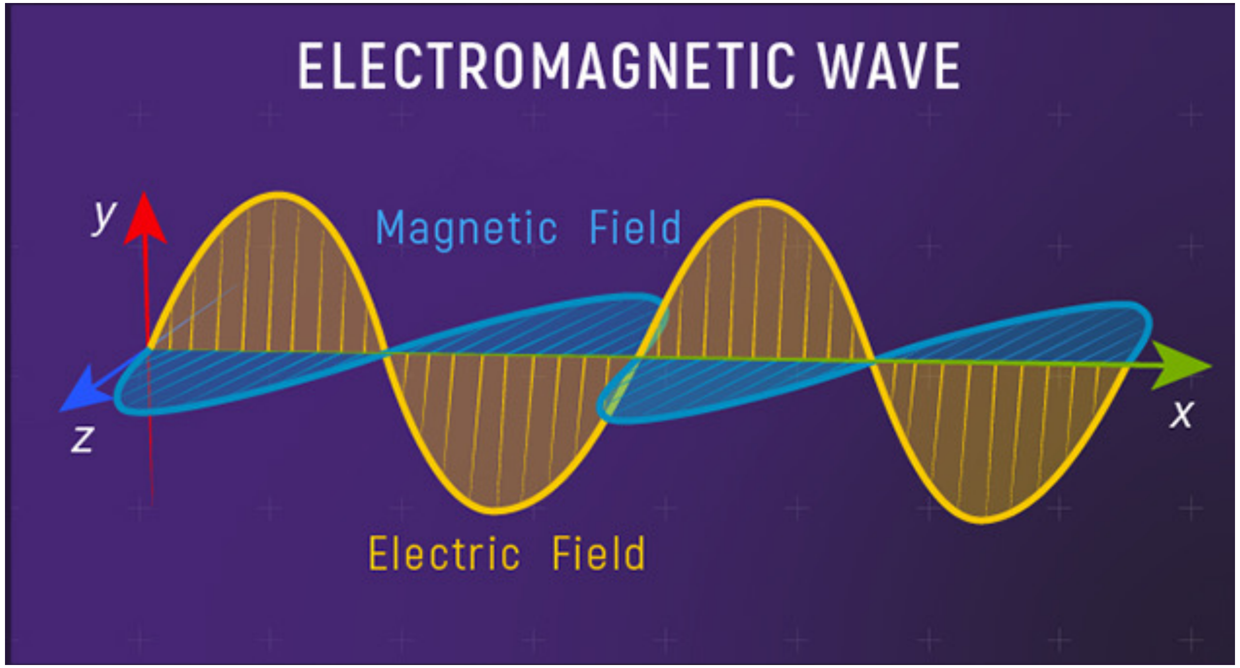
Credit: NASA/STScI.

Light behaves like a **wave** (and also like a particle)



Light is an electromagnetic (EM) wave described by Maxwell's equations → **vector theory** with three components for each field: E_x, E_y, E_z and M_x, M_y, M_z

Light behaves like a **wave** (and also like a particle)



Light is an electromagnetic (EM) wave described by Maxwell's equations → **vector theory** with three components for each field: E_x, E_y, E_z and M_x, M_y, M_z

Under conditions that apply to an HCI instrument*, this can be approximated by a **scalar theory**, where all EM field components follow the same scalar wave equation → light can be represented as a **scalar electric field:**

$$E = E(\vec{r})$$

*Light propagates in a dielectric medium that is linear, isotropic, homogeneous and nondispersive. However, even in an HCI instrument, not all of these are always true.

Light is an E-field with phase and amplitude

$$E(x, y) = A(x, y)e^{i\phi(x,y)}$$

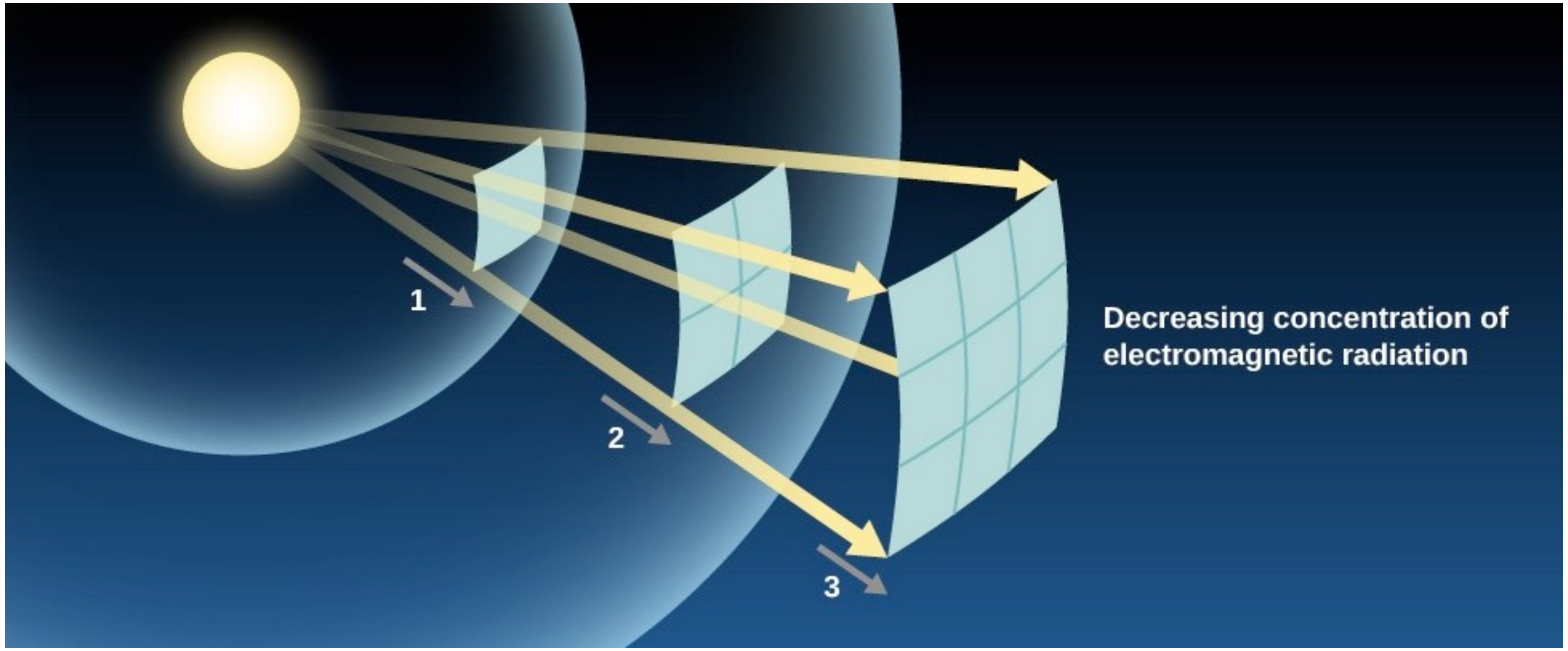
E-field
/wavefront
/wave field

Amplitude

Phase

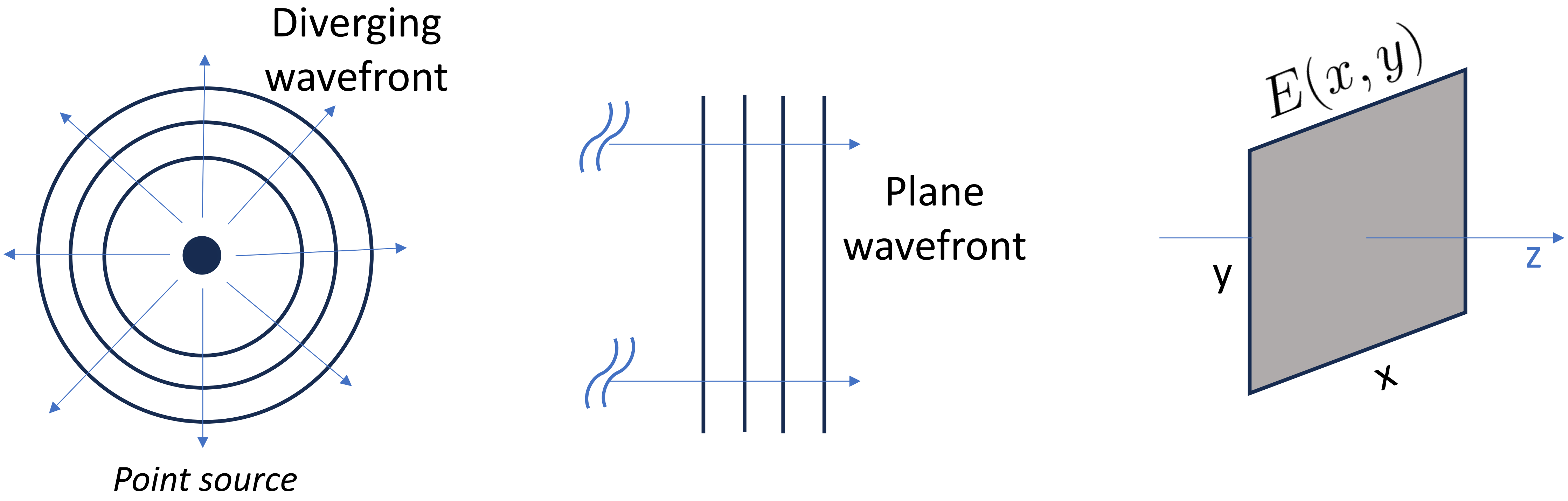
Light propagates in wavefronts

$$E(x, y) = A(x, y)e^{i\phi(x,y)}$$



Light propagates in wavefronts

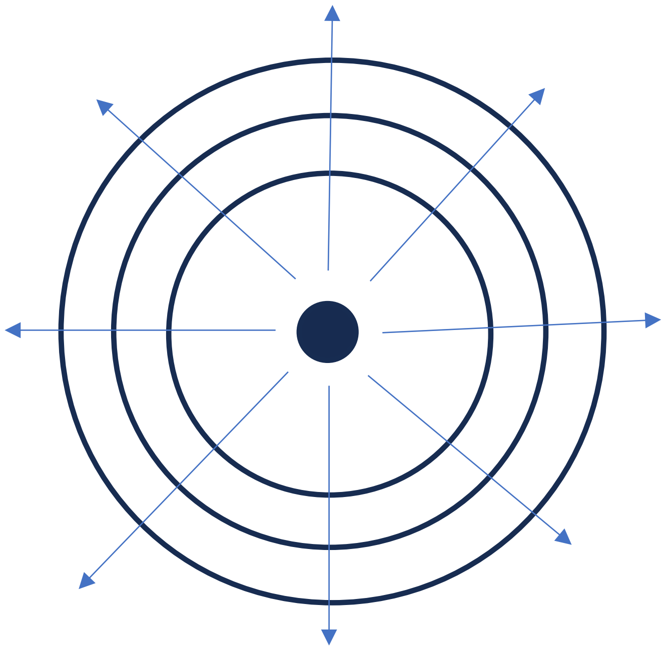
$$E(x,y) = A(x,y)e^{i\phi(x,y)}$$



Light is a scalar field that propagates

$$E(x, y, z, t) = \Re\left\{A(x, y, z)e^{-i\phi(x,y,z)}e^{-i2\pi\nu t}\right\}$$

Propagation to arbitrary positions in space



Point source

Huygens-Fresnel principle propagates from point to point

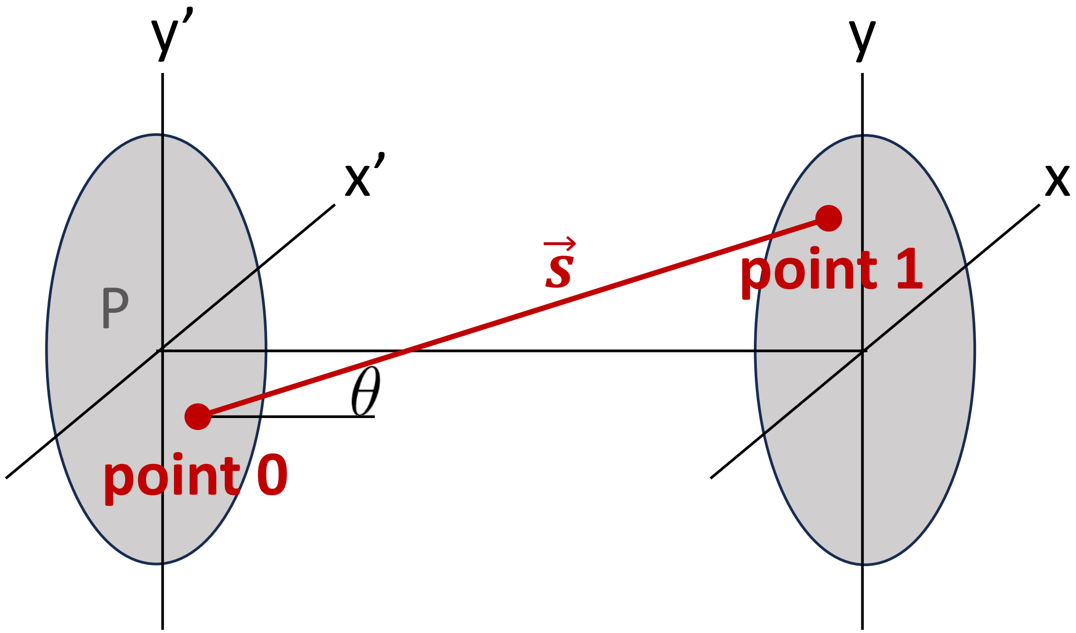
$$E_1(x, y) = \frac{1}{i\lambda} \iint_P E_0(x', y') \frac{e^{ik\vec{s}}}{\vec{s}} \cos(\theta)\, dx' dy'$$

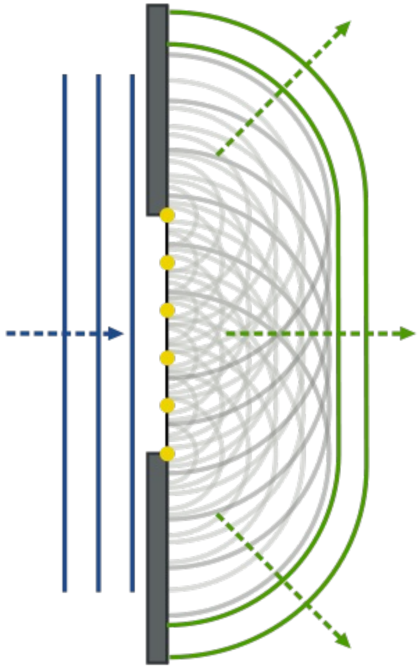
E-field in point 1

E-field in point 0

$$k = \frac{2\pi}{\lambda}$$

Wave number





Describes propagation as sum of wavelets

Fraunhofer integral constrains propagation to far-field

$$E(x,y) \propto \iint_P A(x',y') e^{i\phi} e^{-i\frac{k}{z}(x'x+y'y)} dx'dy'$$

When object sizes in x’ and y’ are negligible with respect to propagation distance z.

Identify Fourier transform in Fraunhofer integral

$$E(x,y) \propto \iint_P \boxed{A(x',y')e^{i\phi}} e^{-i\frac{k}{z}(x'x+y'y)} dx'dy'$$

2D Fourier transform: $$\iint \boxed{f(x,y)} e^{-i\frac{k}{z}(x'x+y'y)} dx'dy'$$

Function to transform

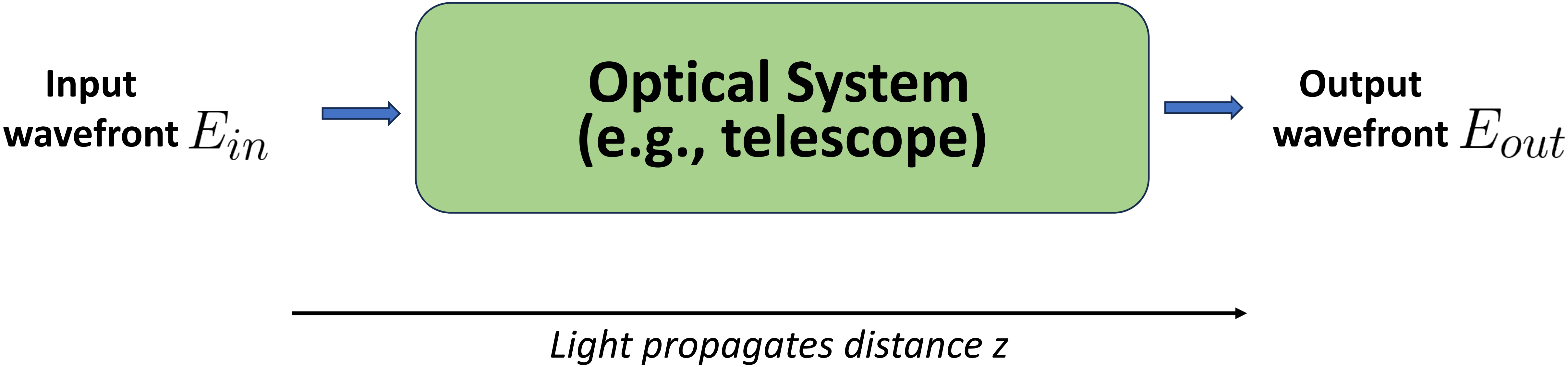
Identify Fourier transform in Fraunhofer integral

$$E(x,y) \propto \iint_P A(x',y') e^{i\phi} e^{-i\frac{k}{z}(x'x+y'y)} dx'dy'$$

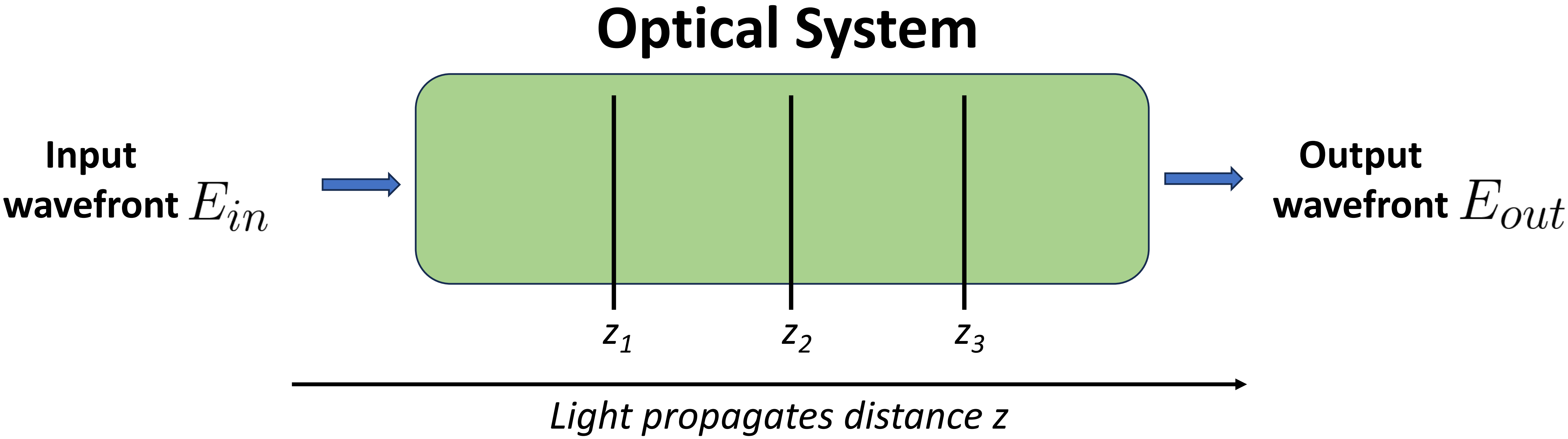
2D Fourier transform: $$\iint f(x,y) e^{-i\frac{k}{z}(x'x+y'y)} dx'dy'$$

$$E(x,y) = \mathcal{F}\{E(x',y')\}$$

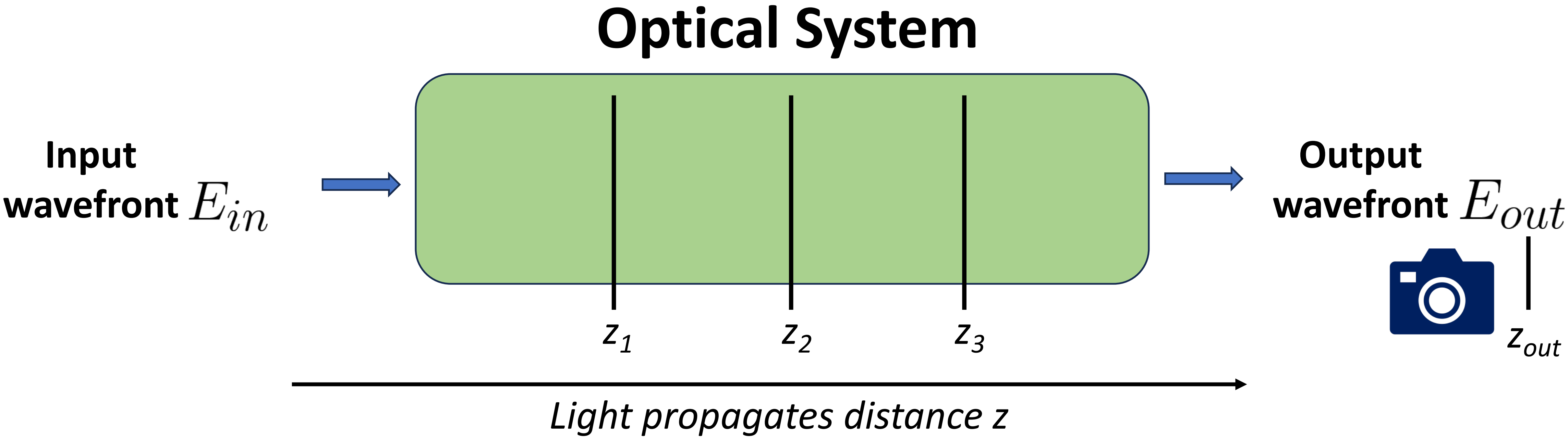
An optical system manipulates wavefronts



We identify relevant optical planes



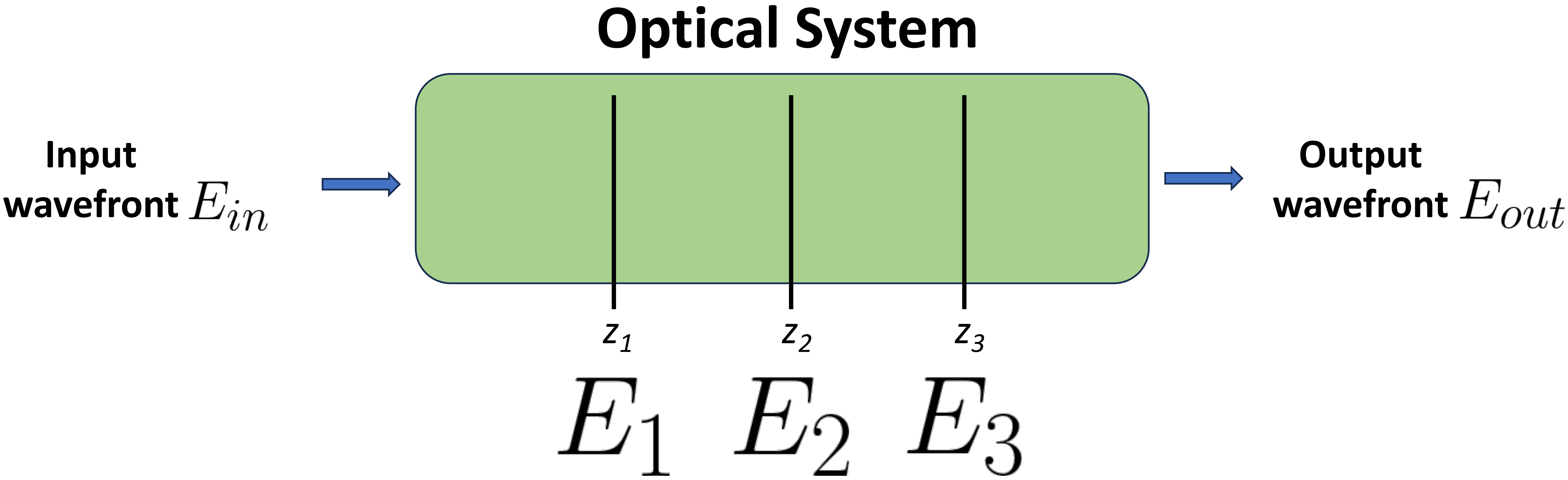
We identify relevant optical planes



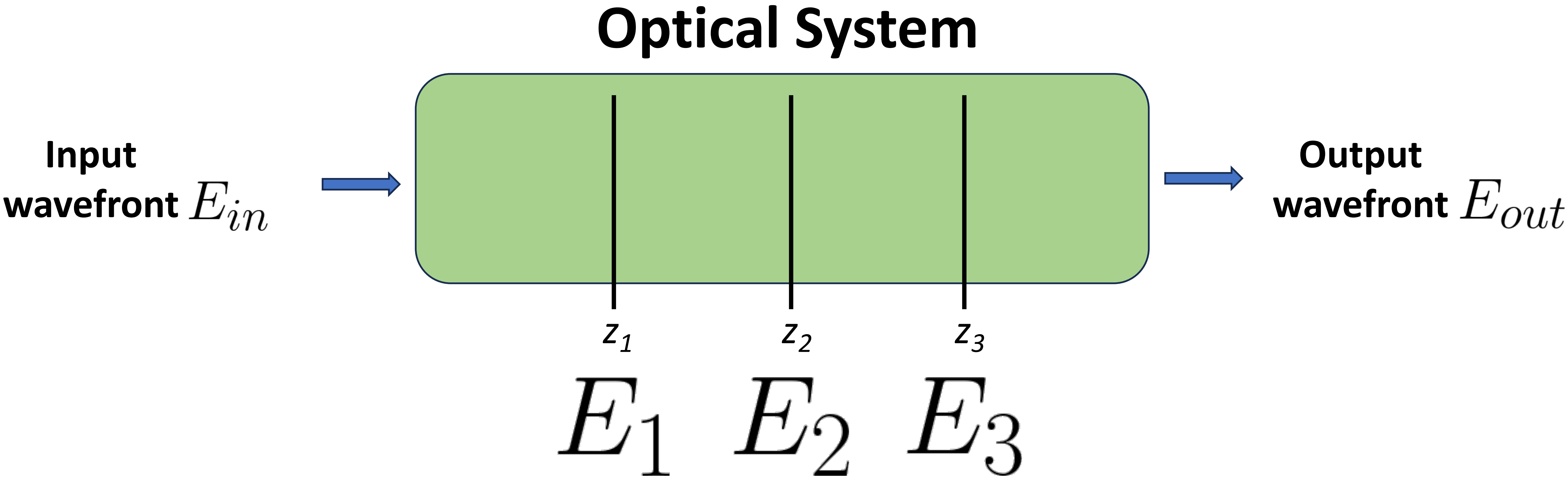
$$I = |E(x, y)|^2$$

Intensity

Each plane holds a relevant wavefront



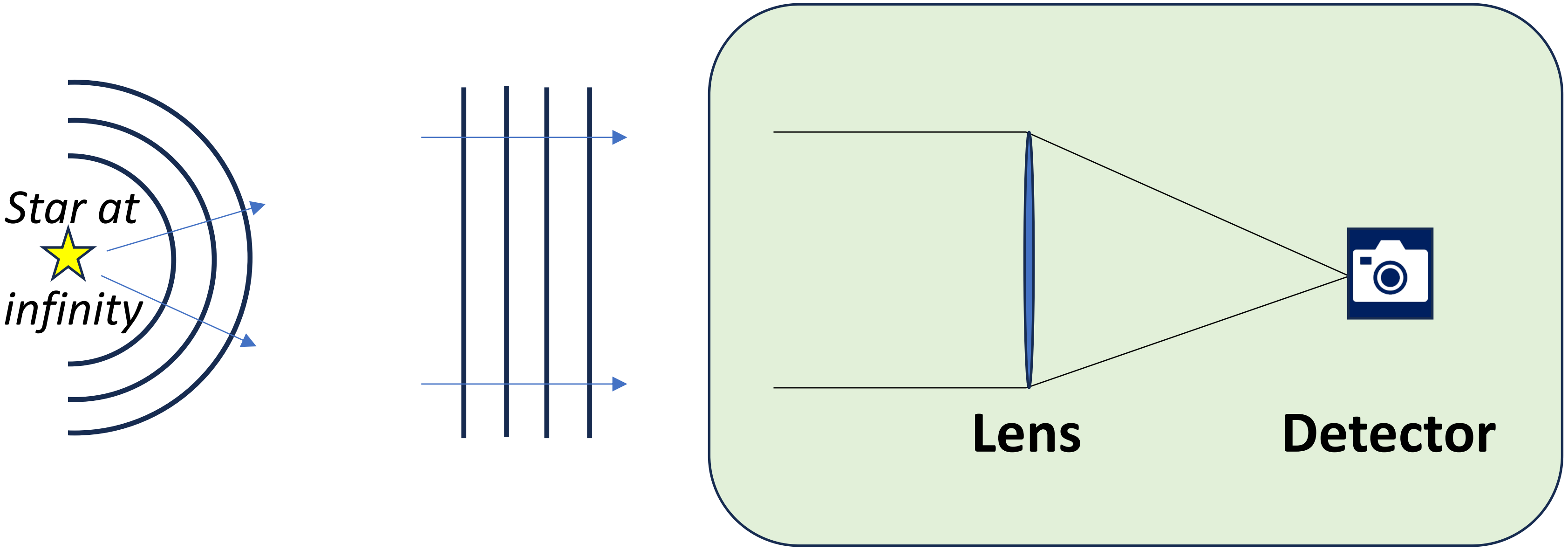
Fourier optics deals with pupil and focal planes



In Fourier optics, we look at wavefronts in pupil planes and focal planes.

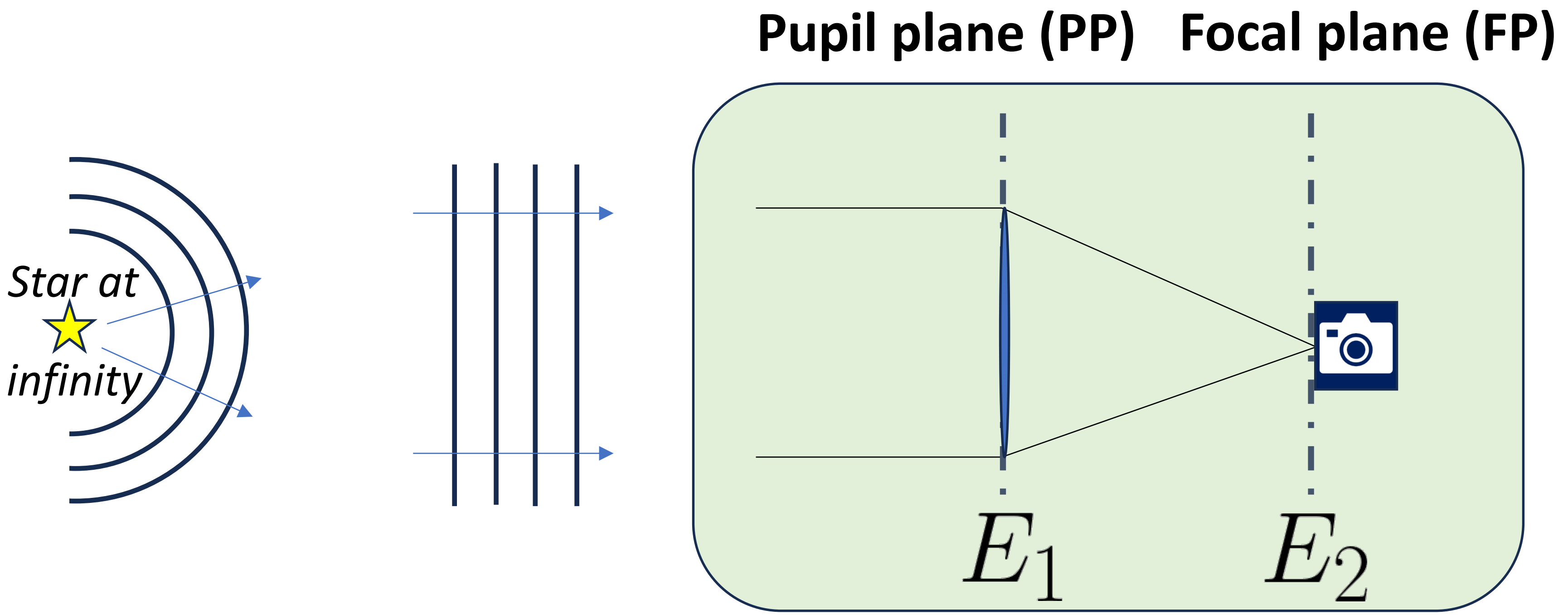
The are Fourier transforms of each other.

Simplest optical system: simple telescope *(e.g., Newtonian telescope)*



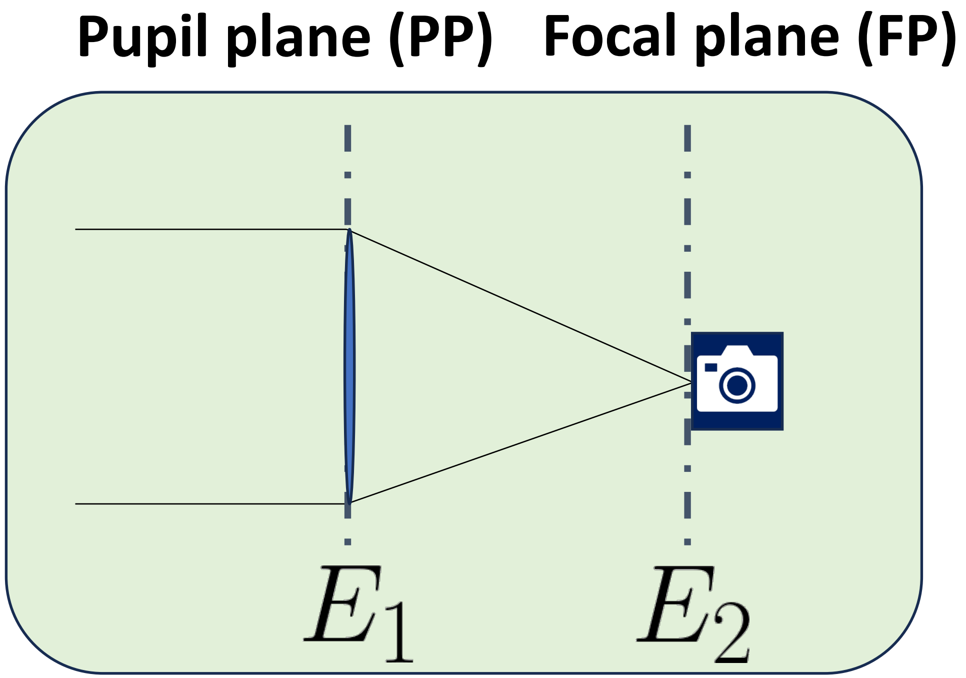
Simplest optical system: simple telescope

(e.g., Newtonian telescope)



Simplest optical system: simple telescope

(e.g., Newtonian telescope)



$$E_1 = A_1 e^{i\phi_1}$$

$$E_2 = A_2 e^{i\phi_2}$$

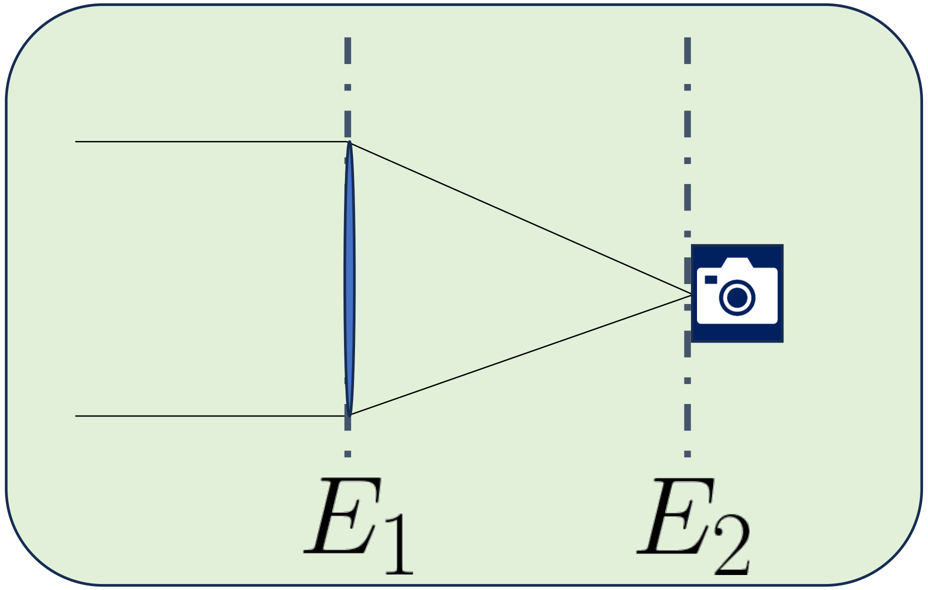
Simplest optical system: simple telescope

(e.g., Newtonian telescope)

Pupil plane

Display complex numbers?

Pupil plane (PP) **Focal plane (FP)**



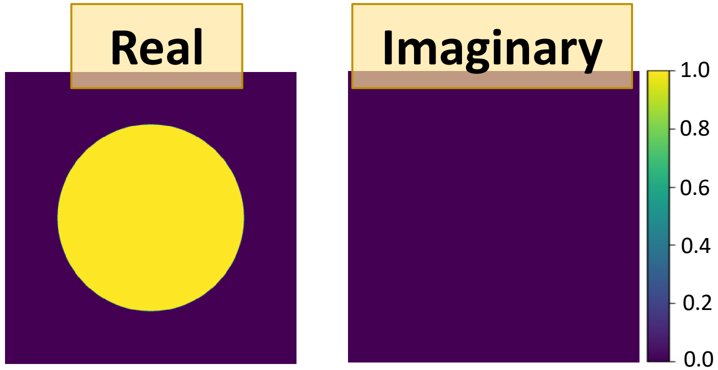
$$E_1 = A_1 e^{i\phi_1}$$

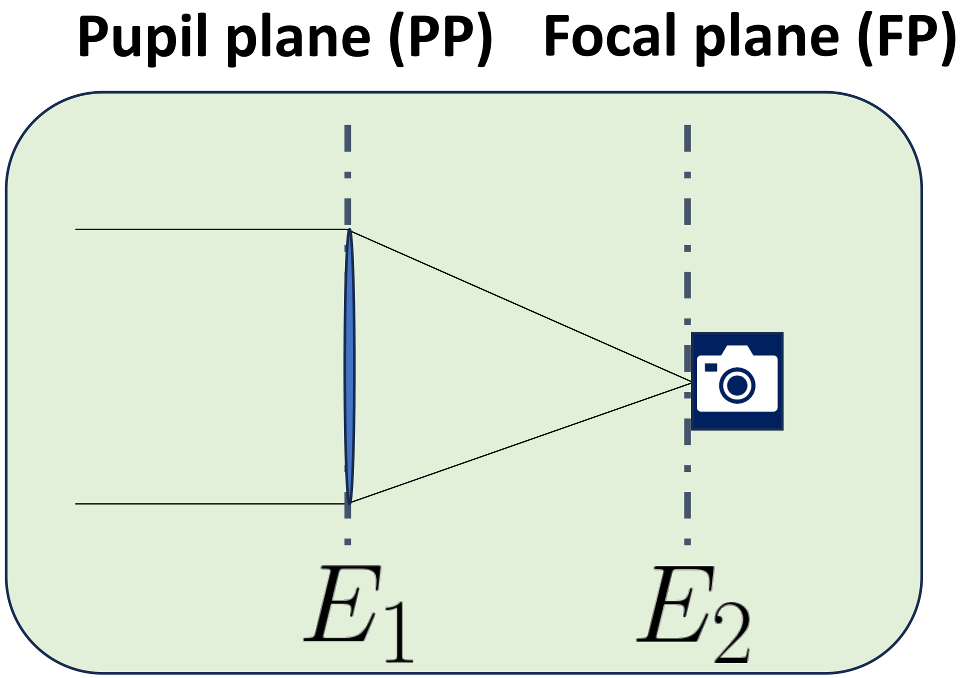
$$E_2 = A_2 e^{i\phi_2}$$

Simplest optical system: simple telescope

(e.g., Newtonian telescope)

Pupil plane





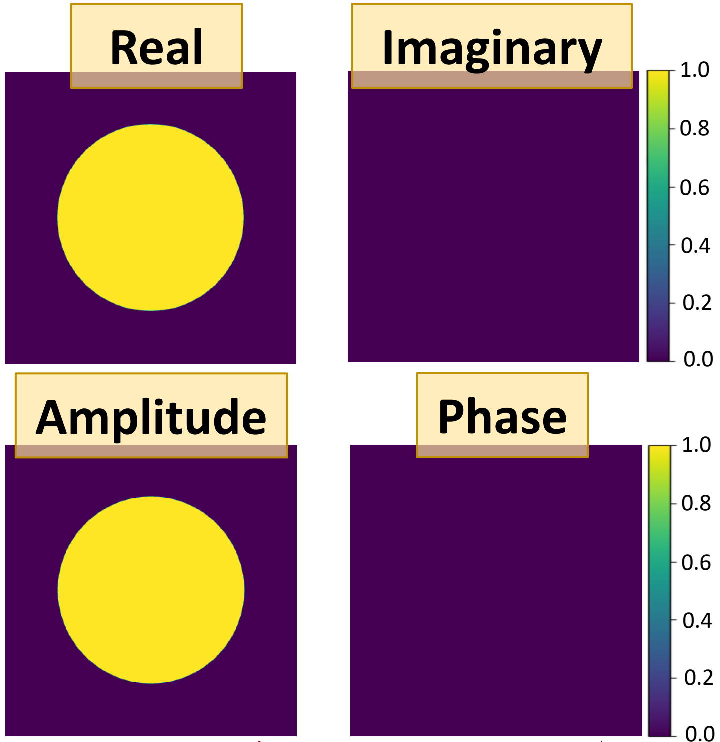
$$E_1 = A_1 e^{i\phi_1}$$

$$E_2 = A_2 e^{i\phi_2}$$

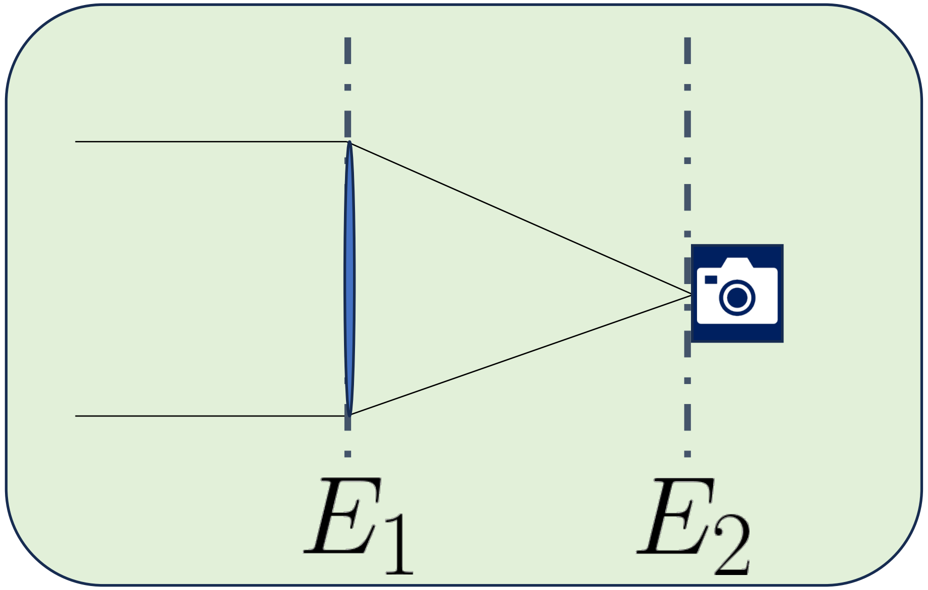
Simplest optical system: simple telescope

(e.g., Newtonian telescope)

Pupil plane



Pupil plane (PP) **Focal plane (FP)**

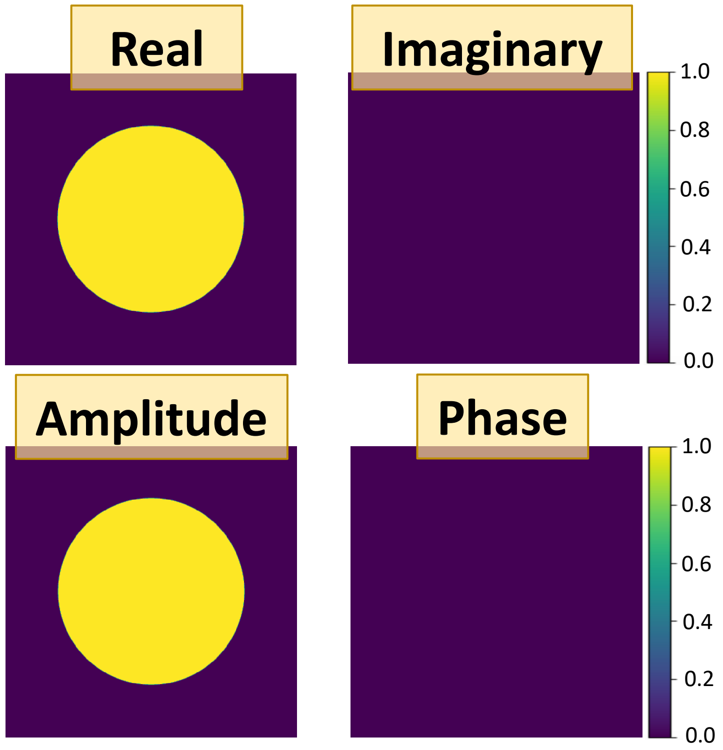


$$E_1 = A_1 e^{i\phi_1}$$

$$E_2 = A_2 e^{i\phi_2}$$

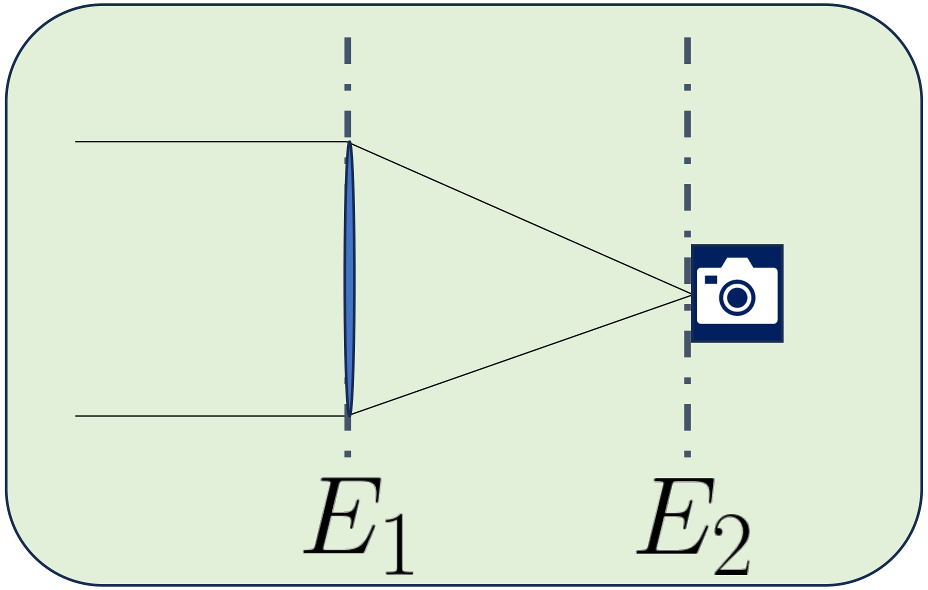
Simplest optical system: simple telescope *(e.g., Newtonian telescope)*

Pupil plane

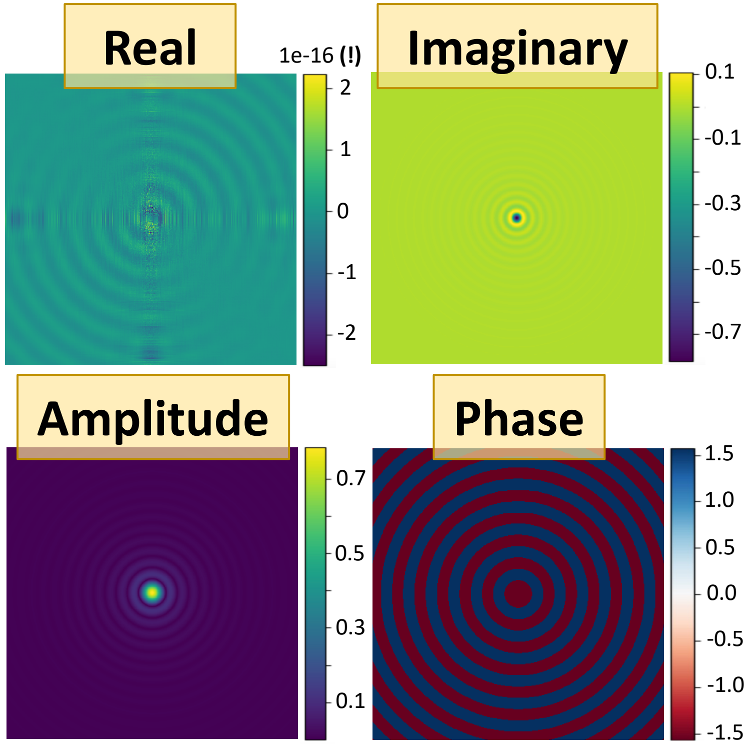


$$E_1 = A_1 e^{i\phi_1}$$

Pupil plane (PP) **Focal plane (FP)**



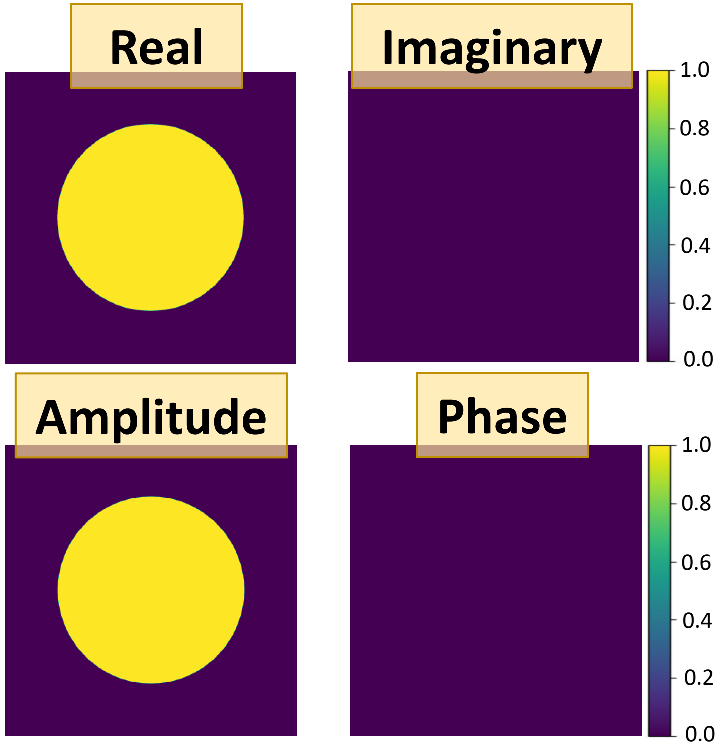
Focal plane

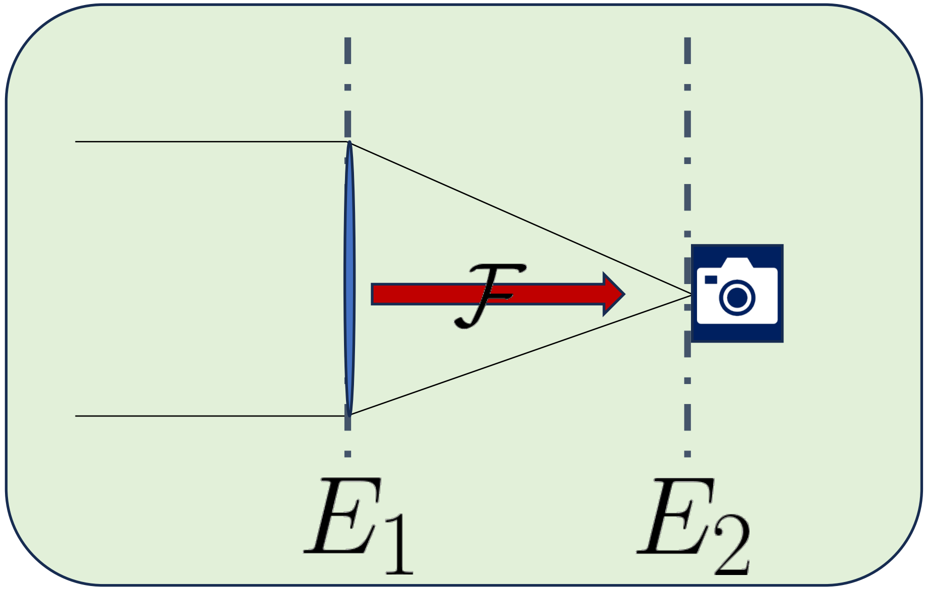


$$E_2 = A_2 e^{i\phi_2}$$

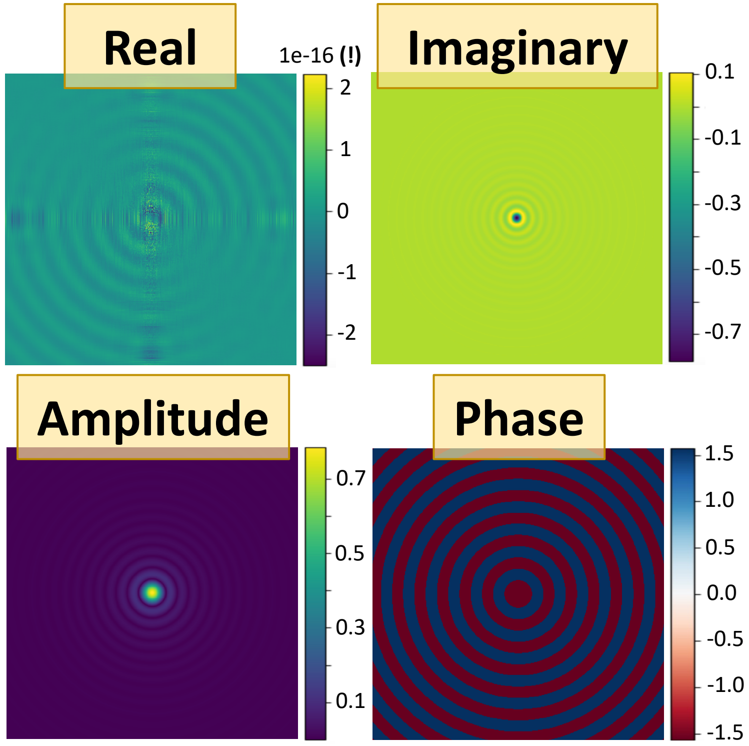
Simplest optical system: one Fourier transform

Pupil plane





Focal plane



$$E_1 = A_1 e^{i\phi_1}$$

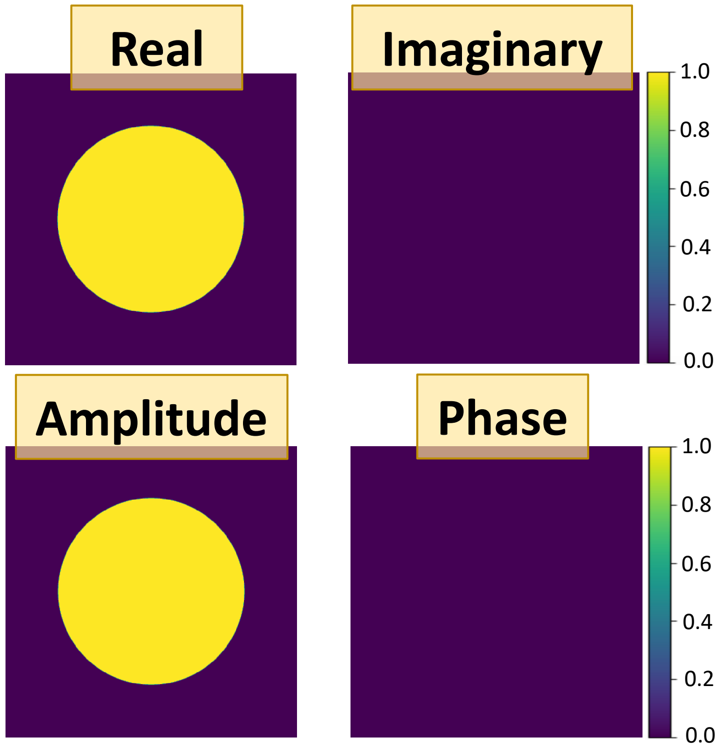
Fourier transform $\mathcal{F}$

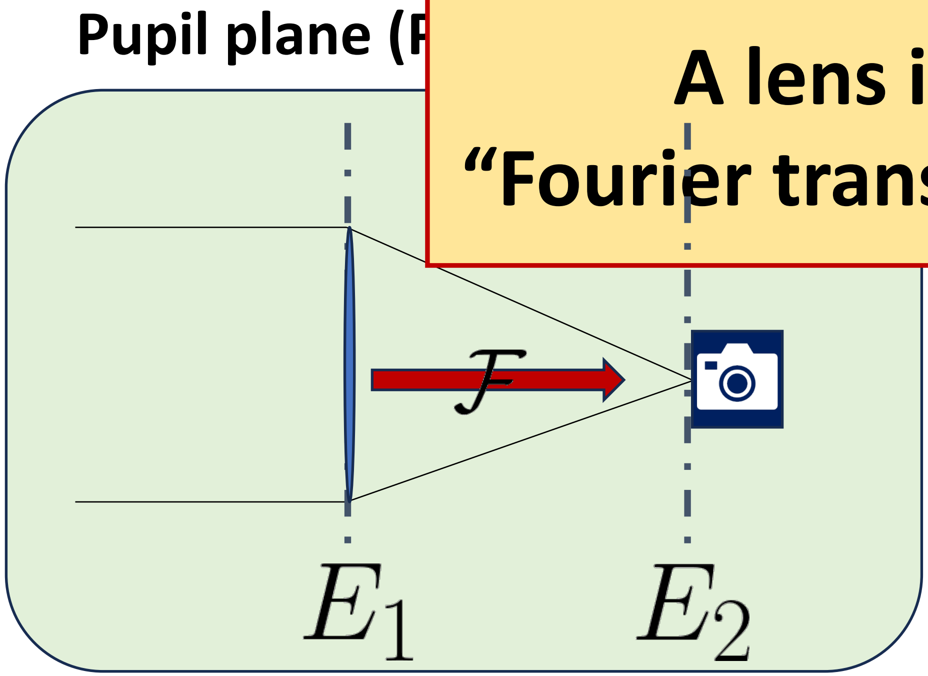
$$E_2 = \mathcal{F}(E_1)$$

$$E_2 = A_2 e^{i\phi_2}$$

Simplest optical system: one Fourier transform

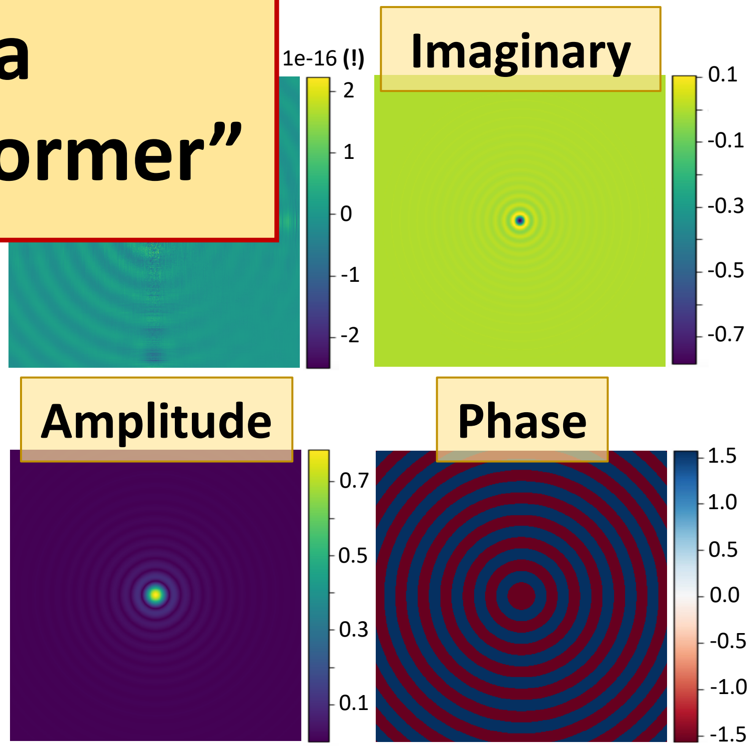
Pupil plane





A lens is a "Fourier transformer"

Focal plane



$$E_1 = A_1 e^{i\phi_1}$$

Fourier transform $\mathcal{F}$

$$E_2 = \mathcal{F}(E_1)$$

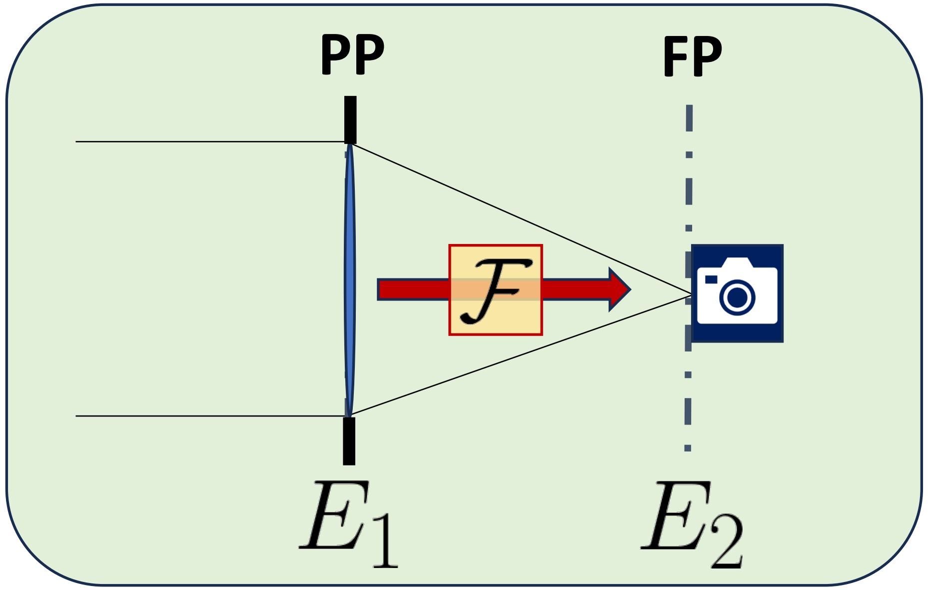
$$E_2 = A_2 e^{i\phi_2}$$

HCI instruments are optical systems and they propagate wavefronts from one optical plane to the next.

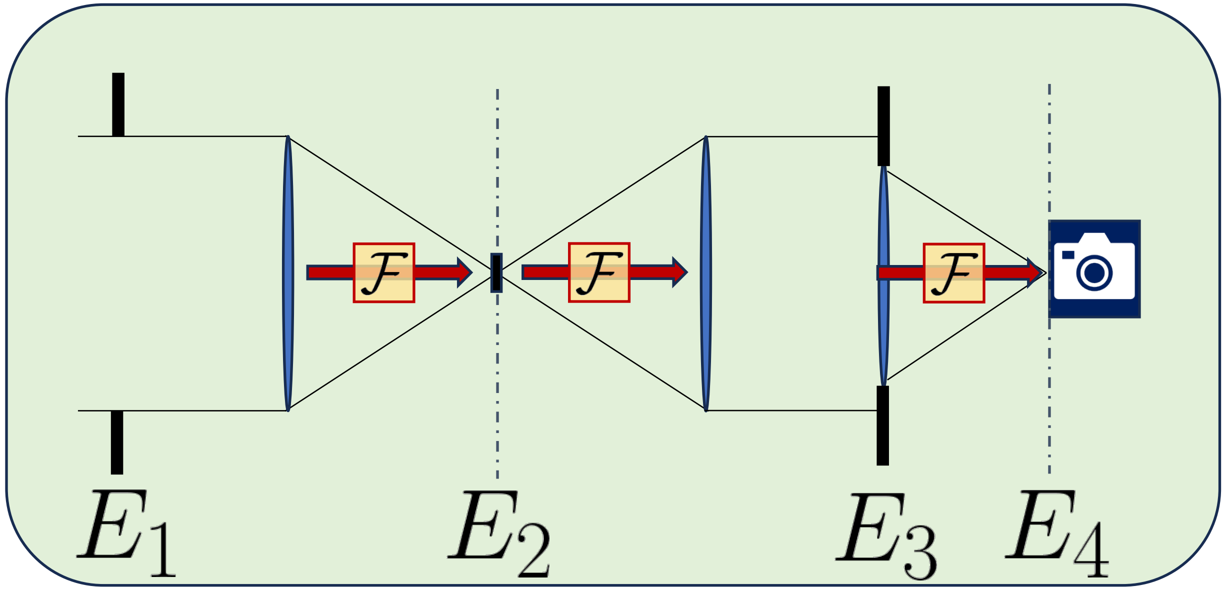
The relationship between pupil and focal planes is a Fourier transform.

→ Pupil planes and focal planes are transformations of each other.

Simple telescope



Lyot Coronagraph



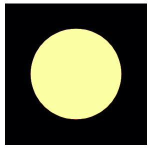
Diffraction, properties of the Fourier transform, resolution

Diffraction patterns, units, angular resoluton, wavelength dependence

Diffractive optics and Fourier optics

Reminder: $I = |E|^2$

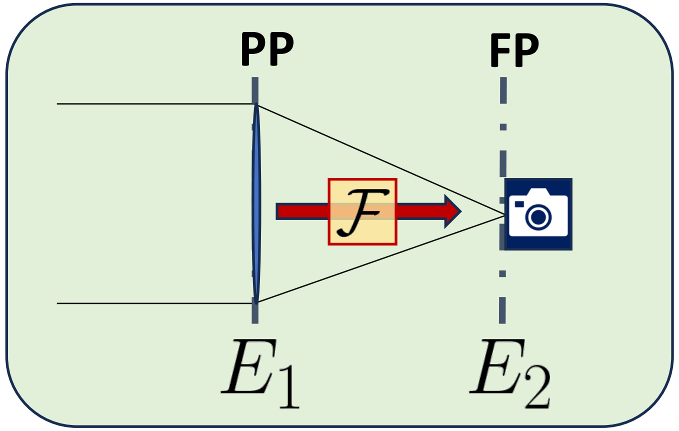
- A simple **telescope pupil** imposes a circular edge that defines the collecting area
- Result is an **Airy function** in the focal plane

$\mathcal{F}\Big($  $\Big) \propto \dfrac{2J_1(\pi\rho D/\lambda z)}{\pi\rho D/\lambda z}$

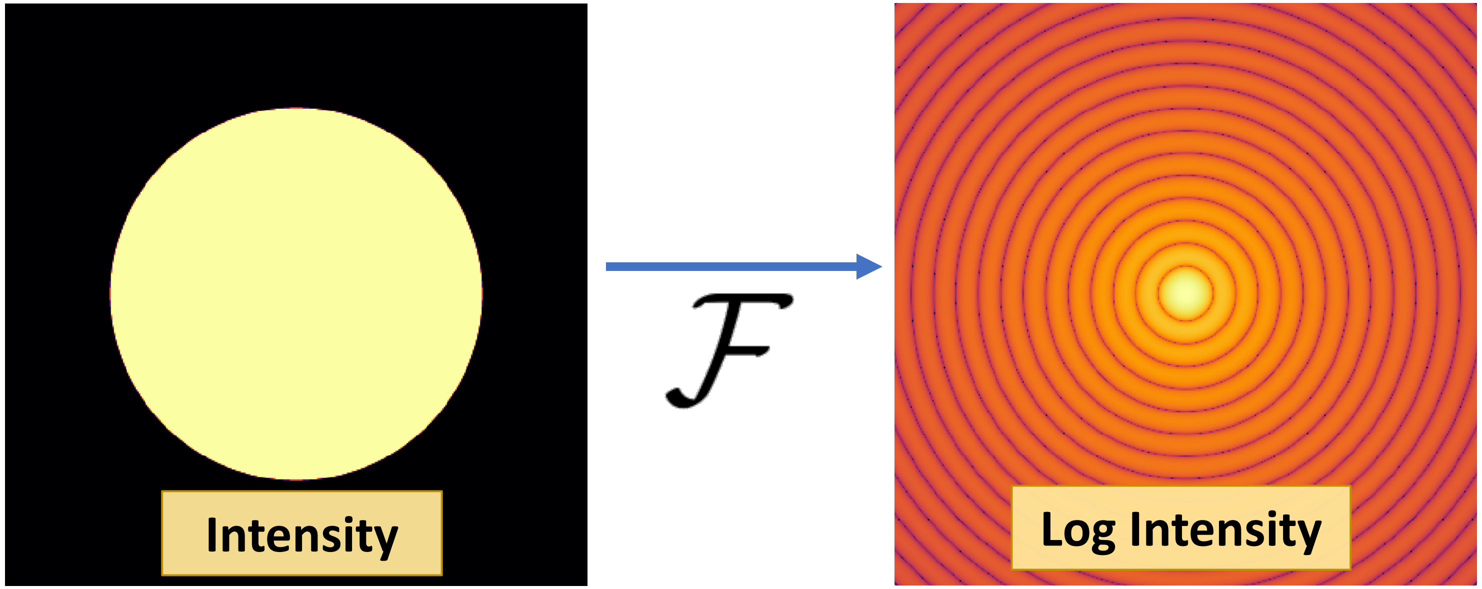
J_1 ...Bessel function of first kind

ρ ...radial distance from optical axis

Simple imager/telescope

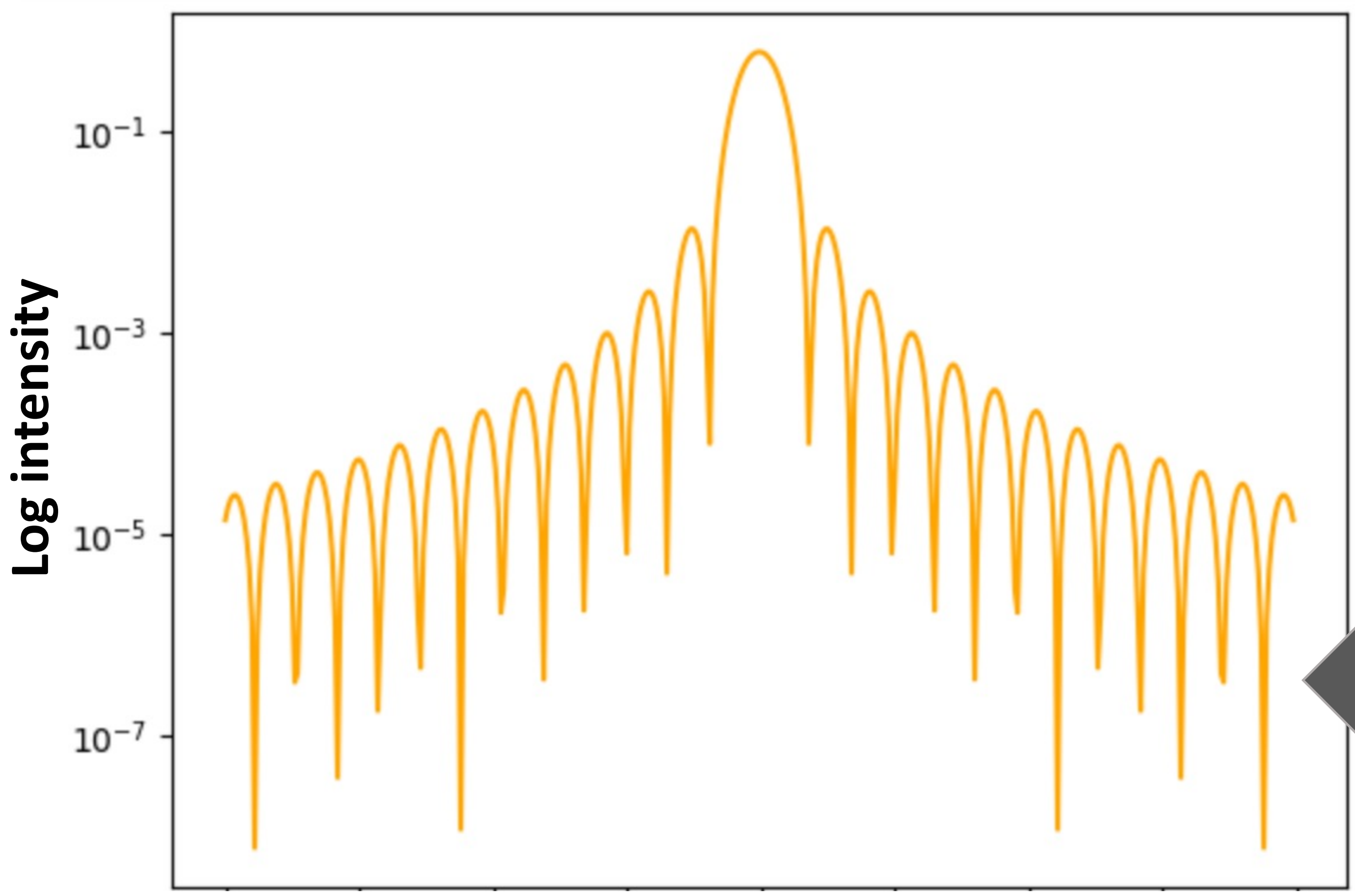


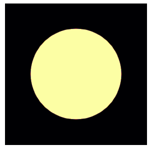
Pupil plane **Focal plane**



Diffractive optics and Fourier optics

Reminder: $I = |E|^2$



$\mathcal{F}\Big($  $\Big) \propto \frac{2J_1(\pi\rho D/\lambda z)}{\pi\rho D/\lambda z}$

J_1 ...Bessel function of first kind

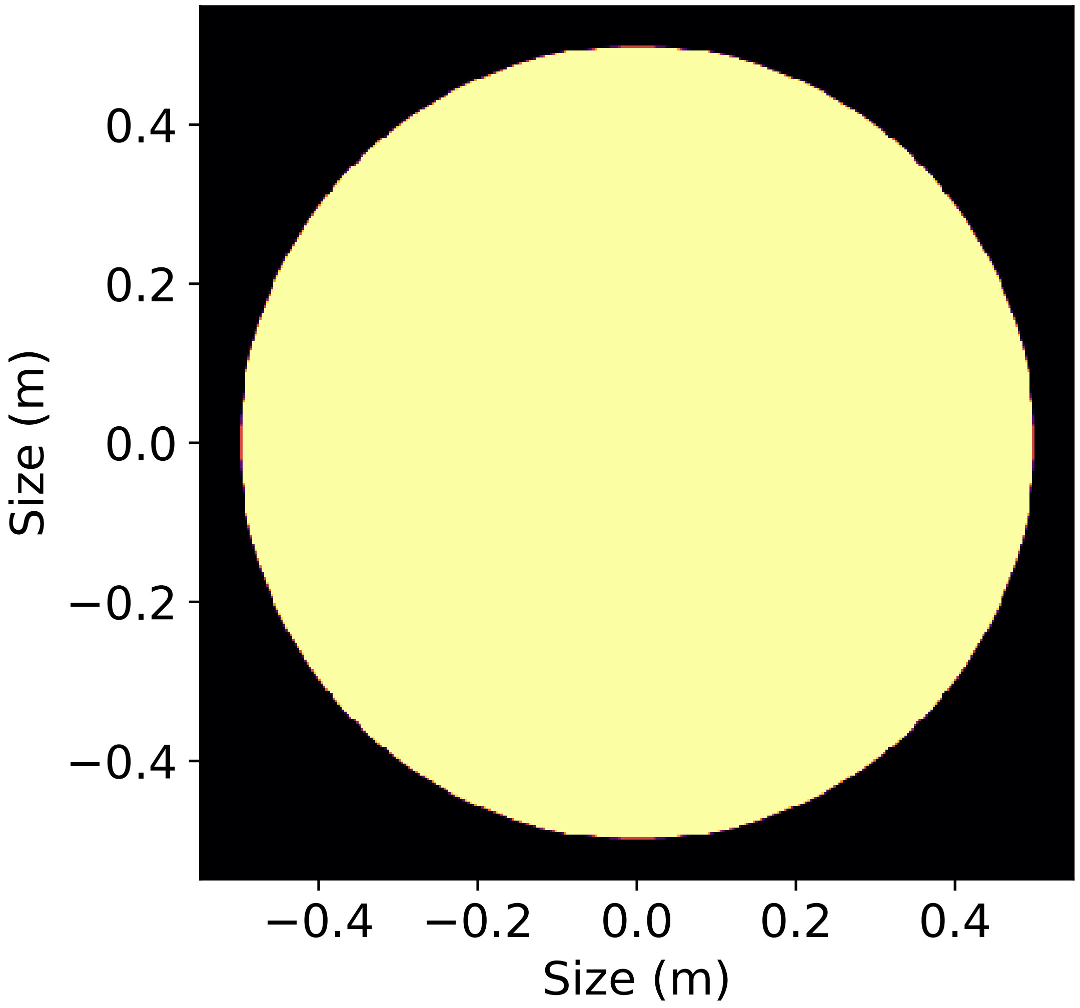
ρ ...radial distance from optical axis

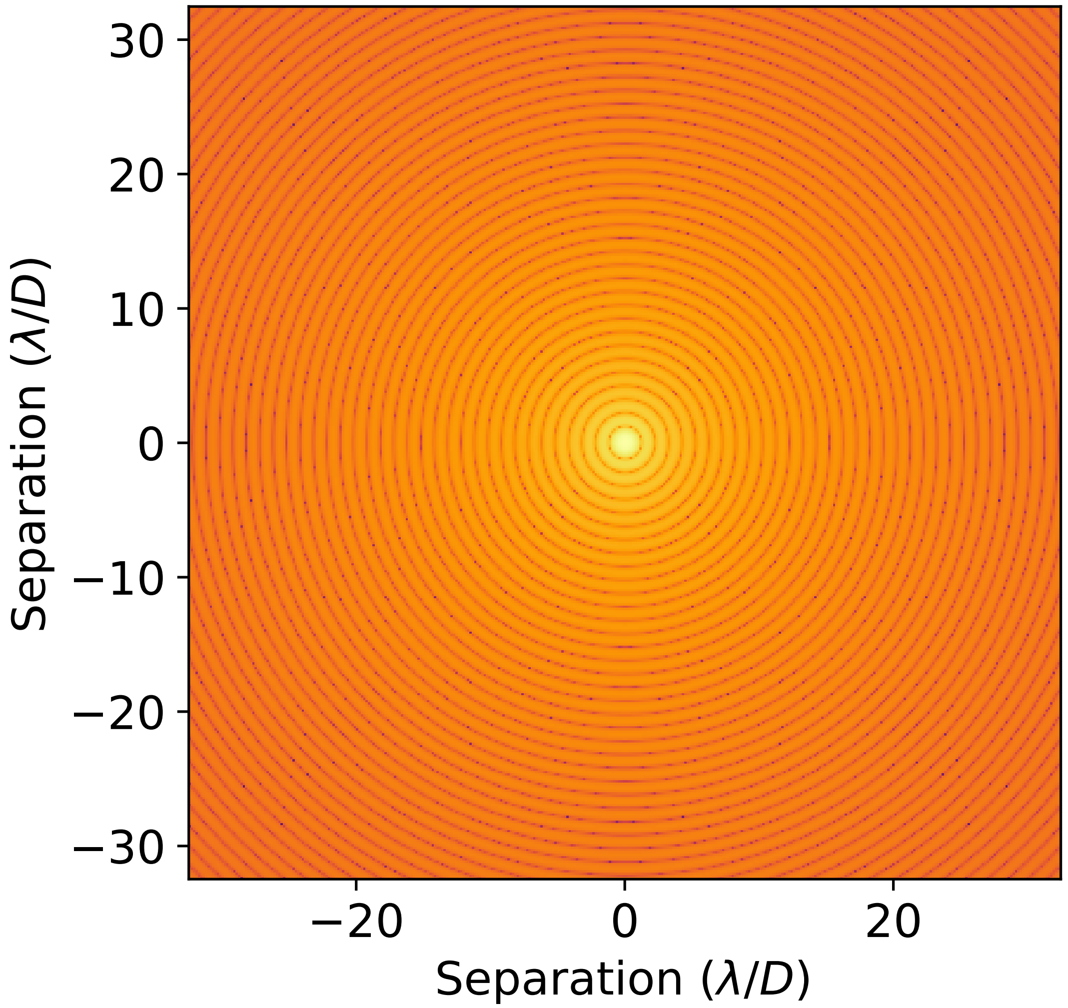
Focal plane

Cut through center

Simple diffraction examples

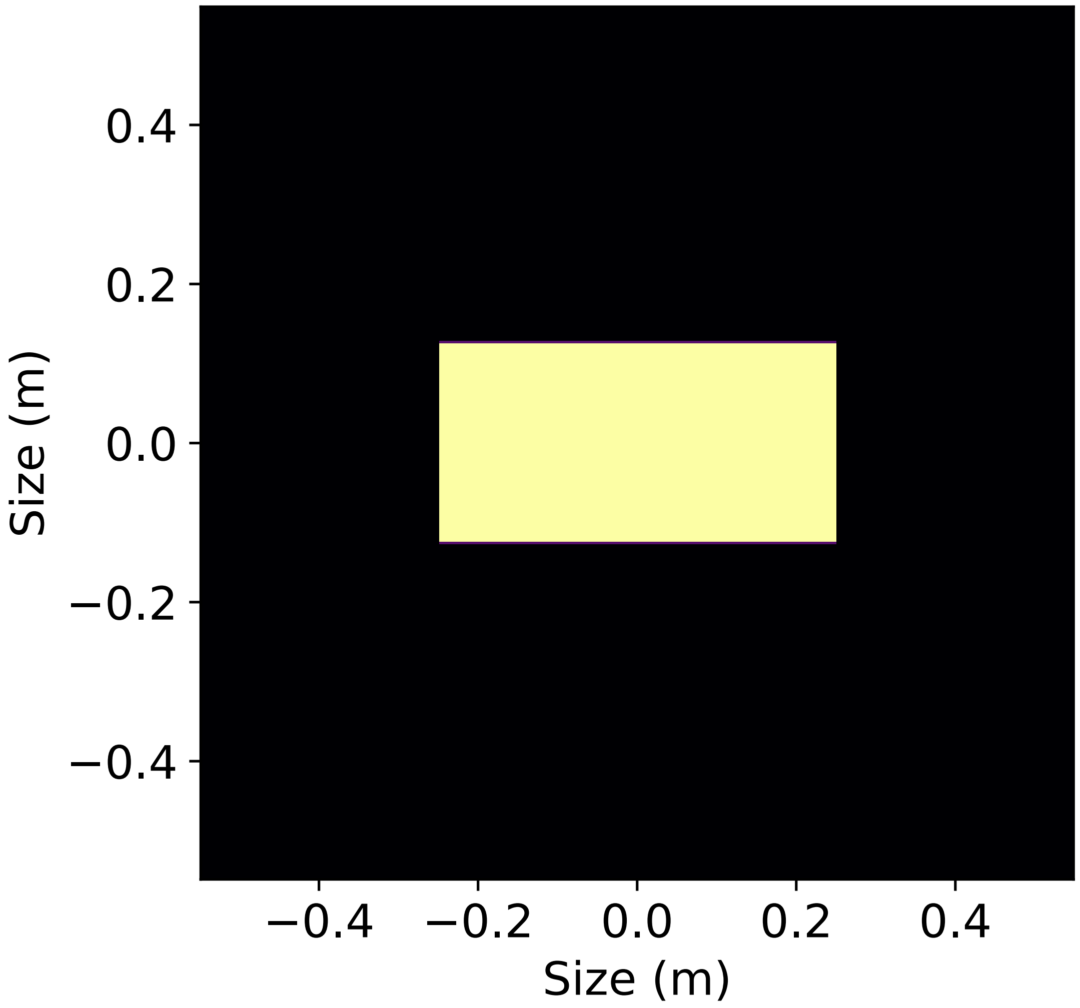
(All apertures with normalized diameter.)

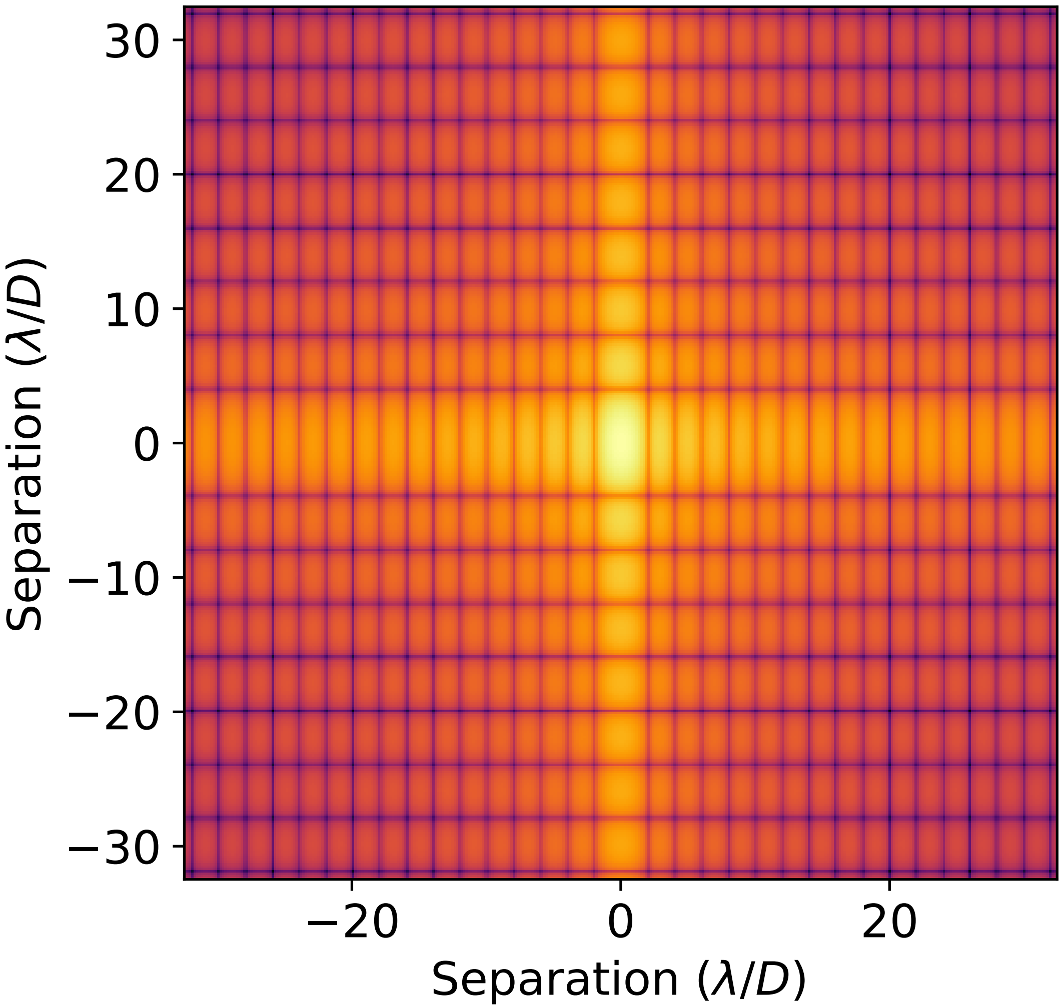




Simple diffraction examples

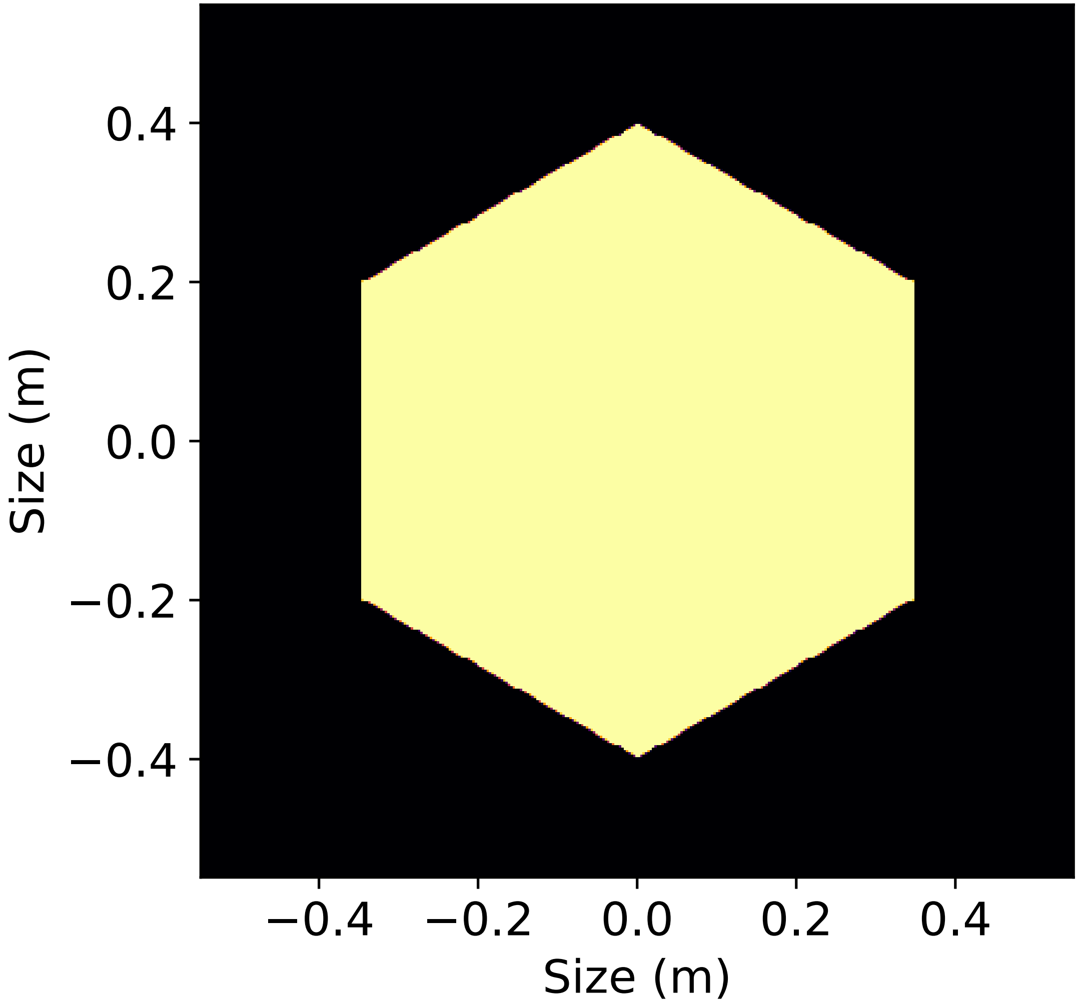
(All apertures with normalized diameter.)

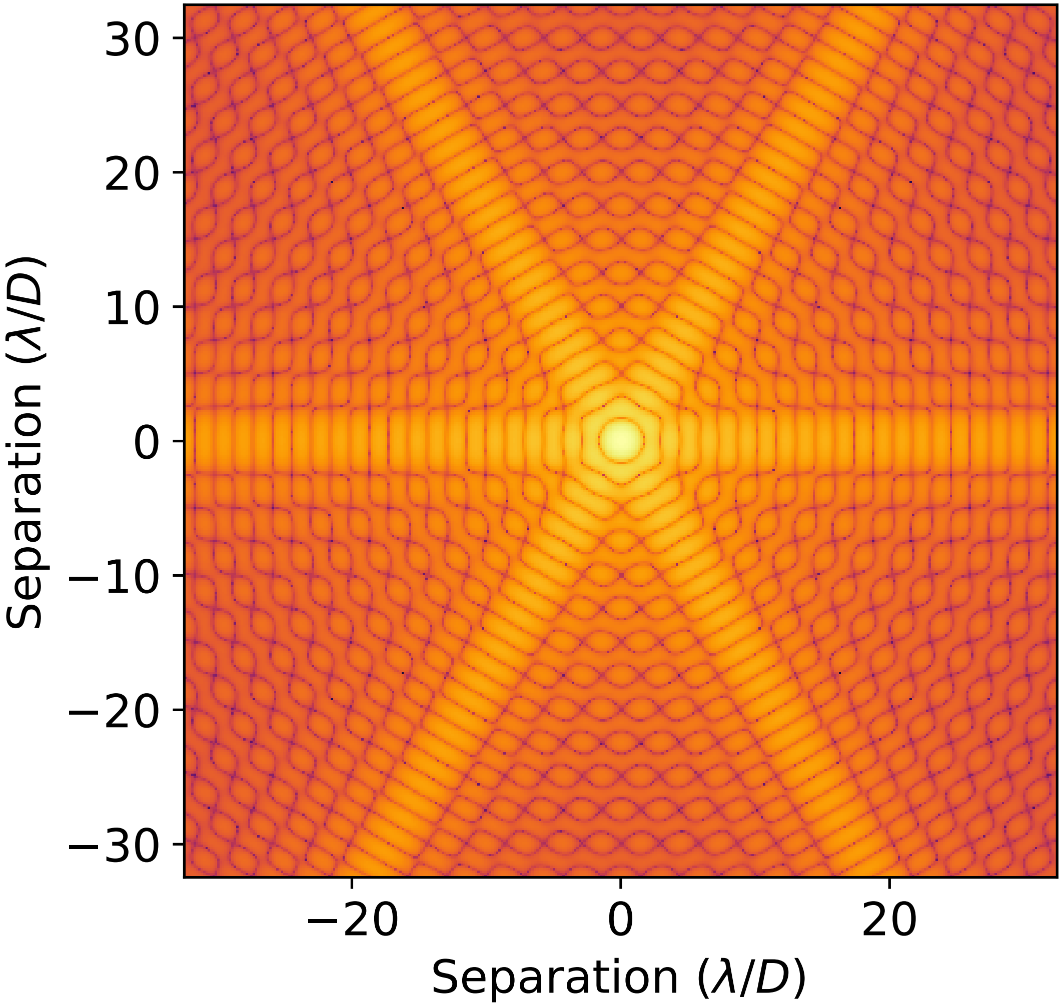




Simple diffraction examples

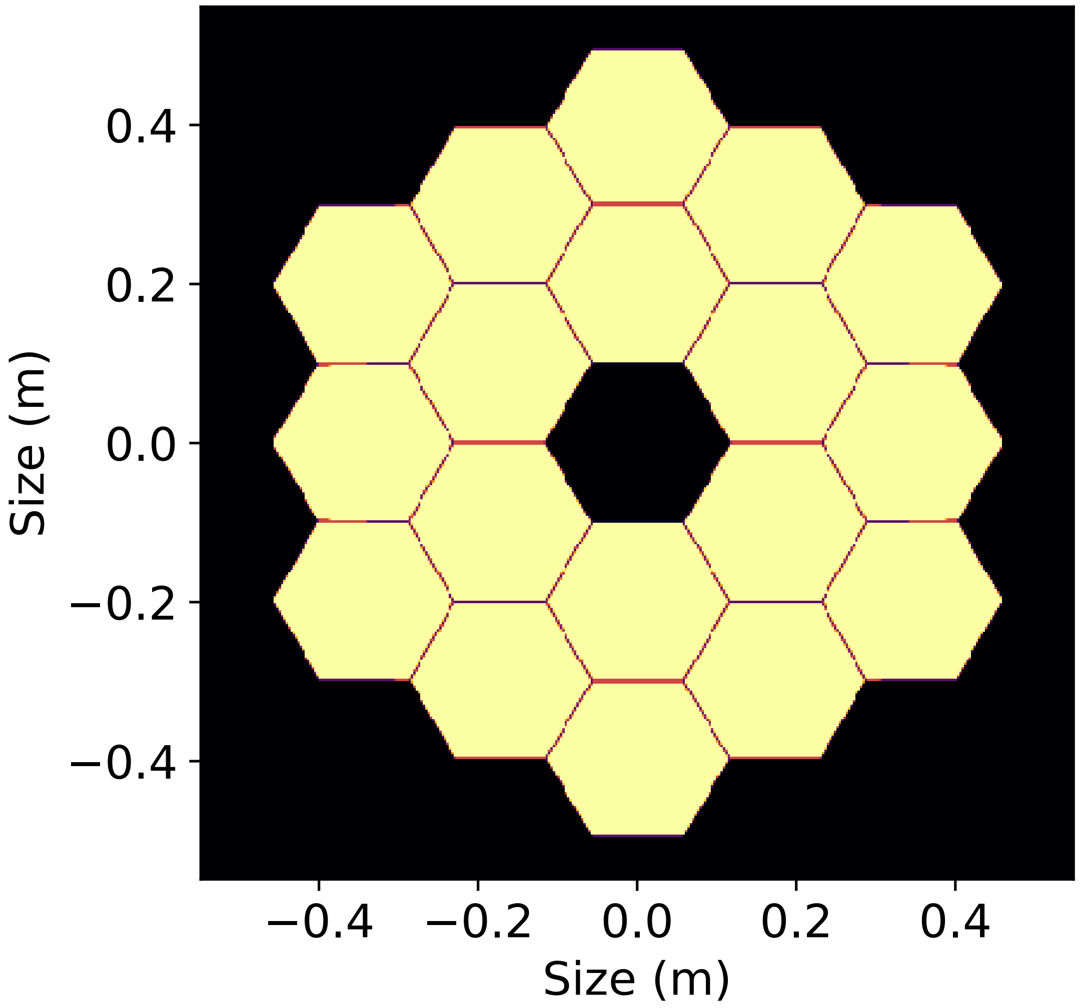
(All apertures with normalized diameter.)

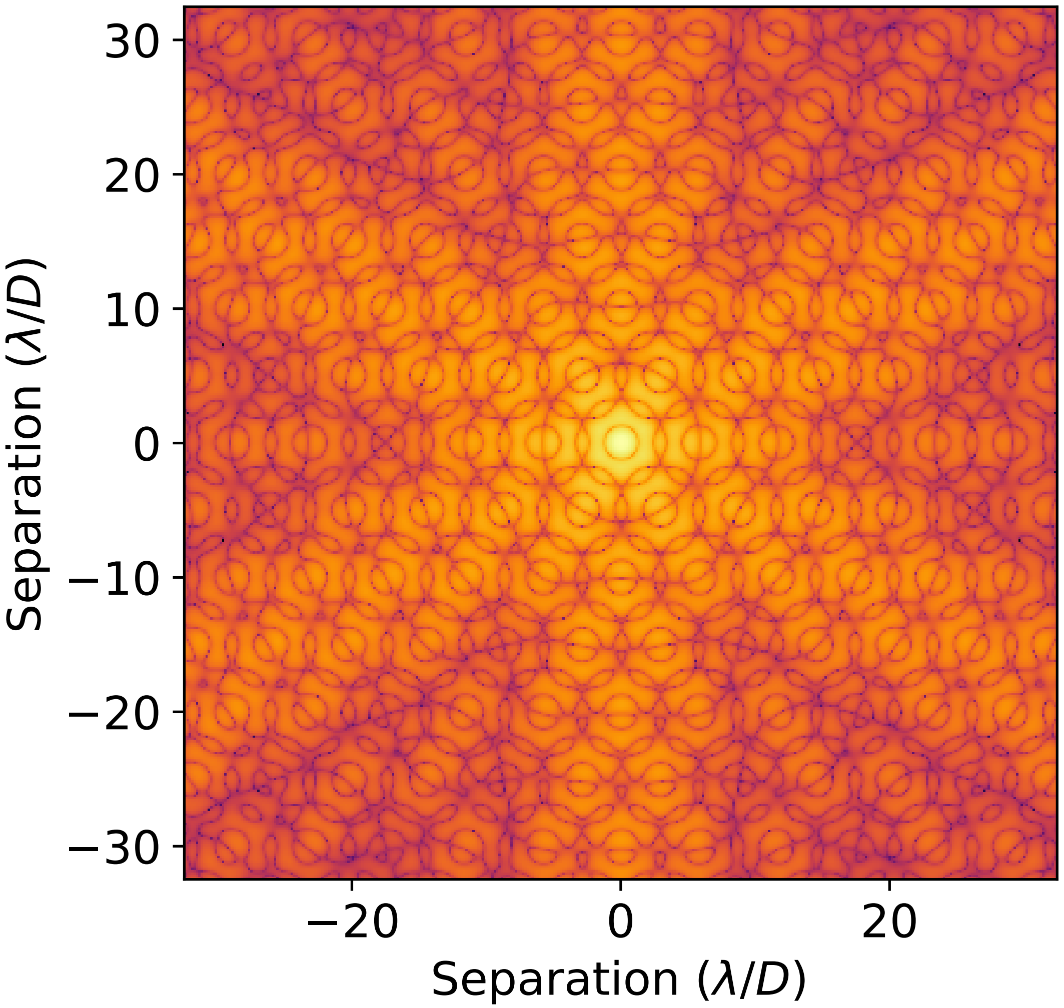




Simple diffraction examples

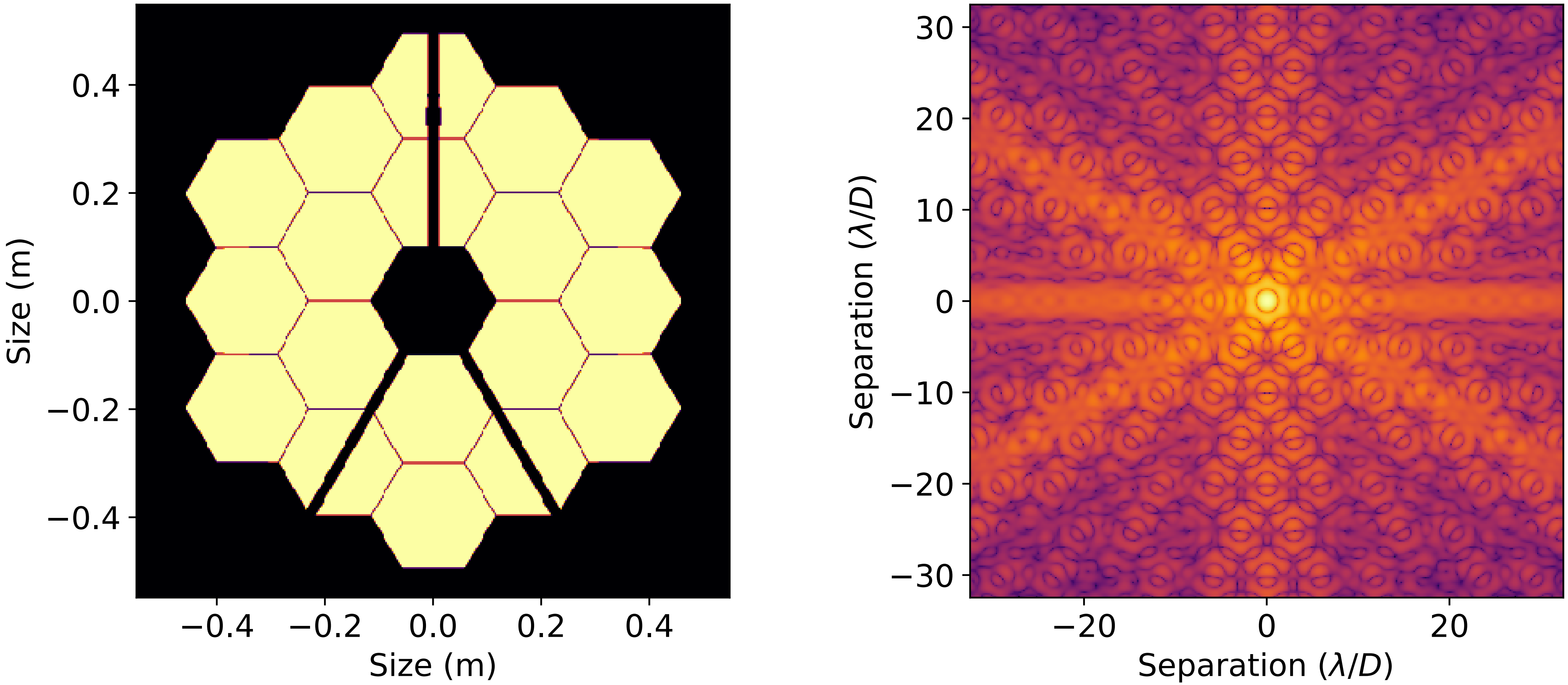
(All apertures with normalized diameter.)





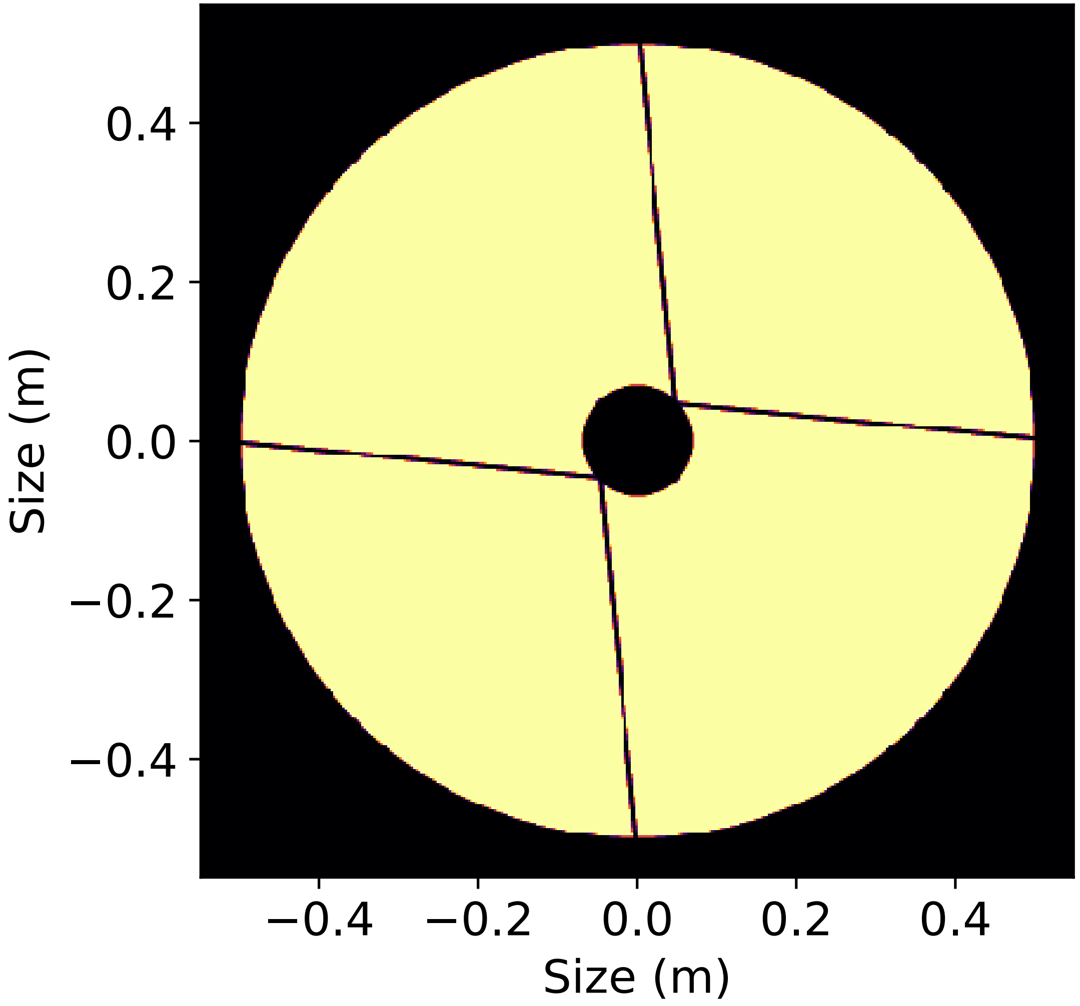
Simple diffraction examples

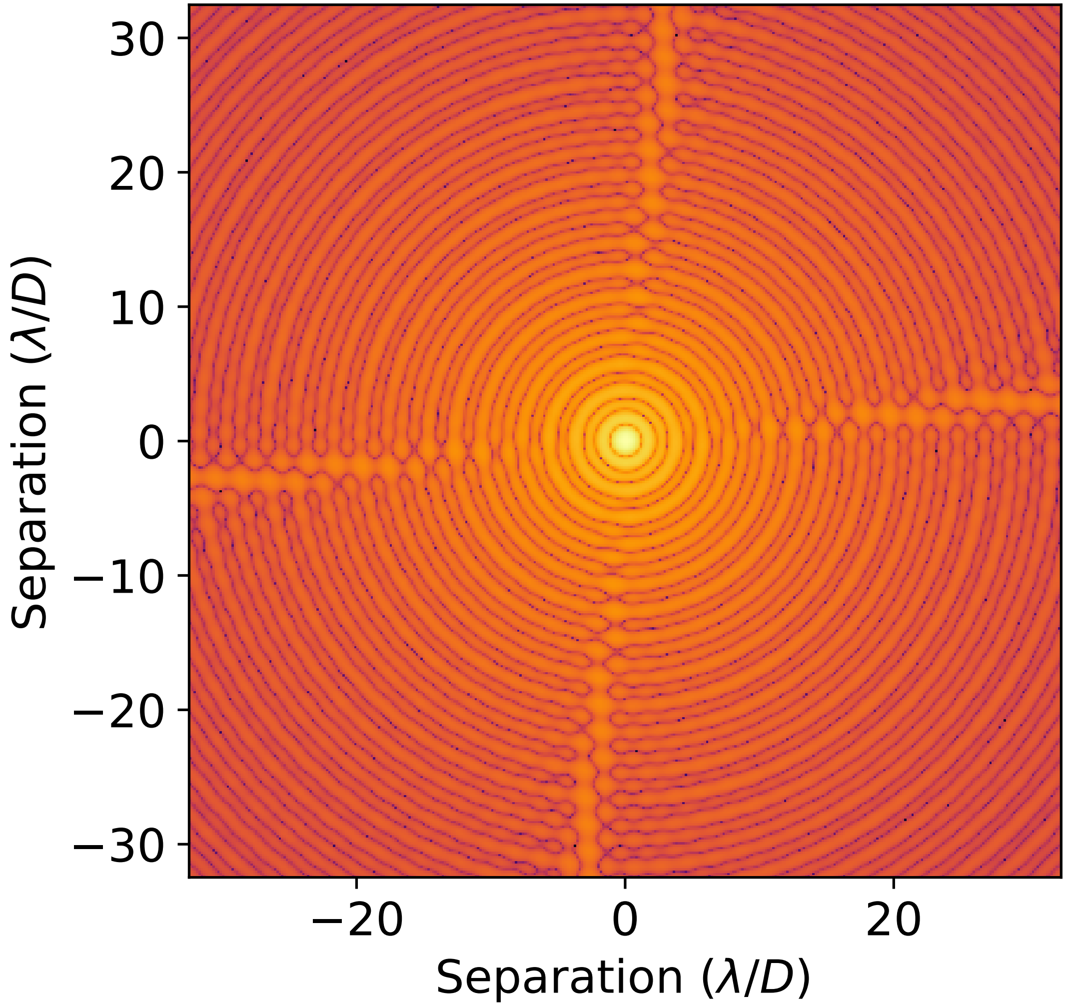
(All apertures with normalized diameter.)



Simple diffraction examples

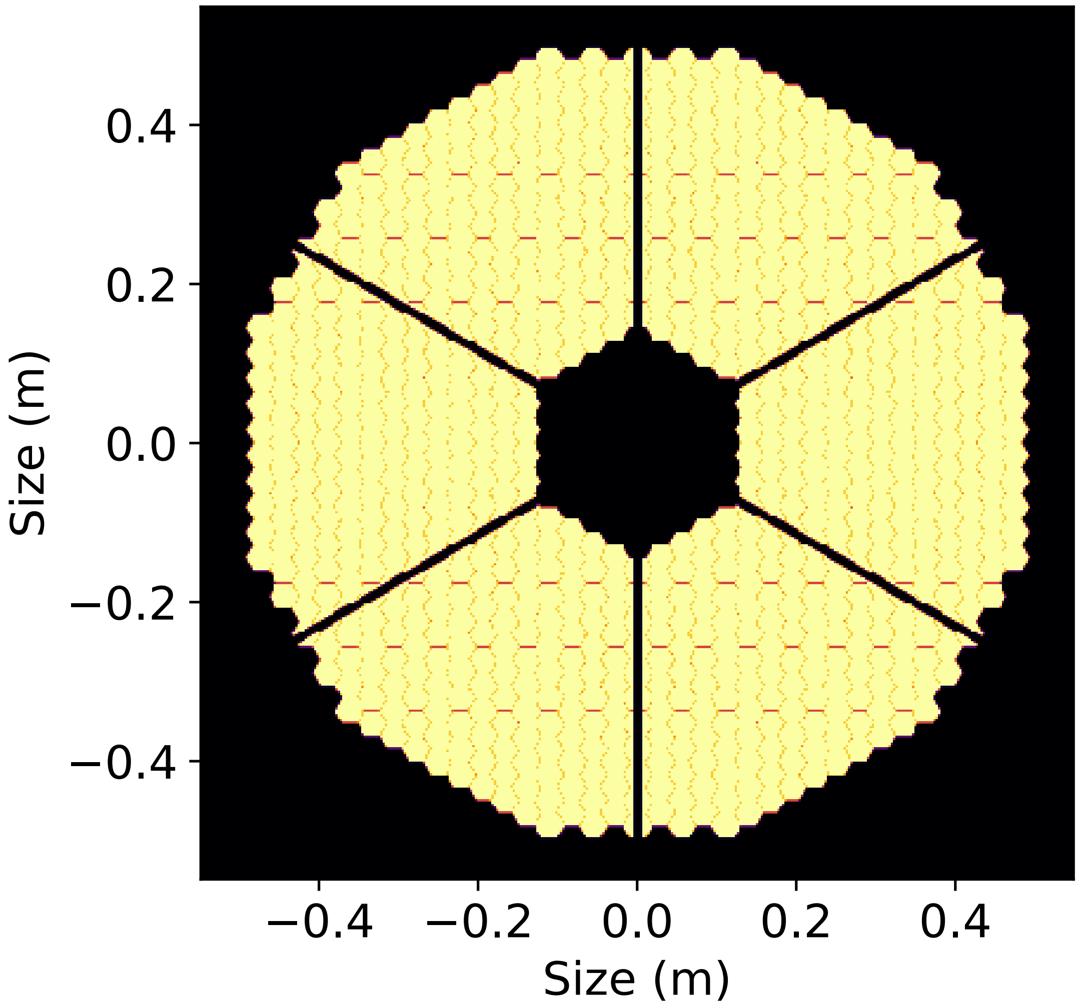
(All apertures with normalized diameter.)

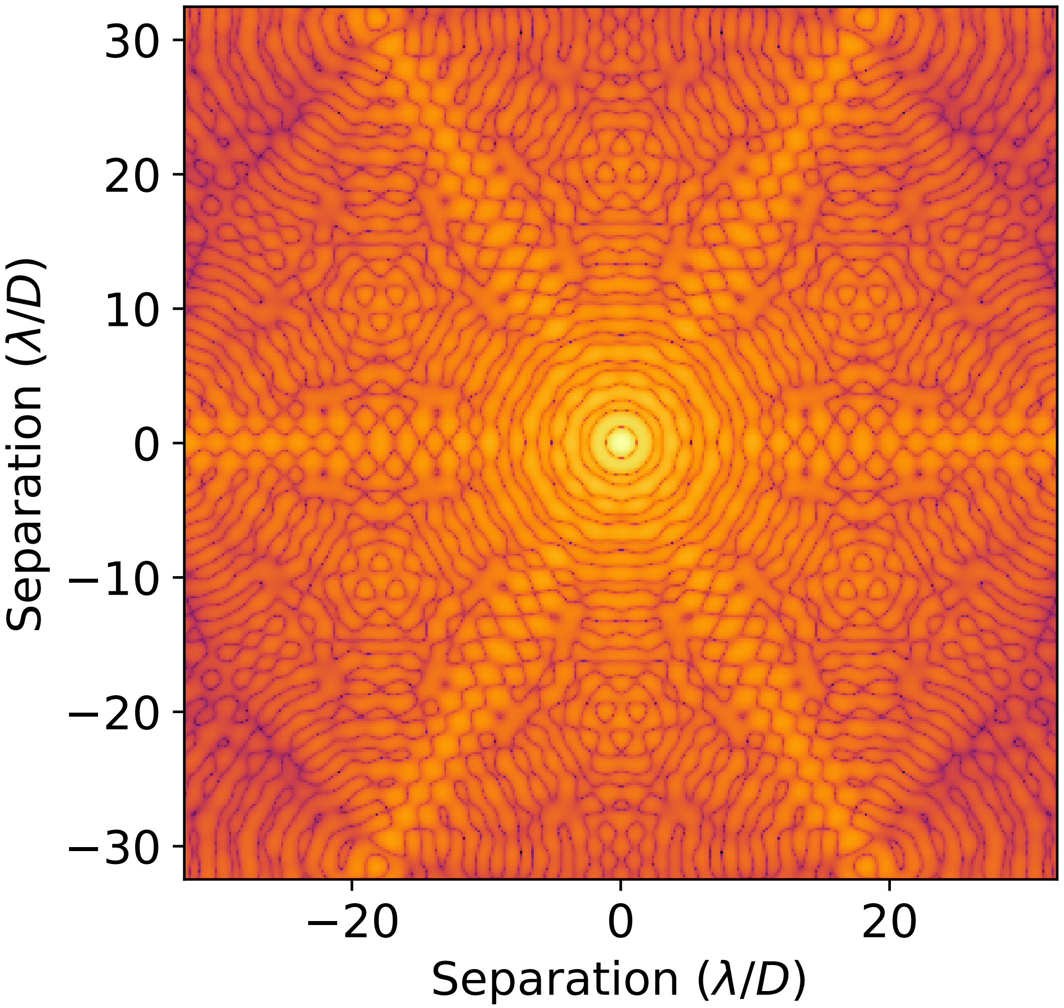




Simple diffraction examples

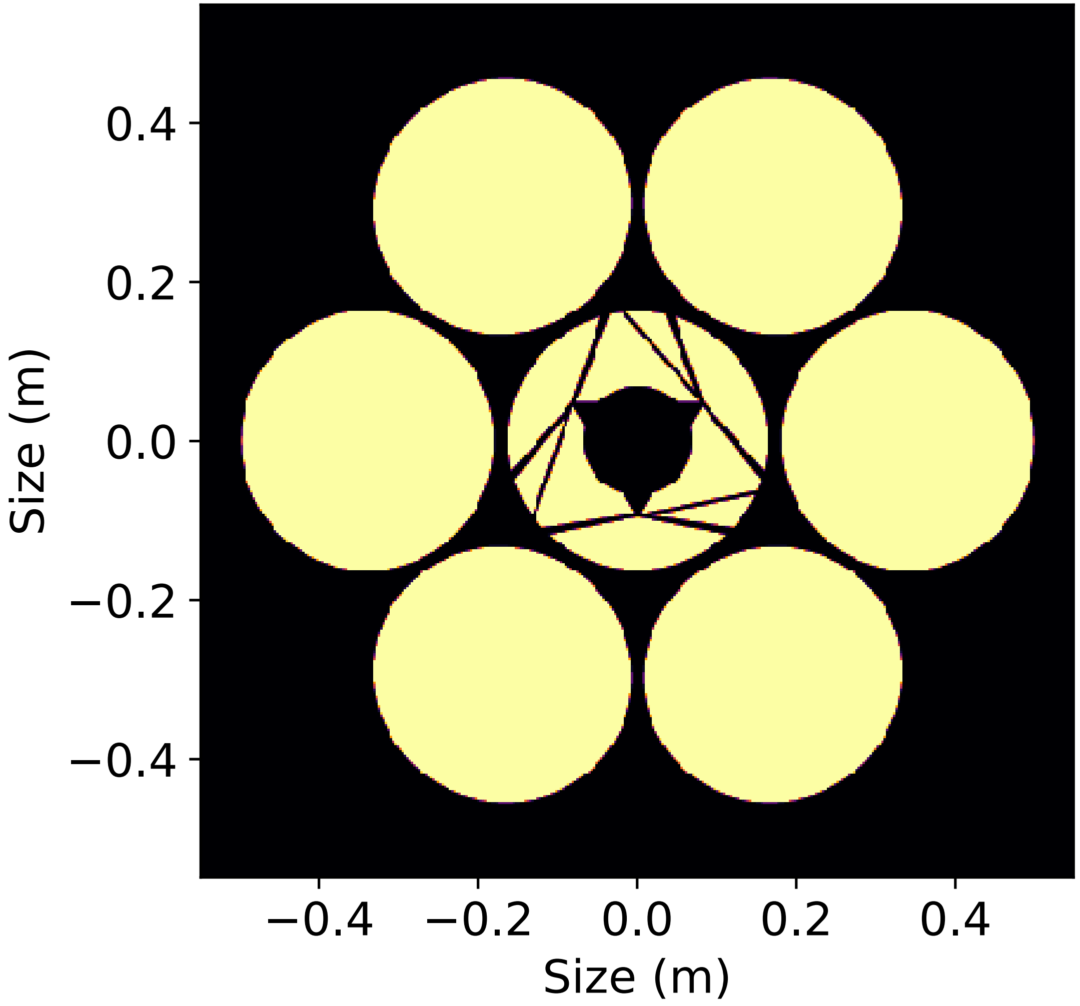
(All apertures with normalized diameter.)

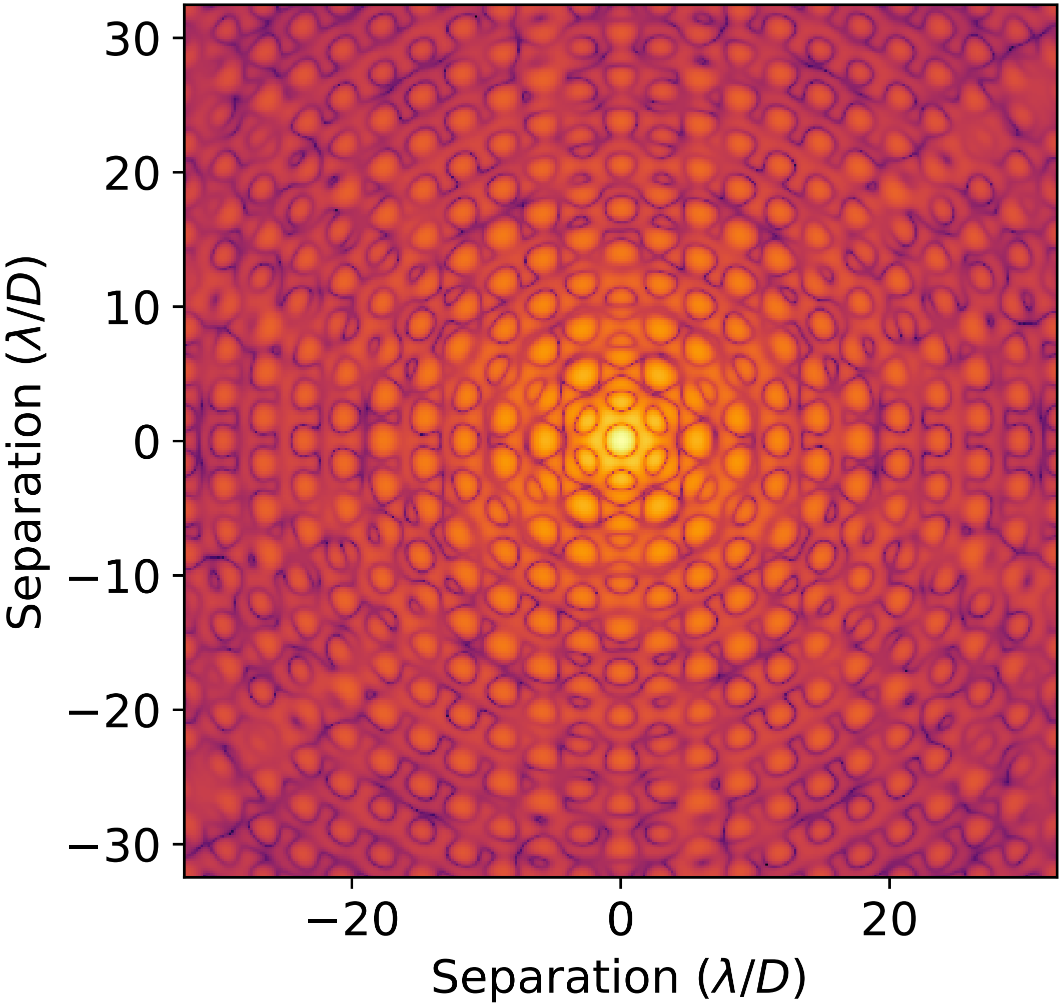




Simple diffraction examples

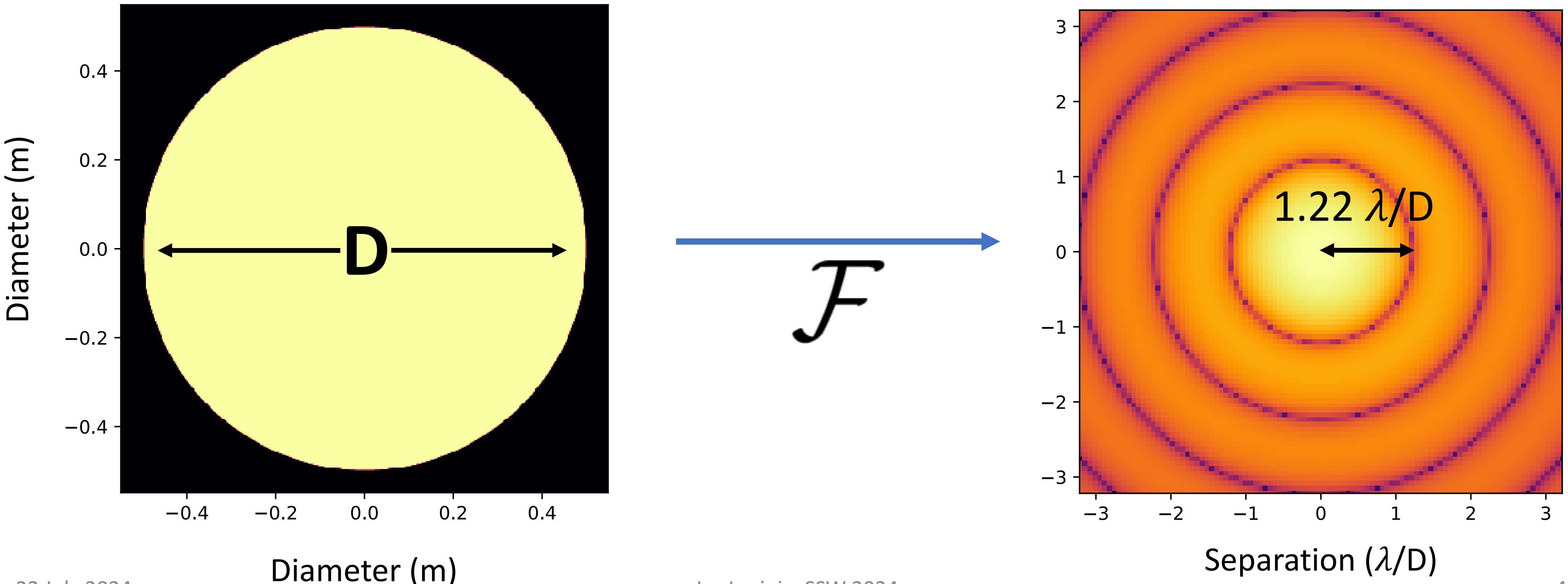
(All apertures with normalized diameter.)





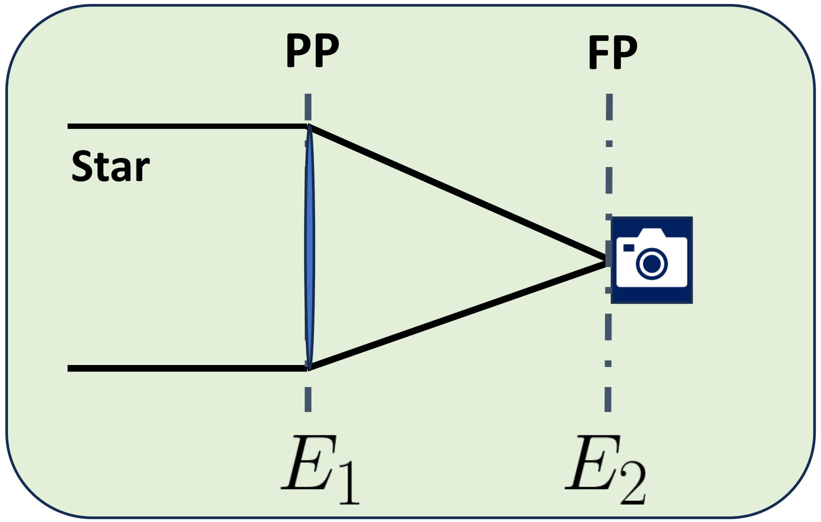
Pupil-plane vs. focal-plane **units**

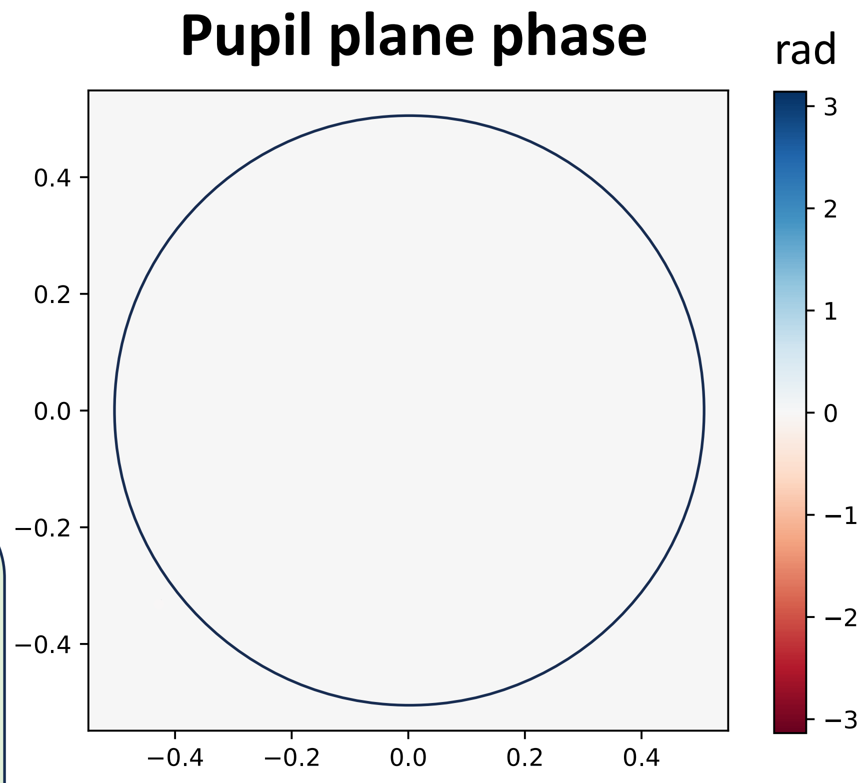
- Focal plane is expressed in terms of **spatial frequencies**
- physical scales (or angular scales) are **inverse of each other**
- The larger the pupil the smaller the core of the PSF

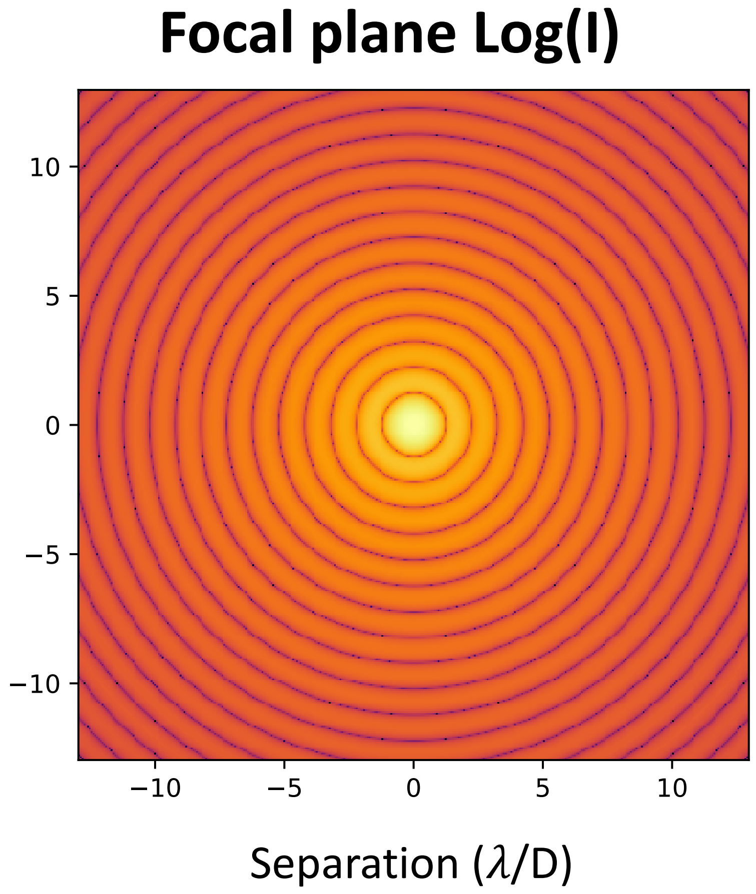


Angle change in pupil → shift in focal plane

Simple imager/telescope



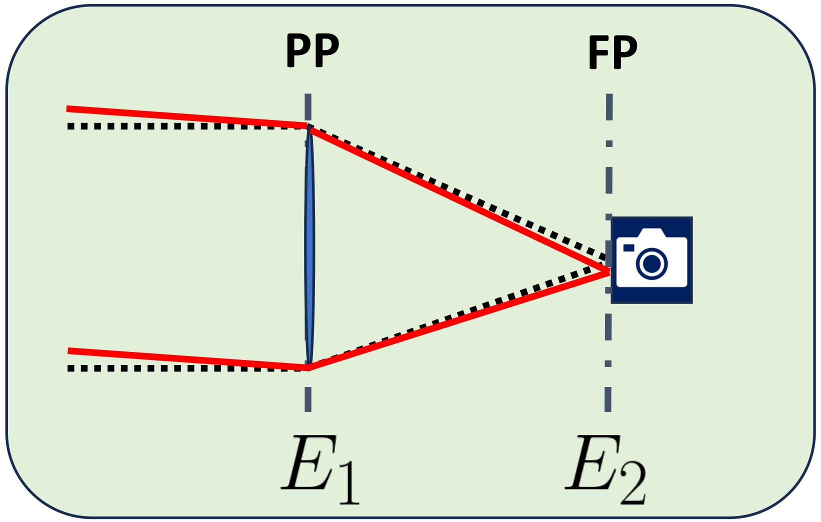


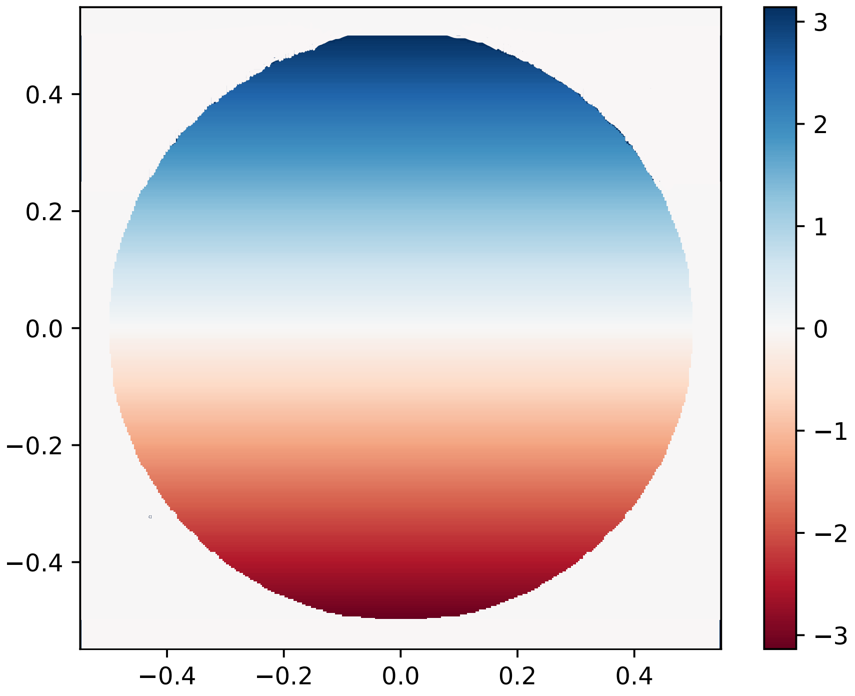


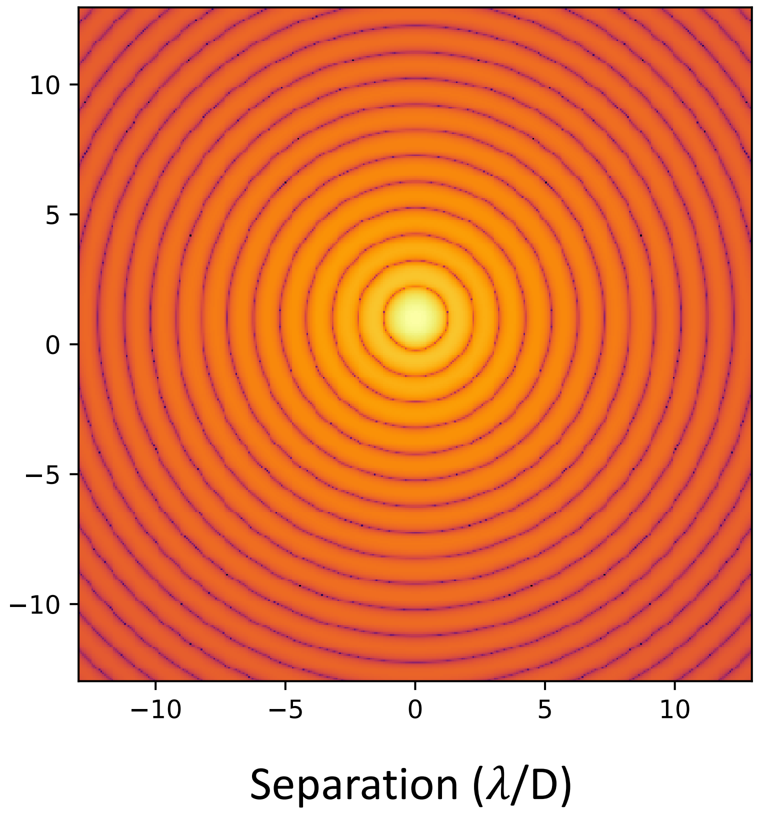
Angle change in pupil → shift in focal plane

→ tip-tilt/jitter !

Simple imager/telescope

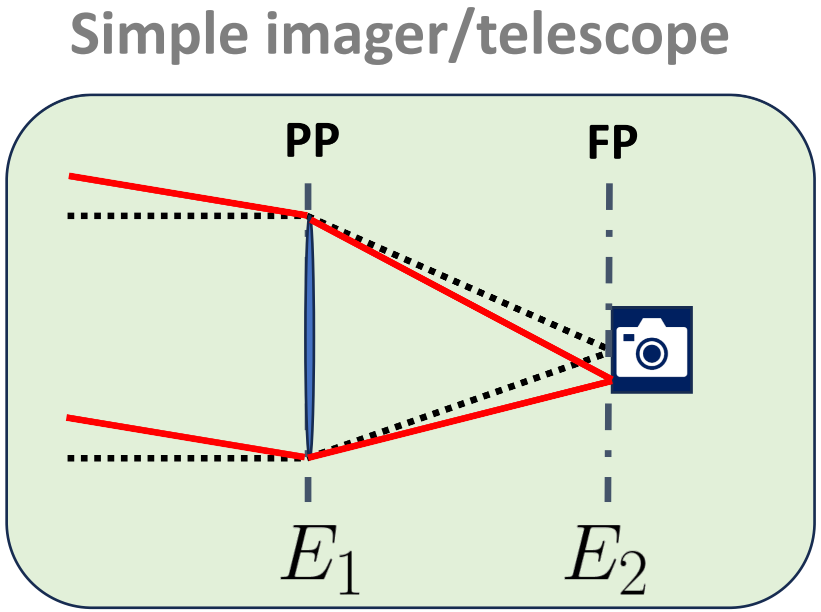


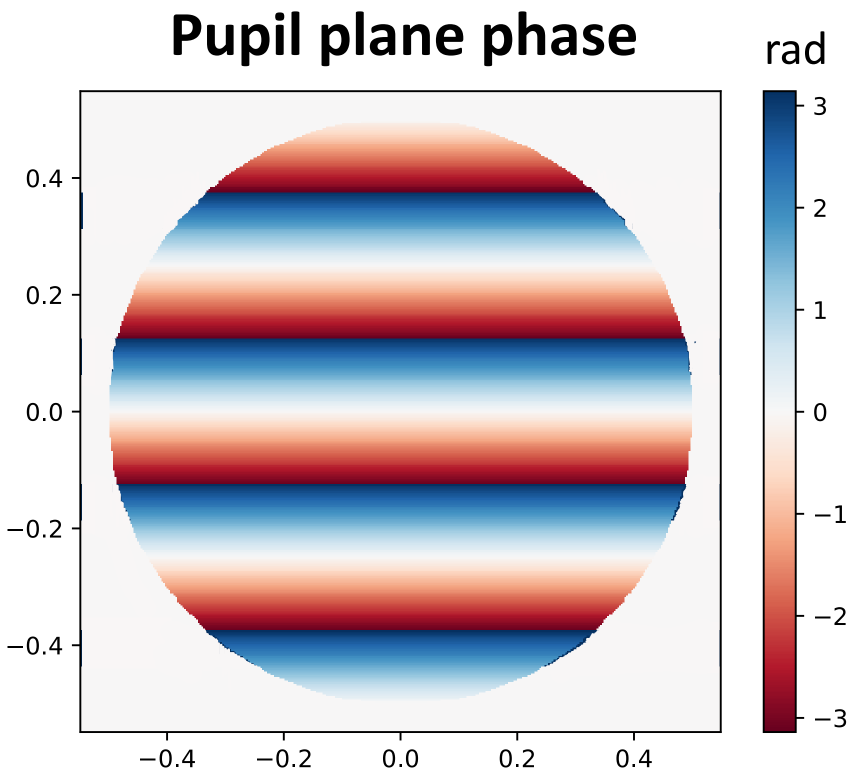


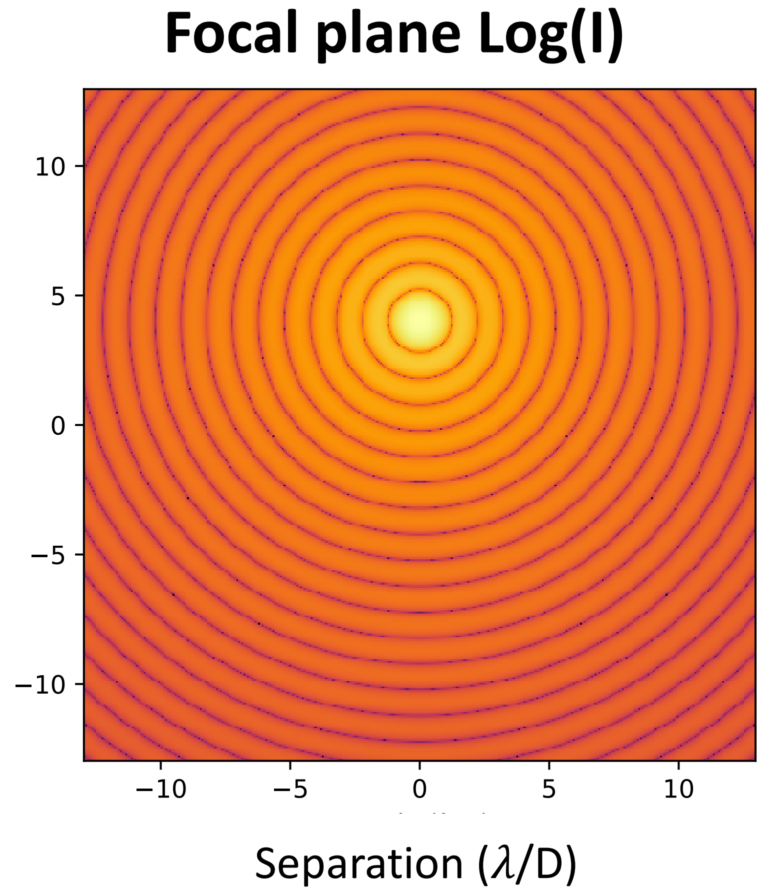


Angle change in pupil → shift in focal plane

→ tip-tilt/jitter !

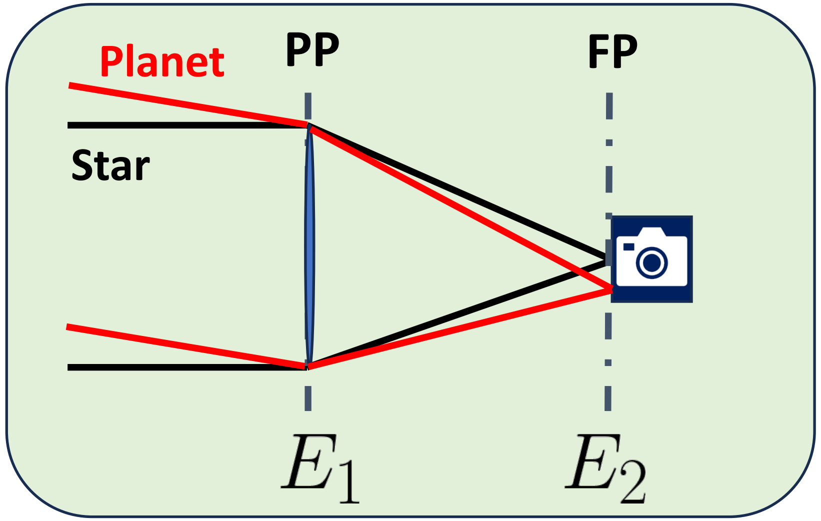






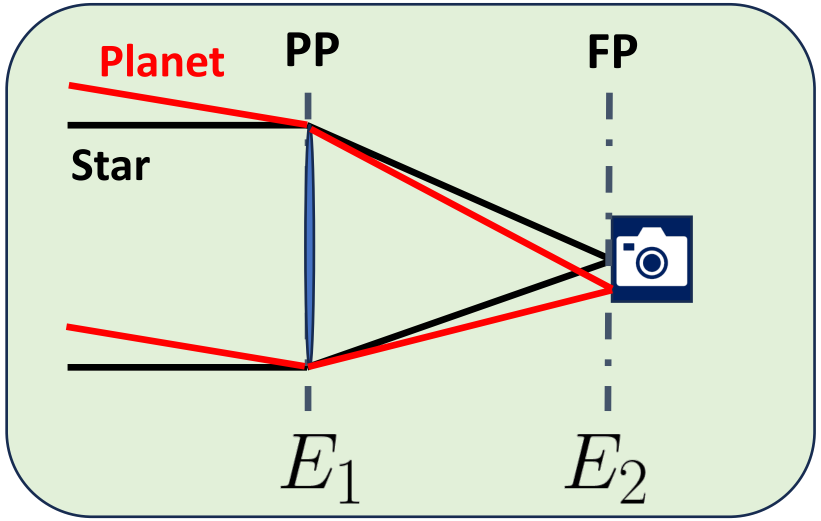
Companions and angular resolution

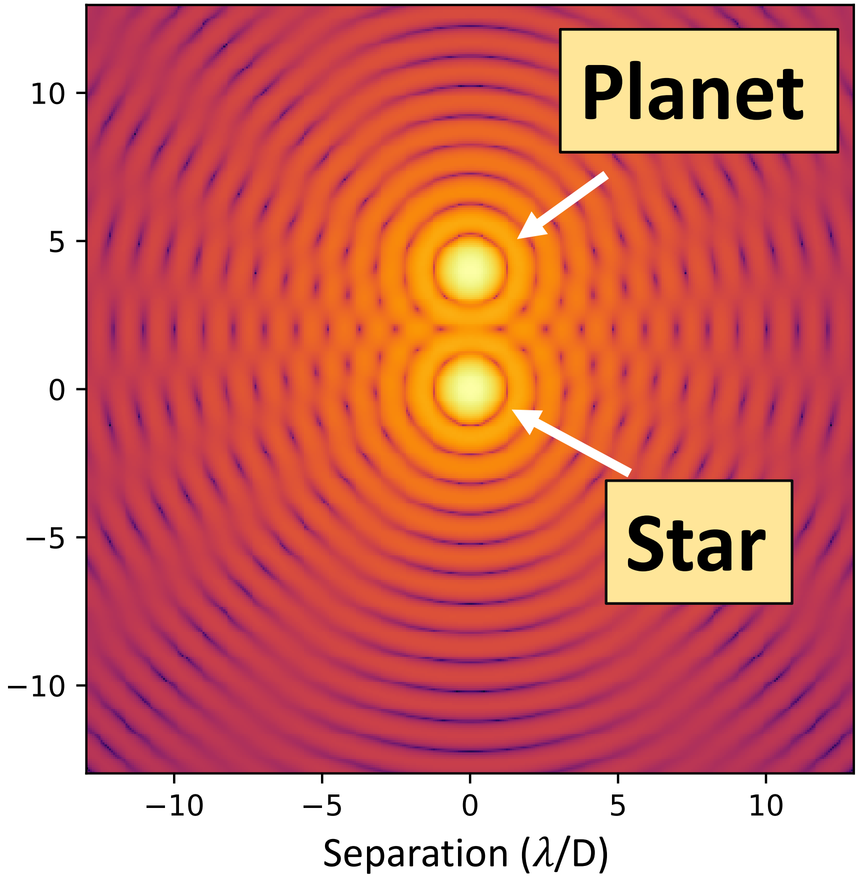
Simple imager/telescope



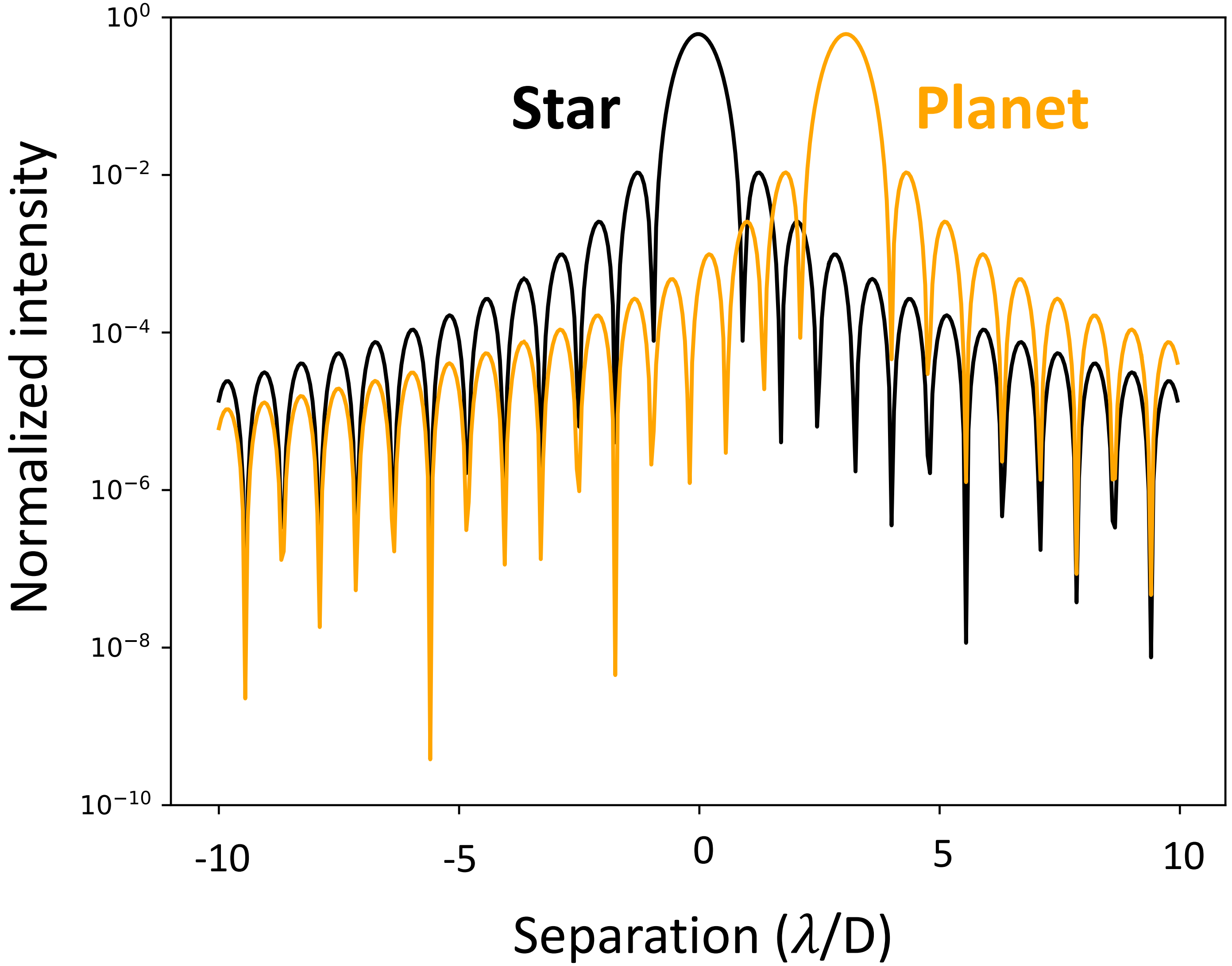
Companions and angular resolution

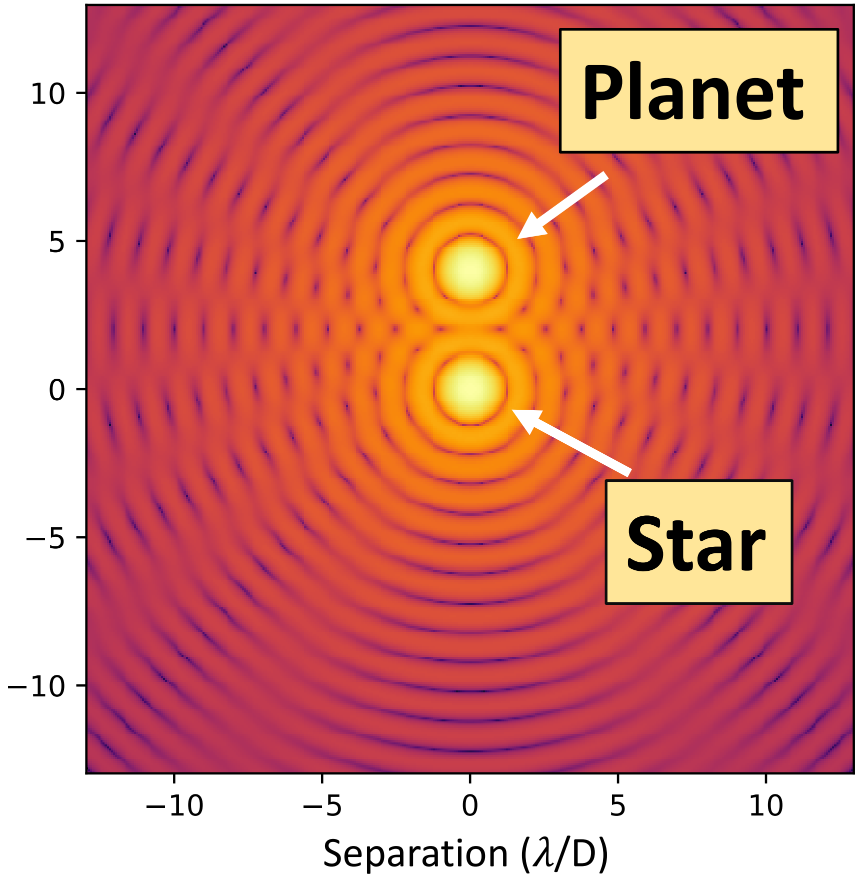
Simple imager/telescope





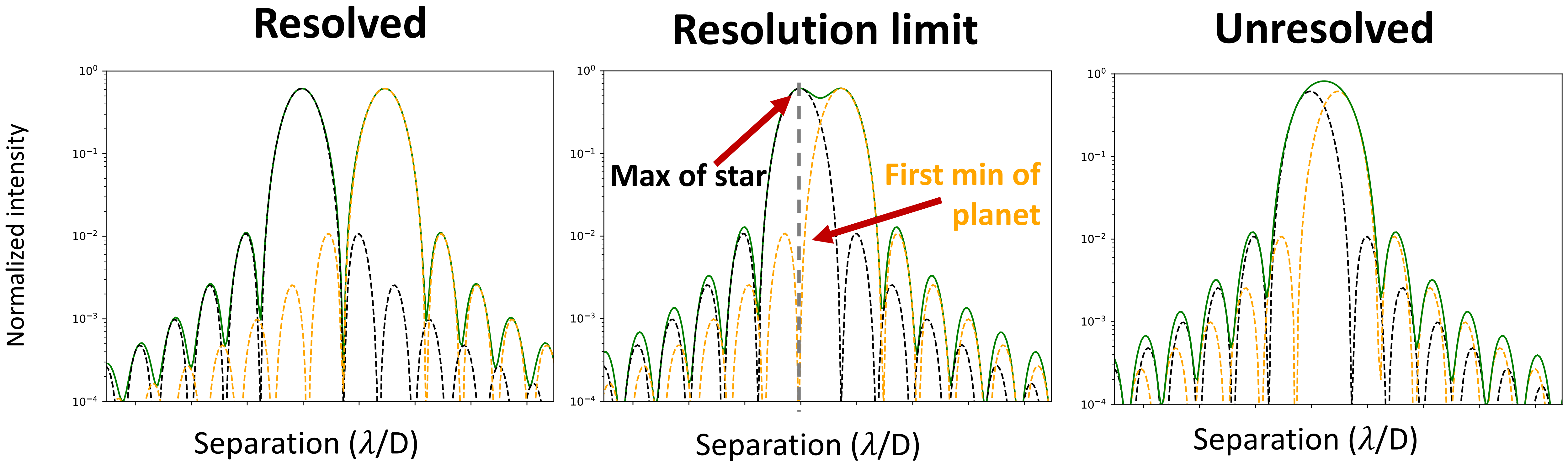
Companions and angular resolution



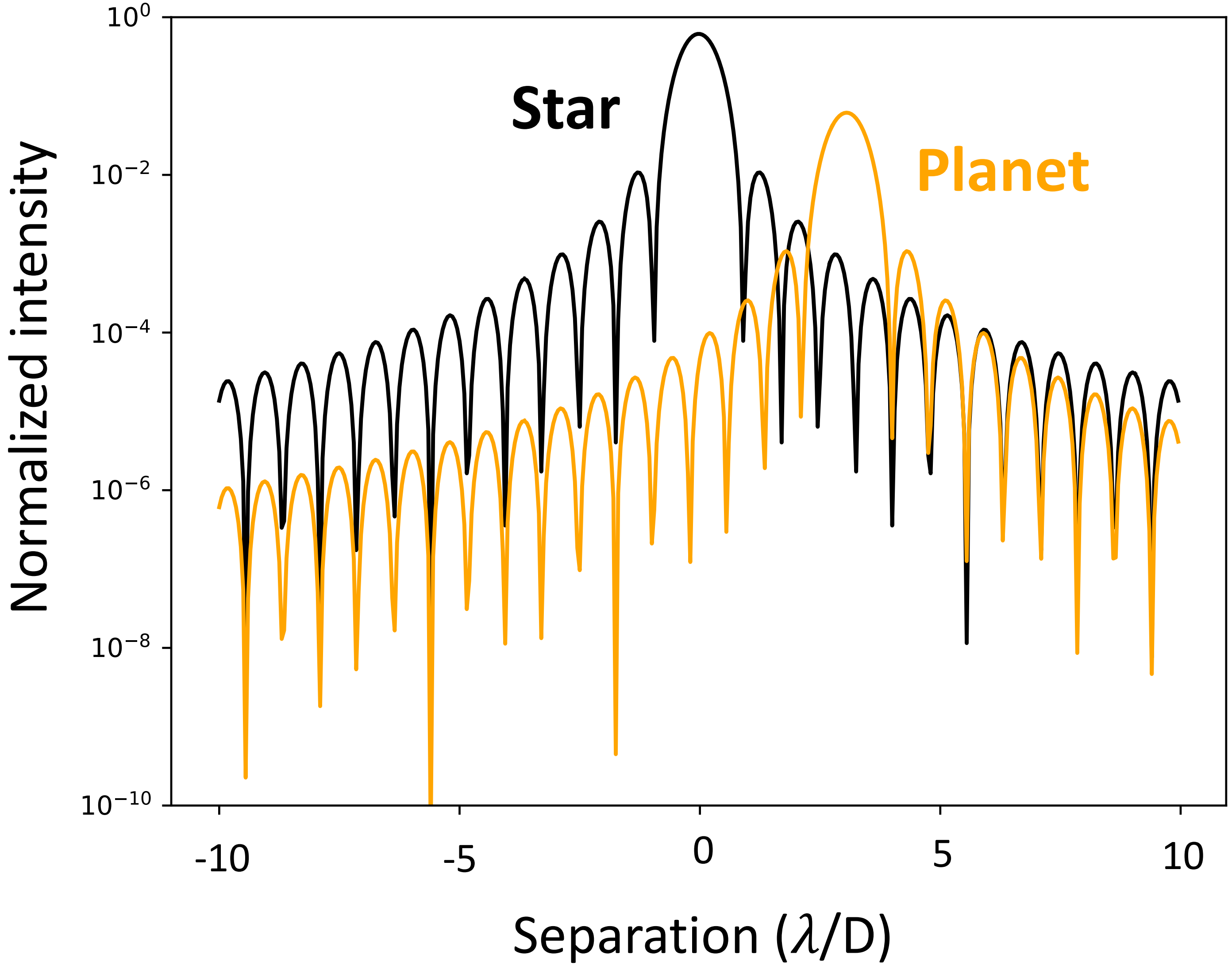


Rayleigh criterion for angular resolution

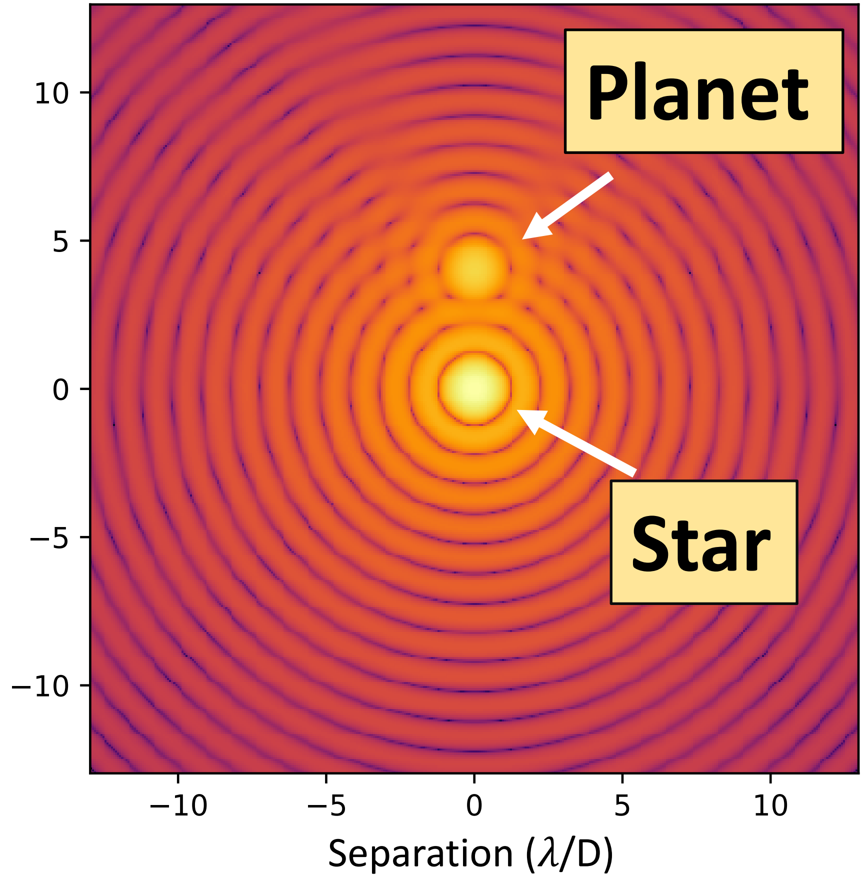
- - Star　　- -Planet　　- Sum



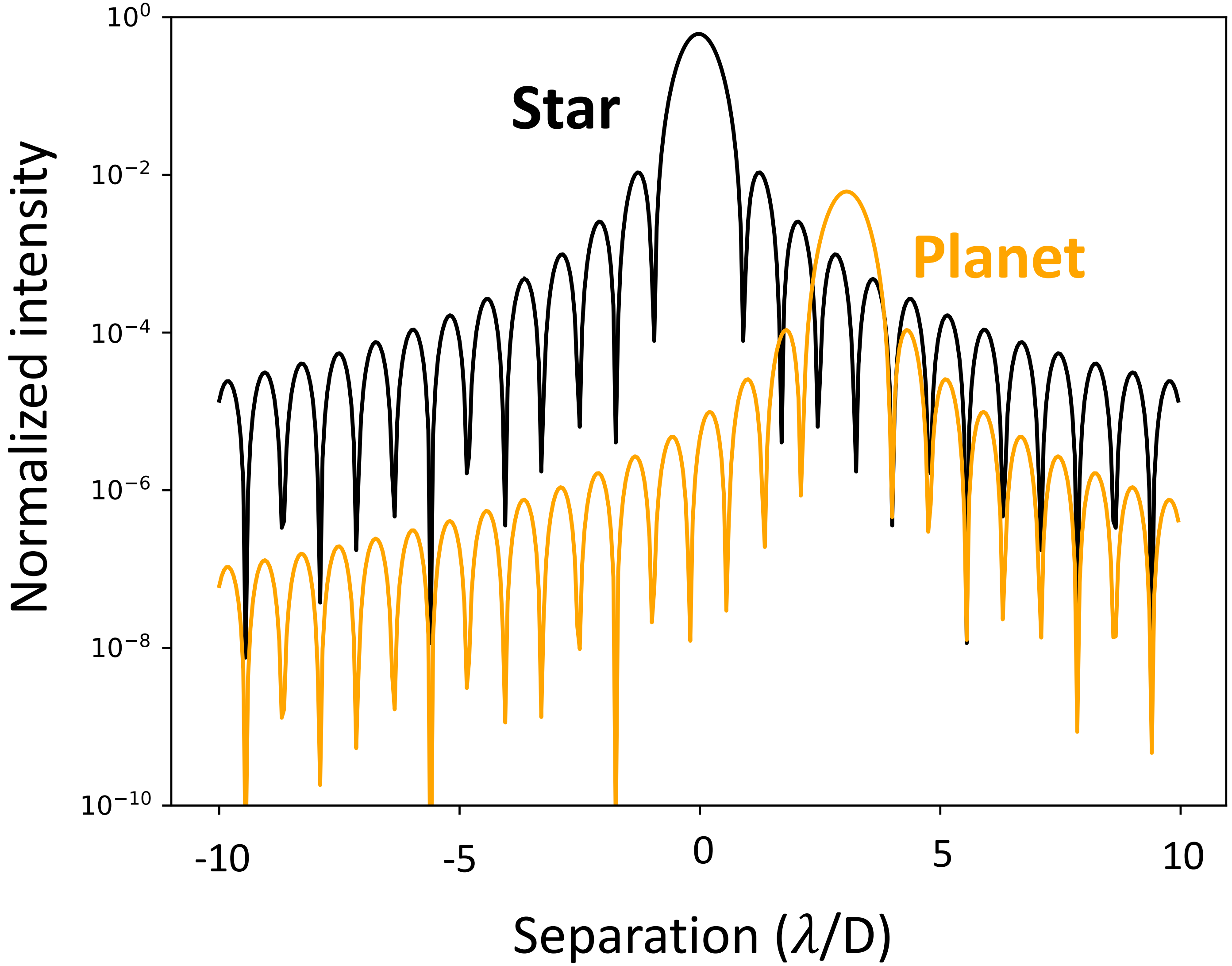
Faint companions and angular resolution



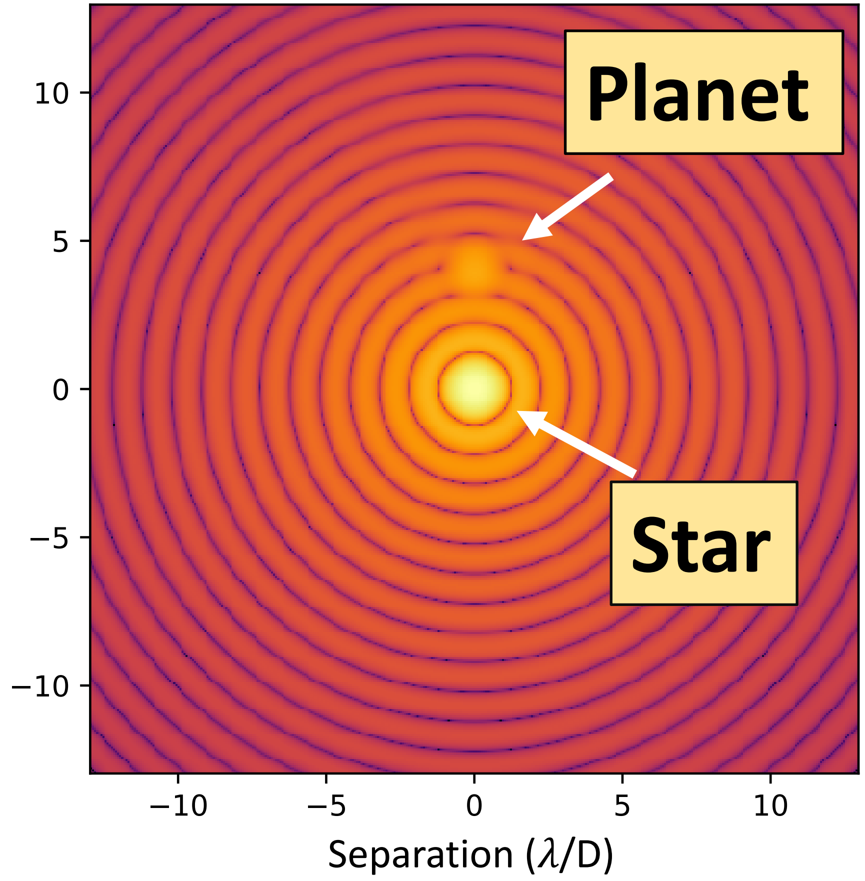
Two PSFs but one is fainter!



Faint companions and angular resolution

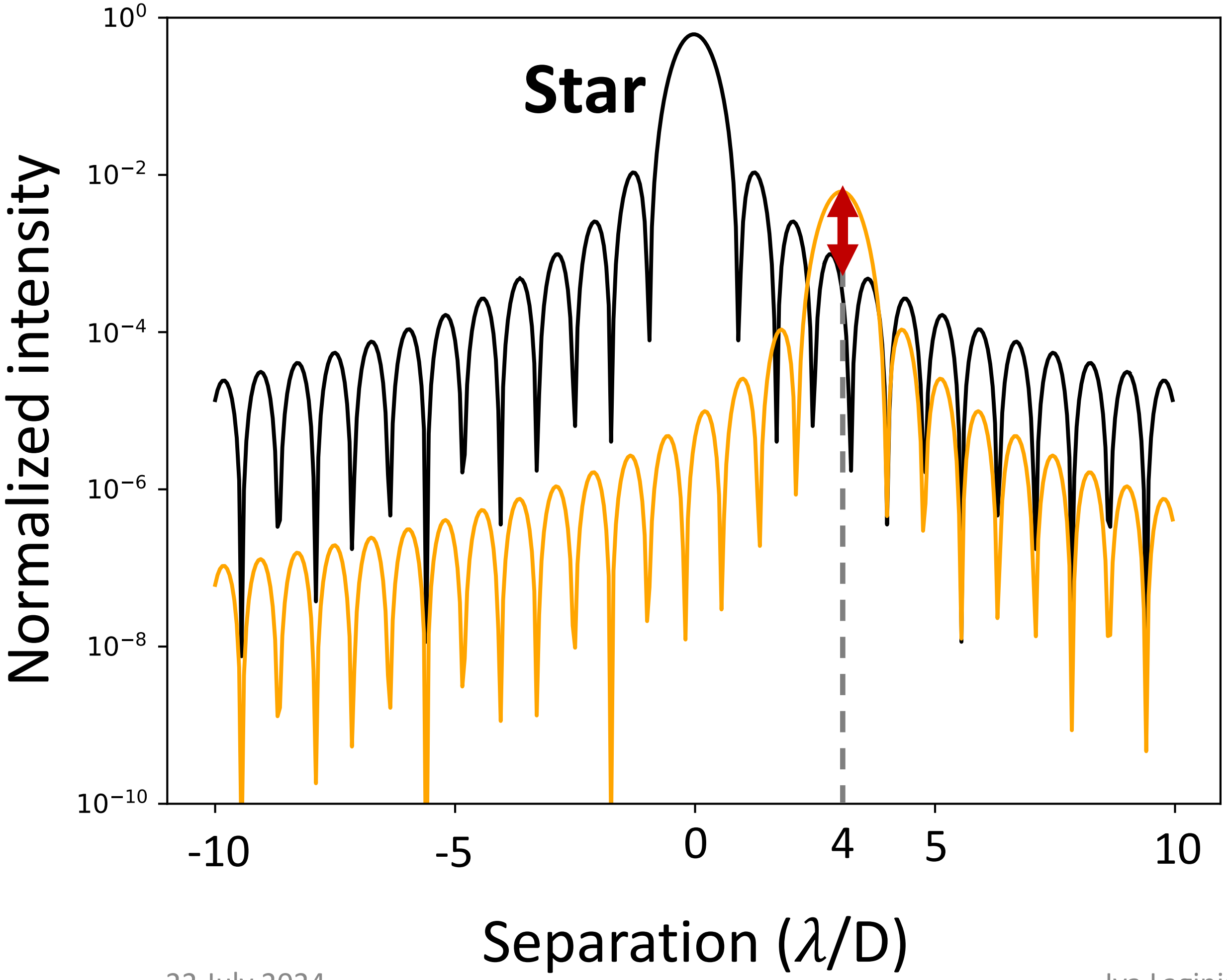


Two PSFs but one is fainter!



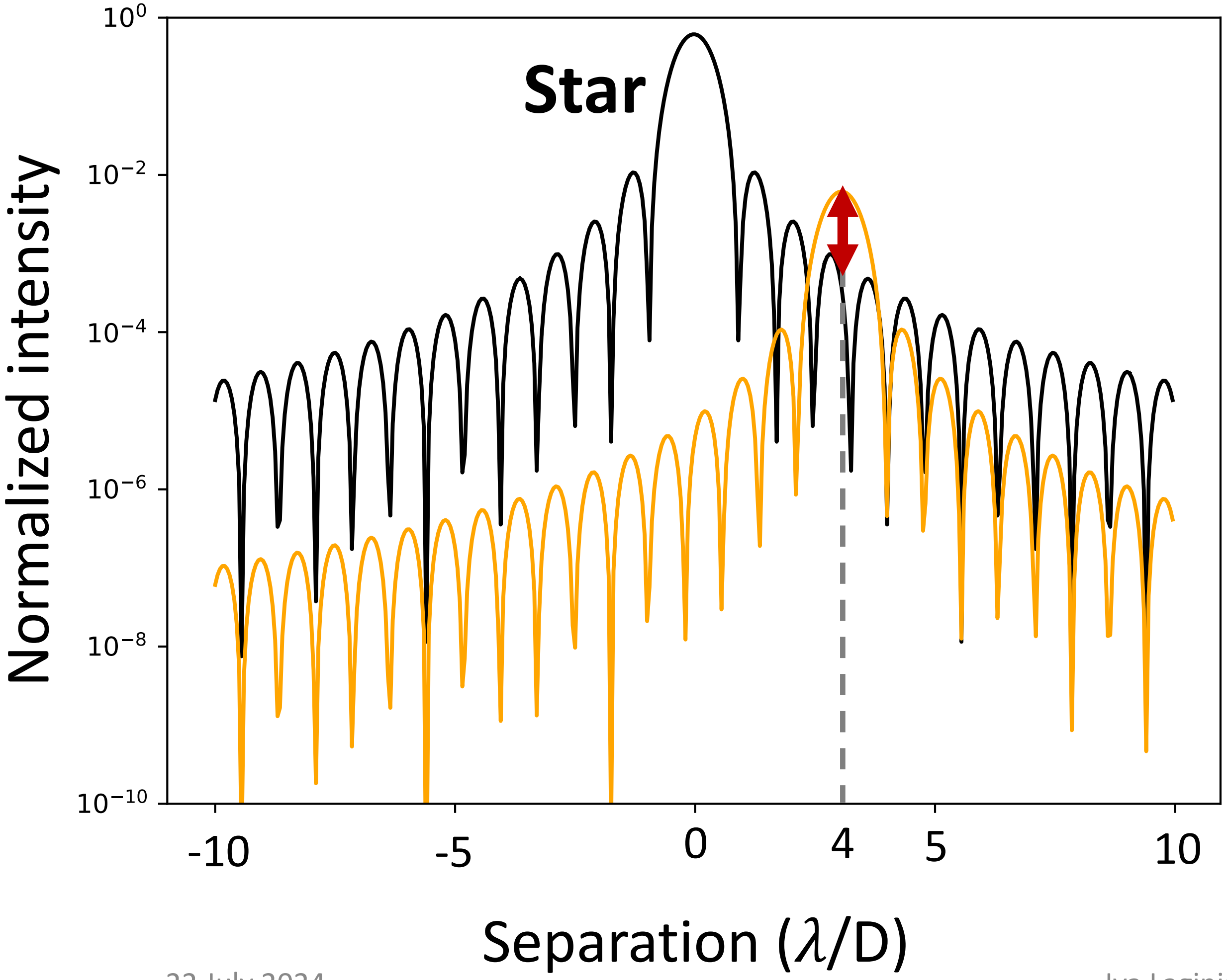
Faint companions at small angular separations

10^{-2} planet at 4 λ/D

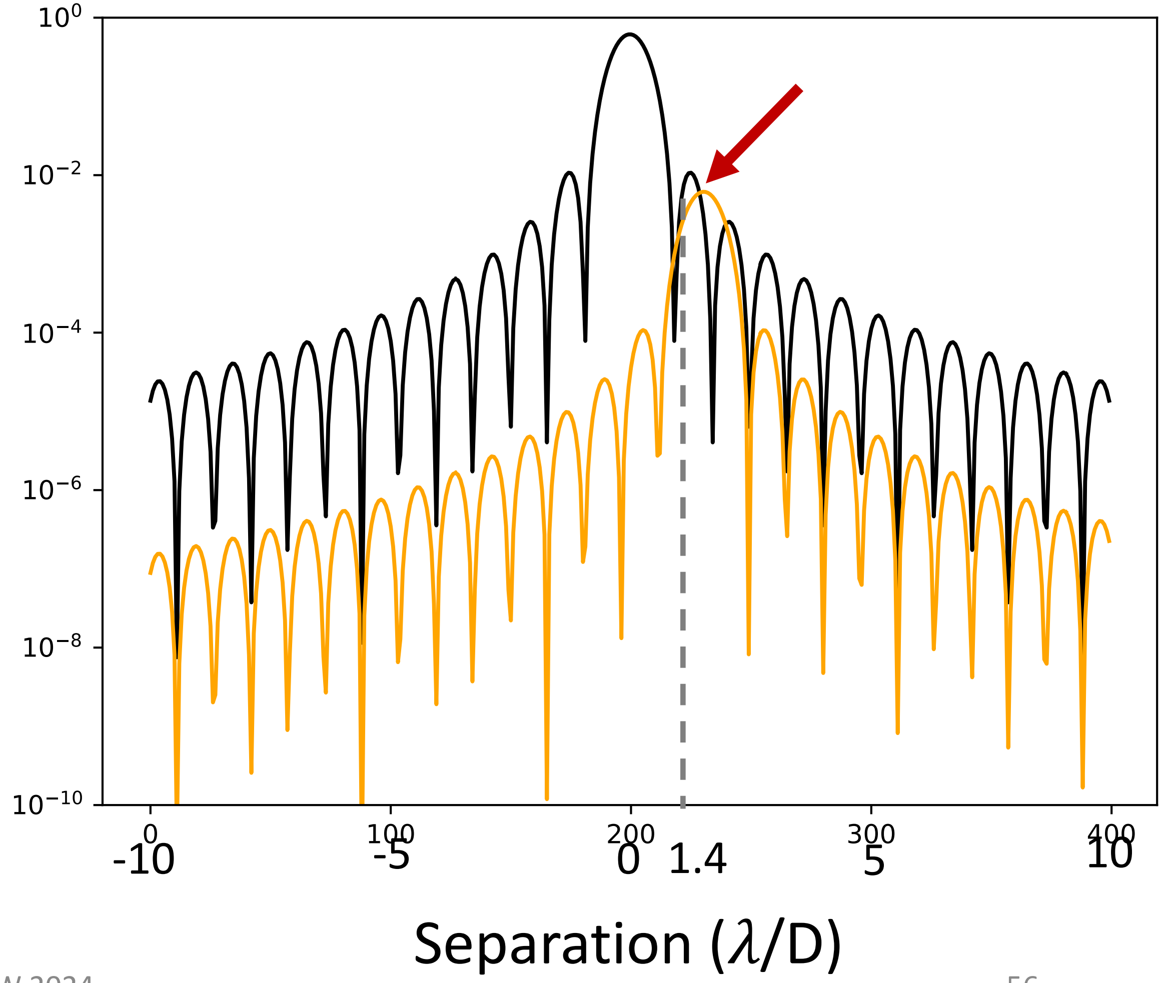


Faint companions at small angular separations

10^{-2} planet at 4 λ/D

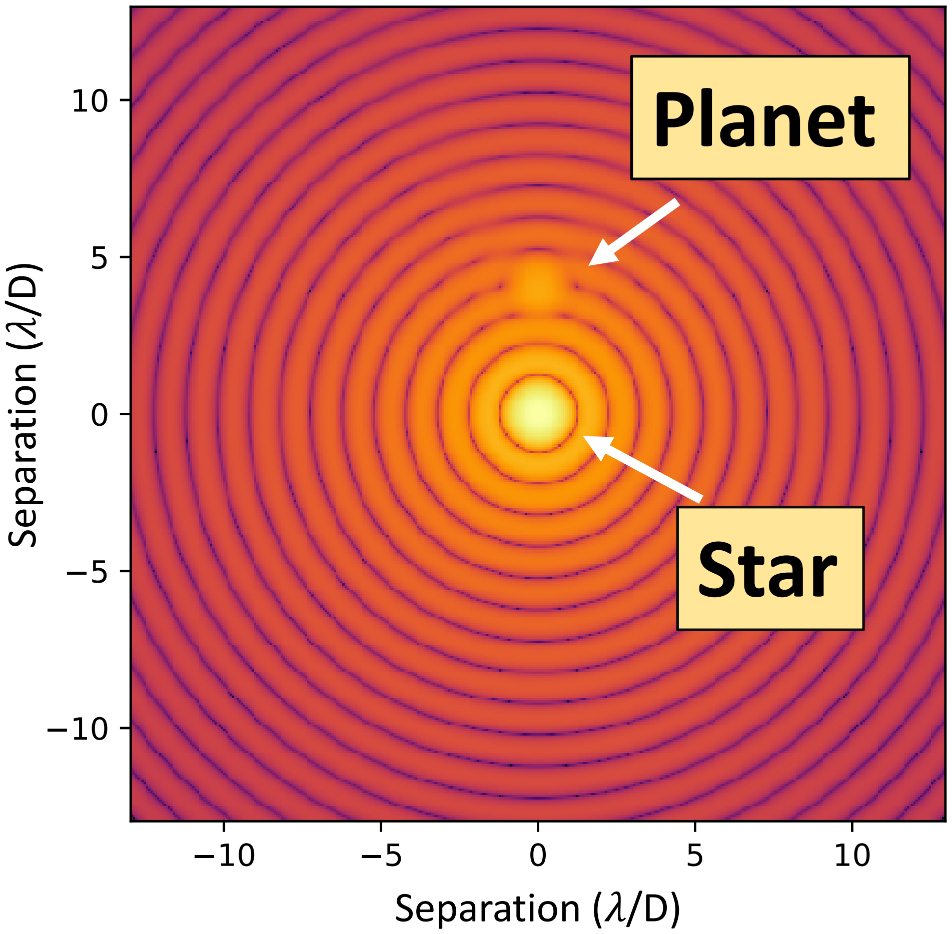


10^{-2} planet at 2 λ/D

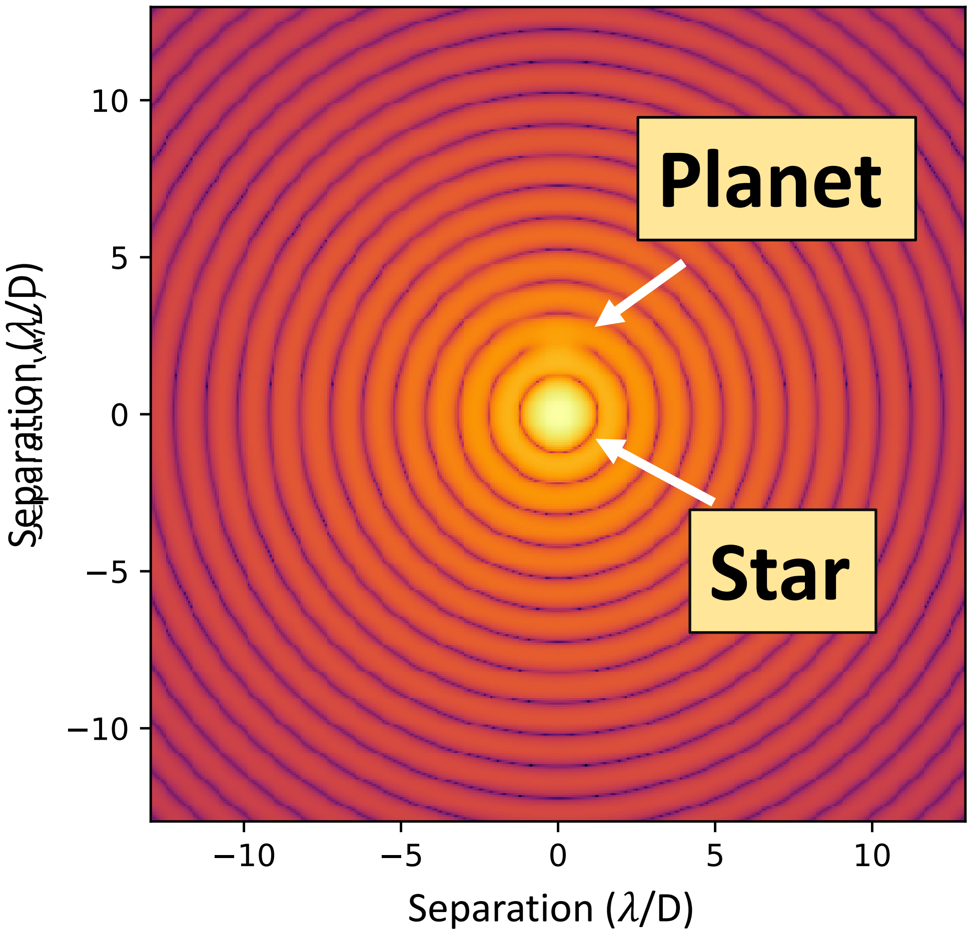


Faint companions at small angular separations

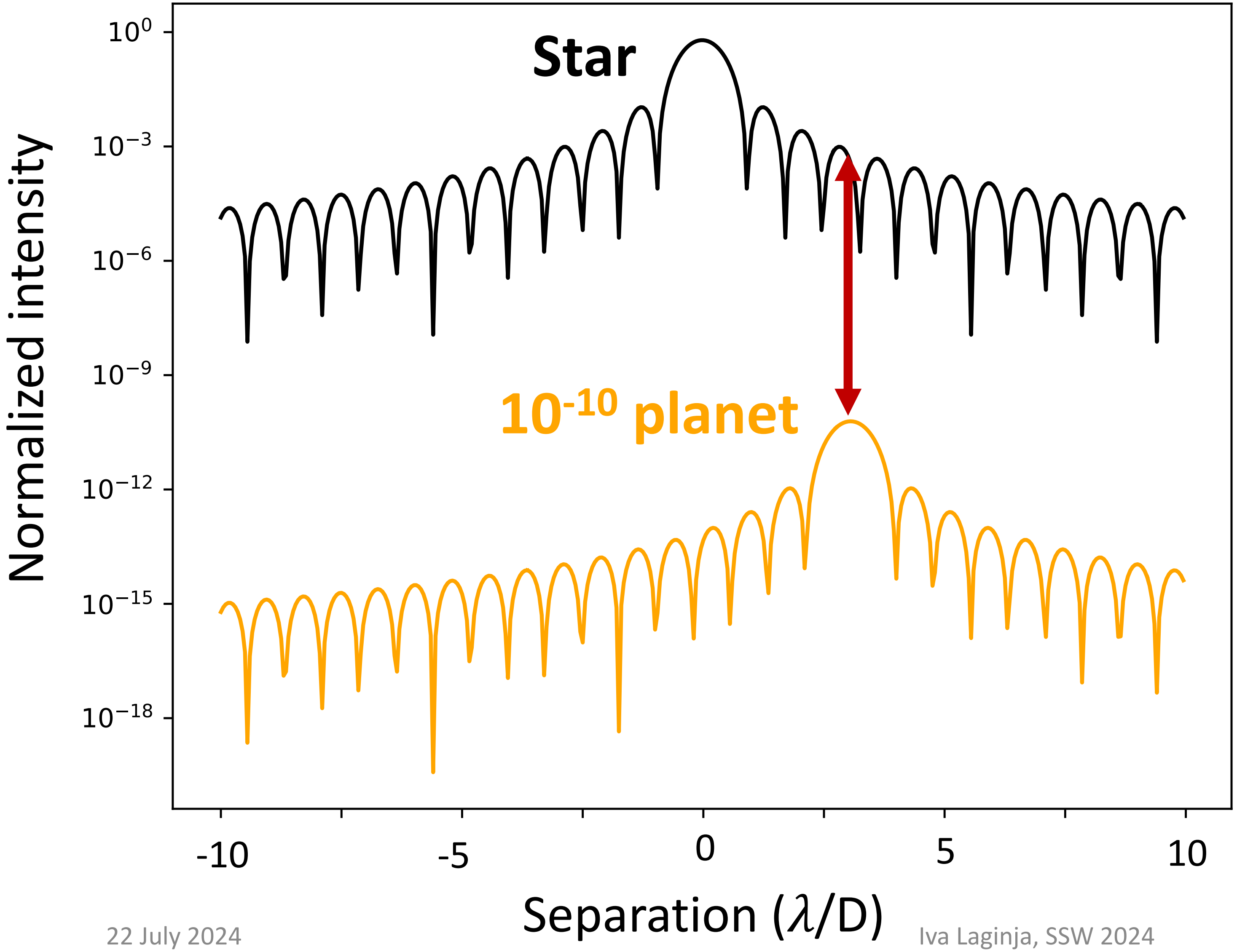
10^{-2} planet at 4 λ/D



10^{-2} planet at 2 λ/D

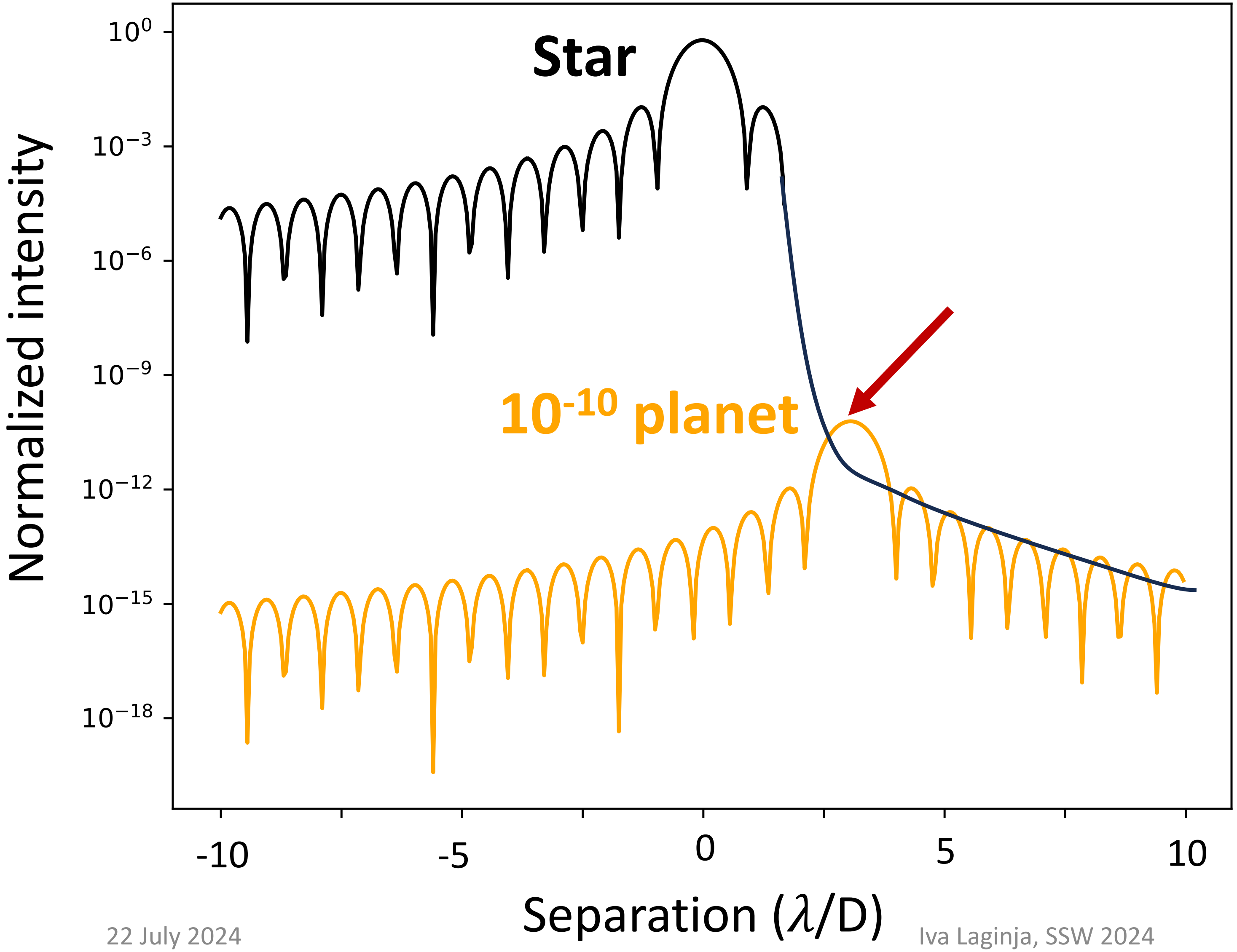


Faint companions at small angular separations



- Planets with worse flux ratio (=fainter planets) even harder to image in stellar light
- The closer the planet to optical axis, the harder to image

Faint companions at small angular separations



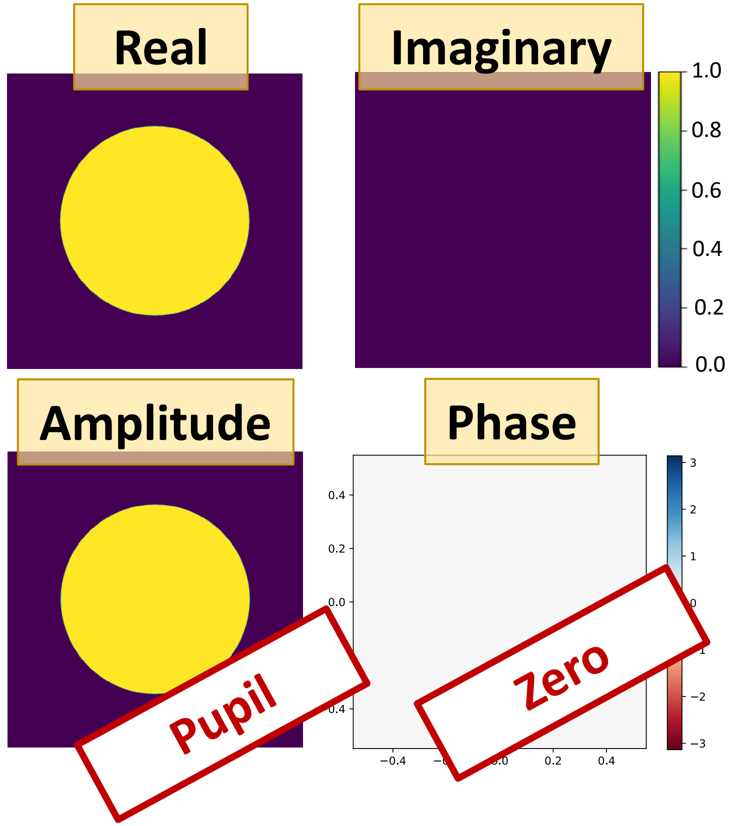
- Planets with worse flux ratio (=fainter planets) even harder to image in stellar light
- The closer the planet to optical axis, the harder to image
- **Starlight suppression** techniques needed → **coronagraphy** (see next talk)

Optical aberrations

Amplitude and phase aberrations, sources of aberrations, aberrations by spatial frequency content

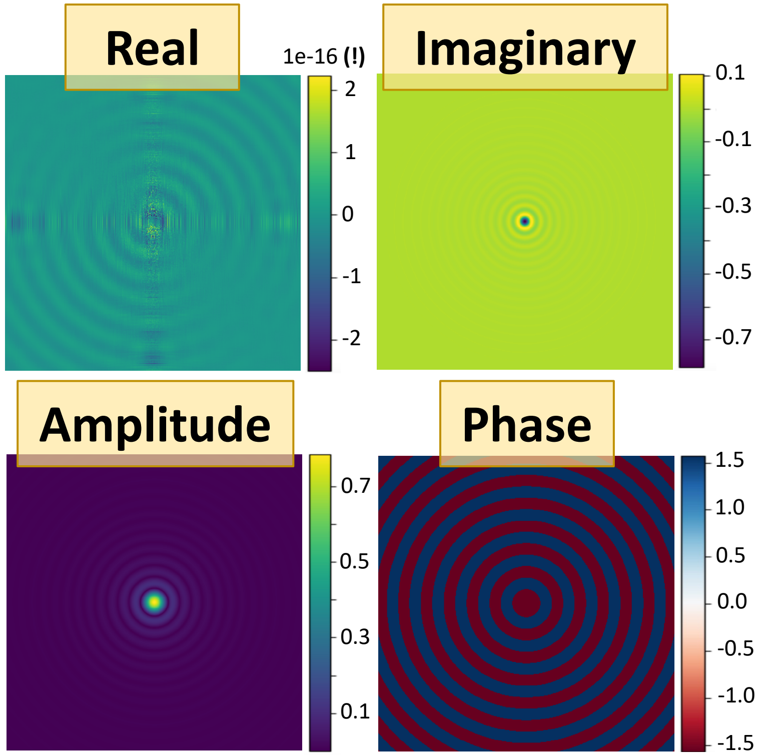
Perfect wavefront

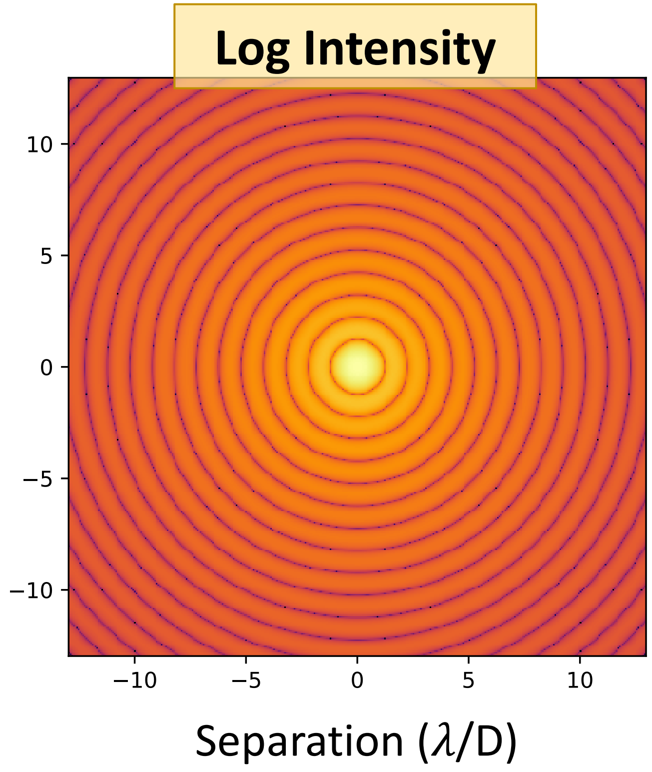
Pupil plane



$$E_{pup} = Ae^{i\phi} = A$$

Focal plane



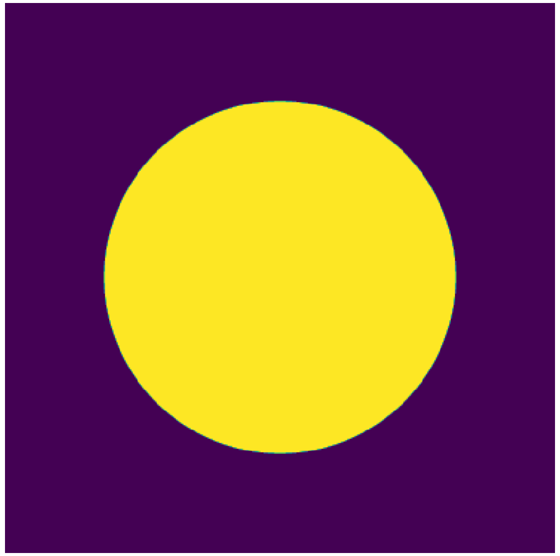


"Perfect" PSF

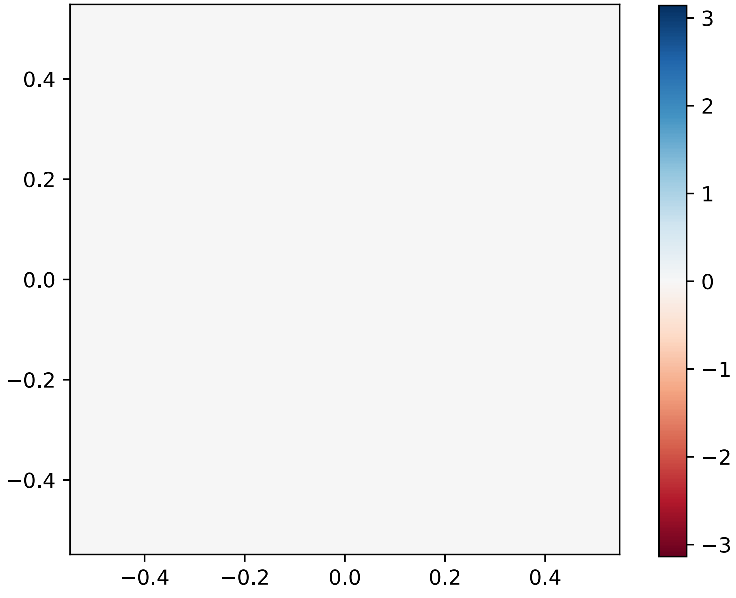
Aberrations can occur in amplitude or phase

Pupil plane

Amplitude

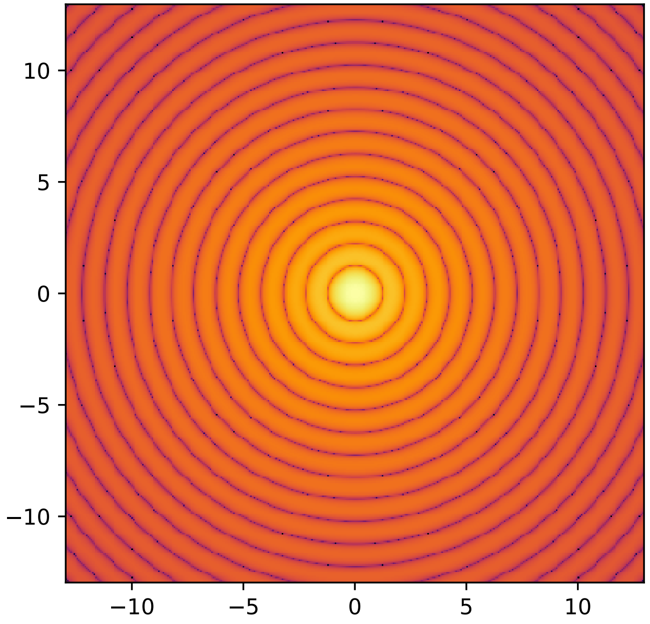


Phase



Focal plane

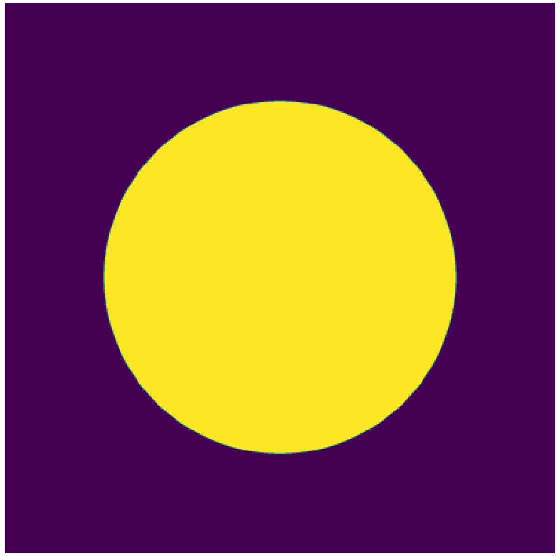
Log Intensity



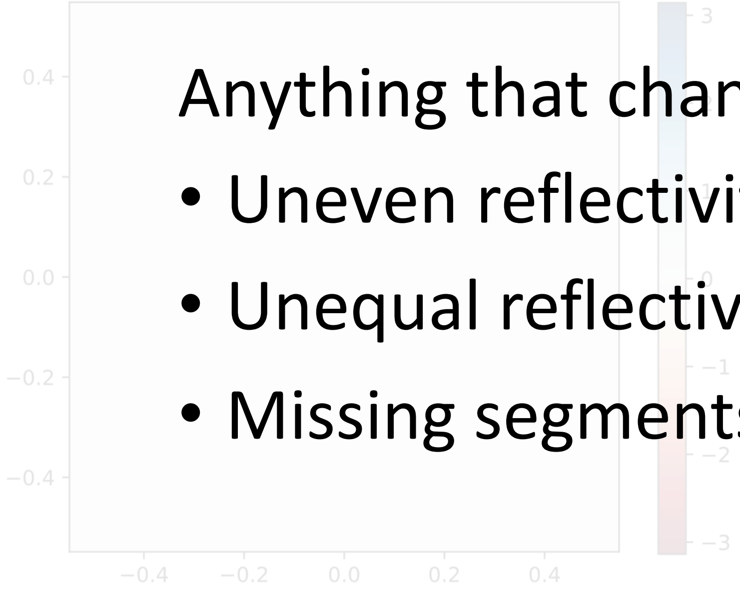
$$E_{pup} = Ae^{\alpha + i\phi}$$

Amplitude aberrations

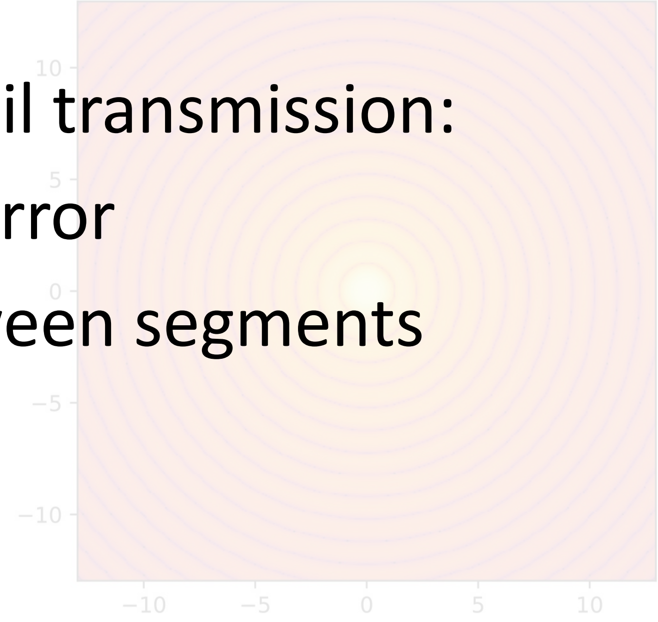
Amplitude



Phase



Log Intensity



Anything that changes pupil transmission:

- Uneven reflectivity on mirror
- Unequal reflectivity between segments
- Missing segments

$$E_{pup} = Ae^{\alpha + i\phi}$$

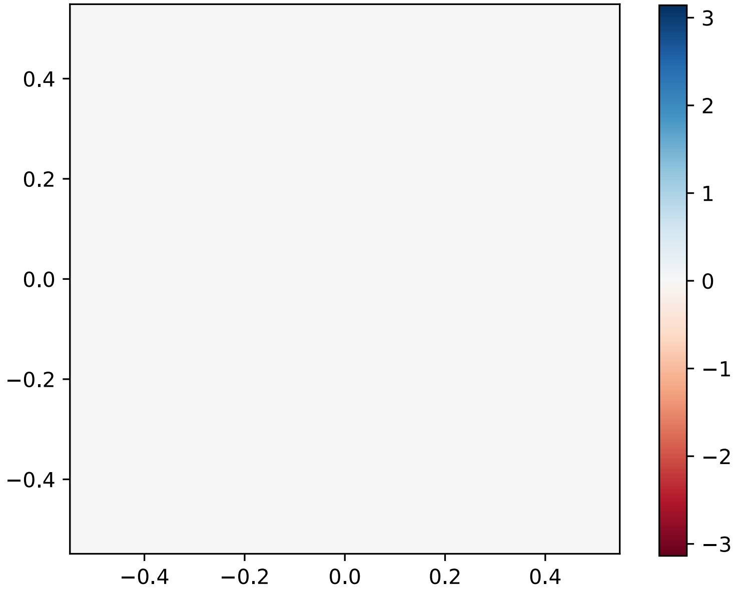
Phase aberrations

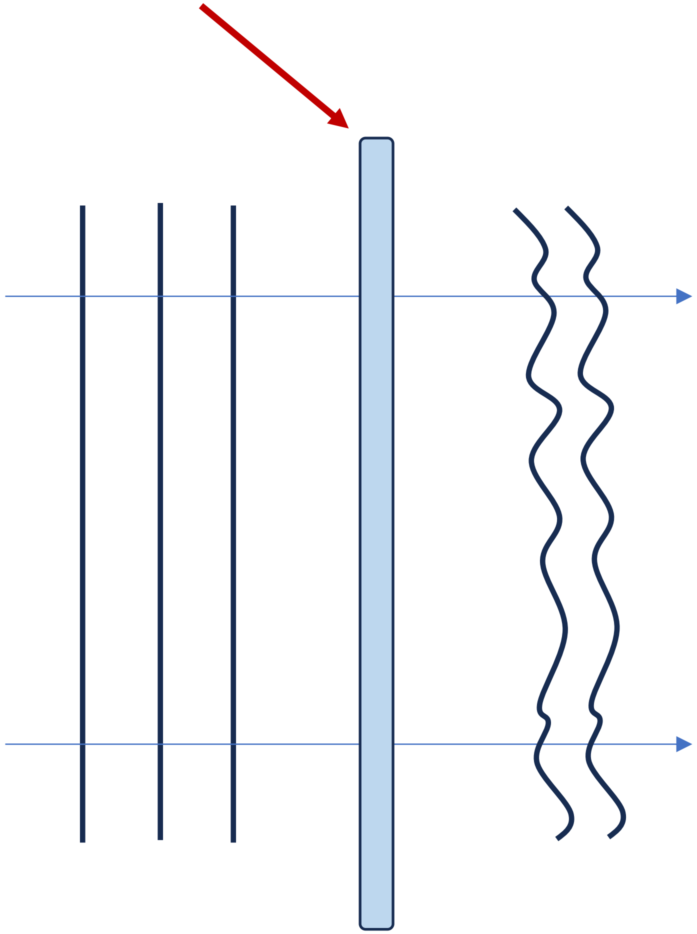
Medium introducing phase aberrations,
e.g. atmosphere, faulty optic

Pupil plane

Amplitude

Phase

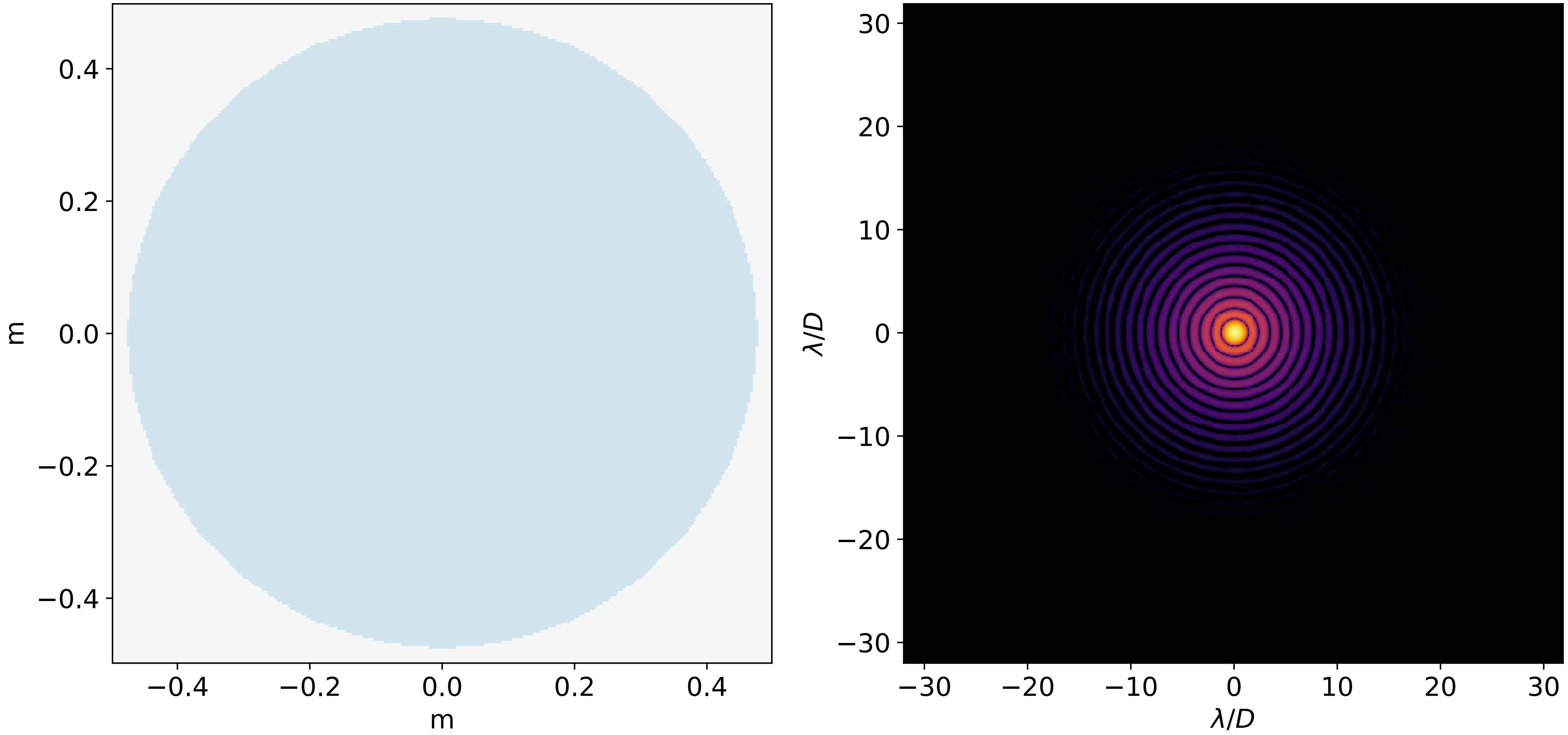




$$E_{pup} = Ae^{i\phi}$$

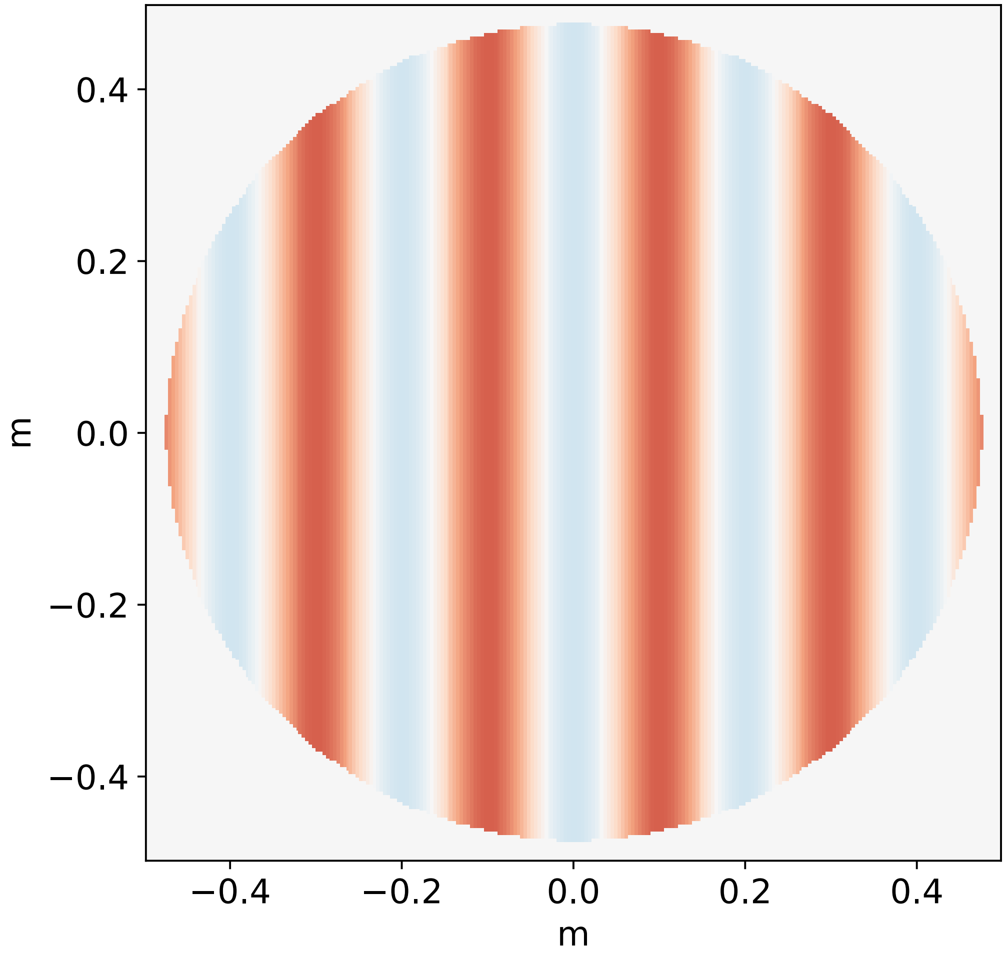
Phase ripples across pupil

No aberration

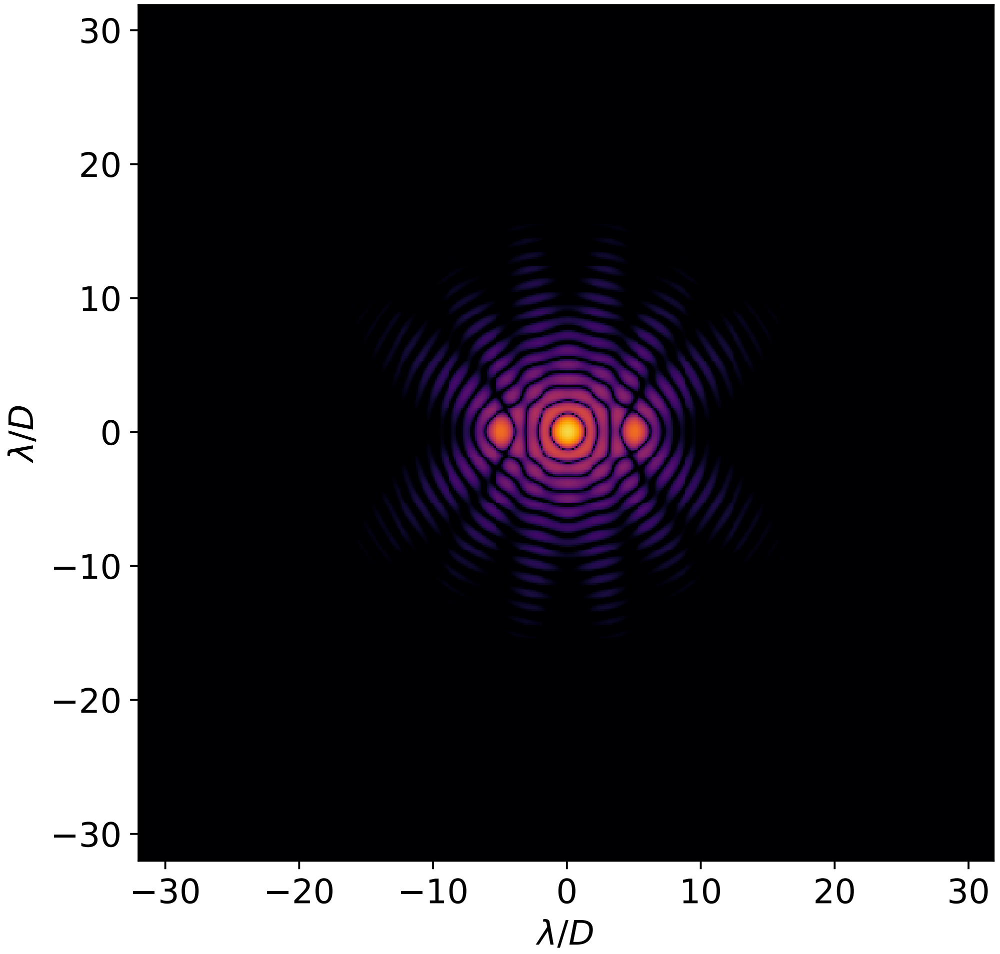


Phase ripples across pupil

5 cycles per pupil

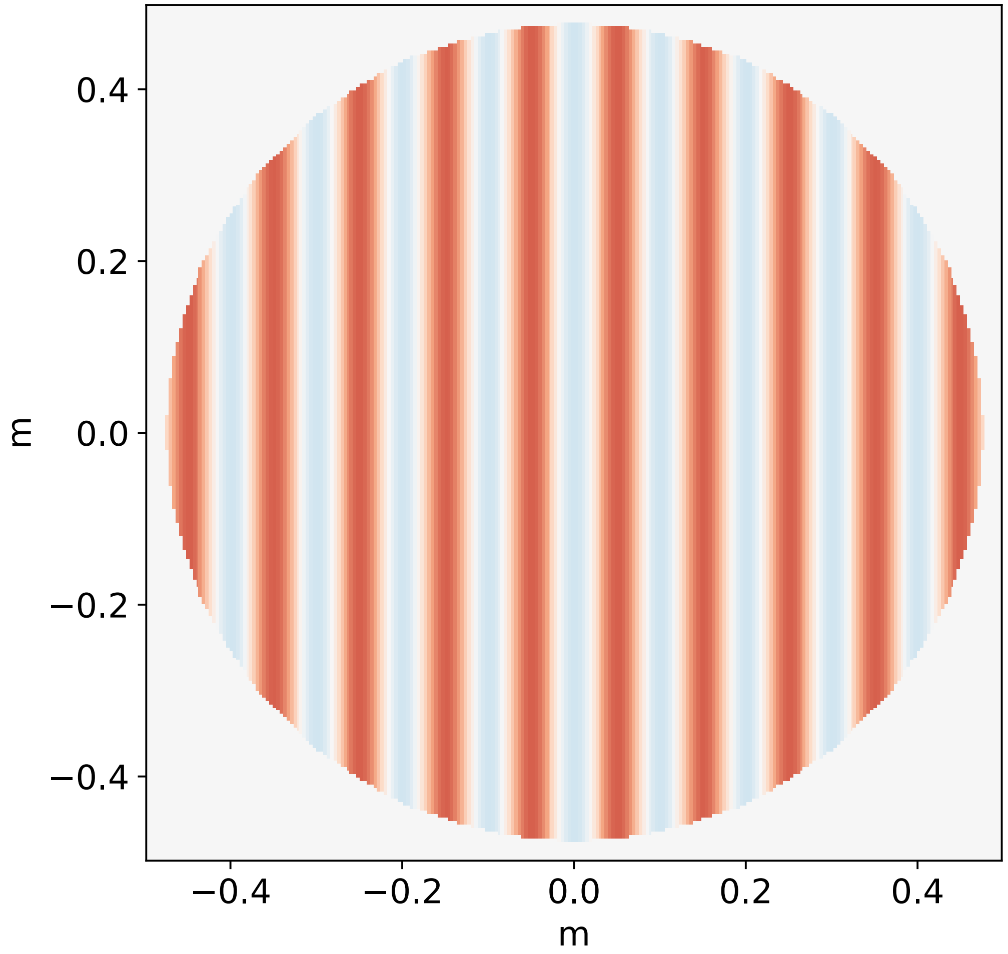


Pair of speckles at 5 λ/D

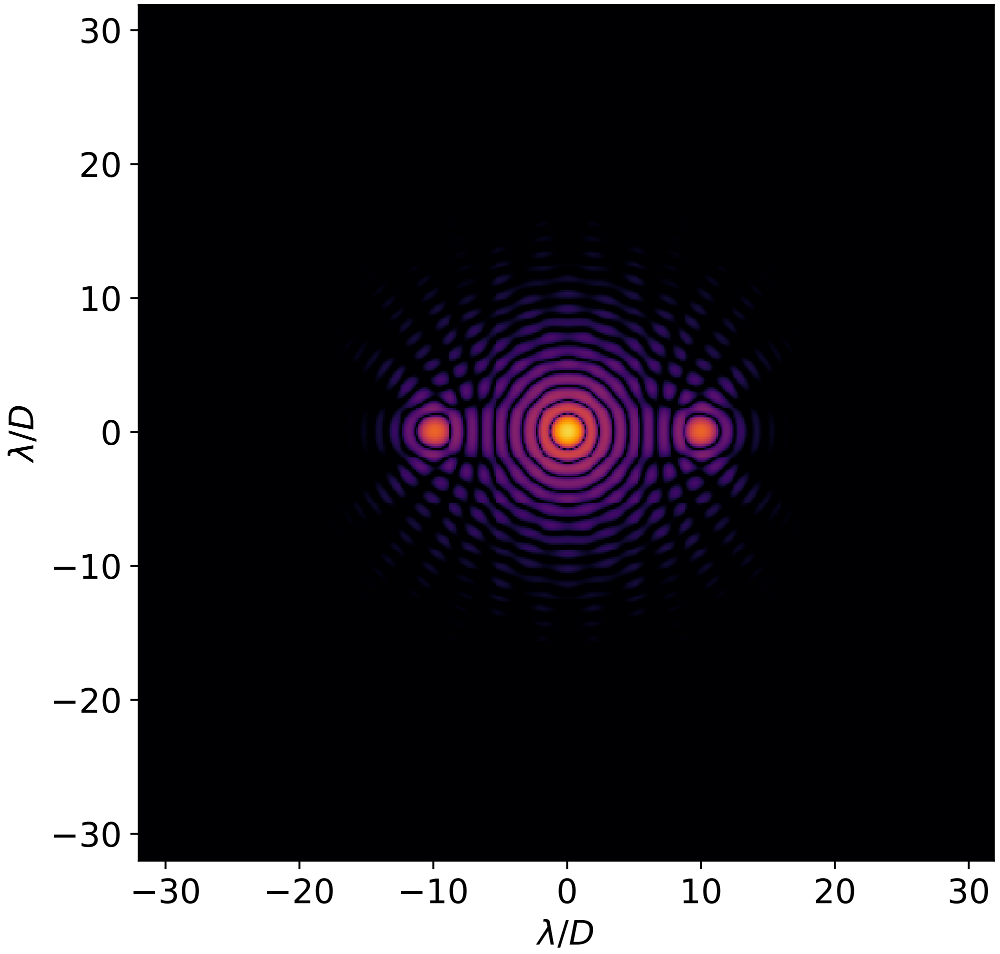


Phase ripples across pupil

10 cycles per pupil

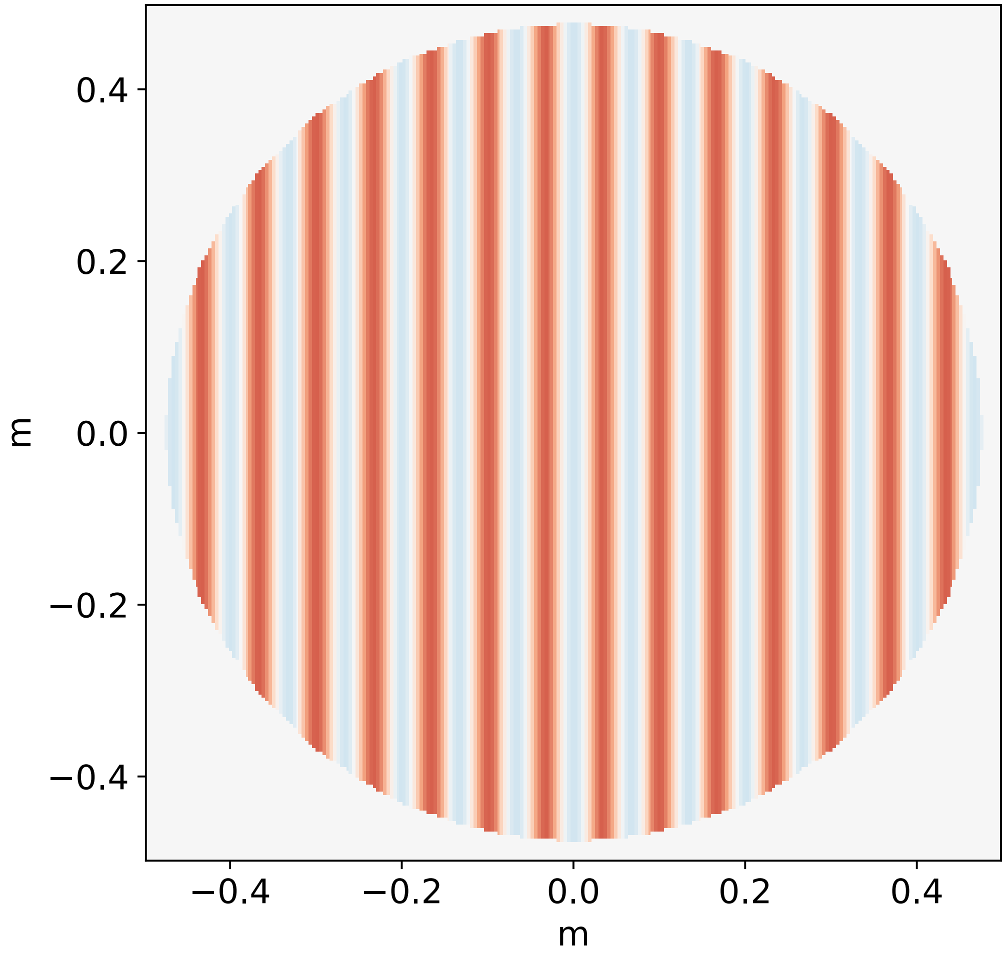


Pair of speckles at 10 λ/D

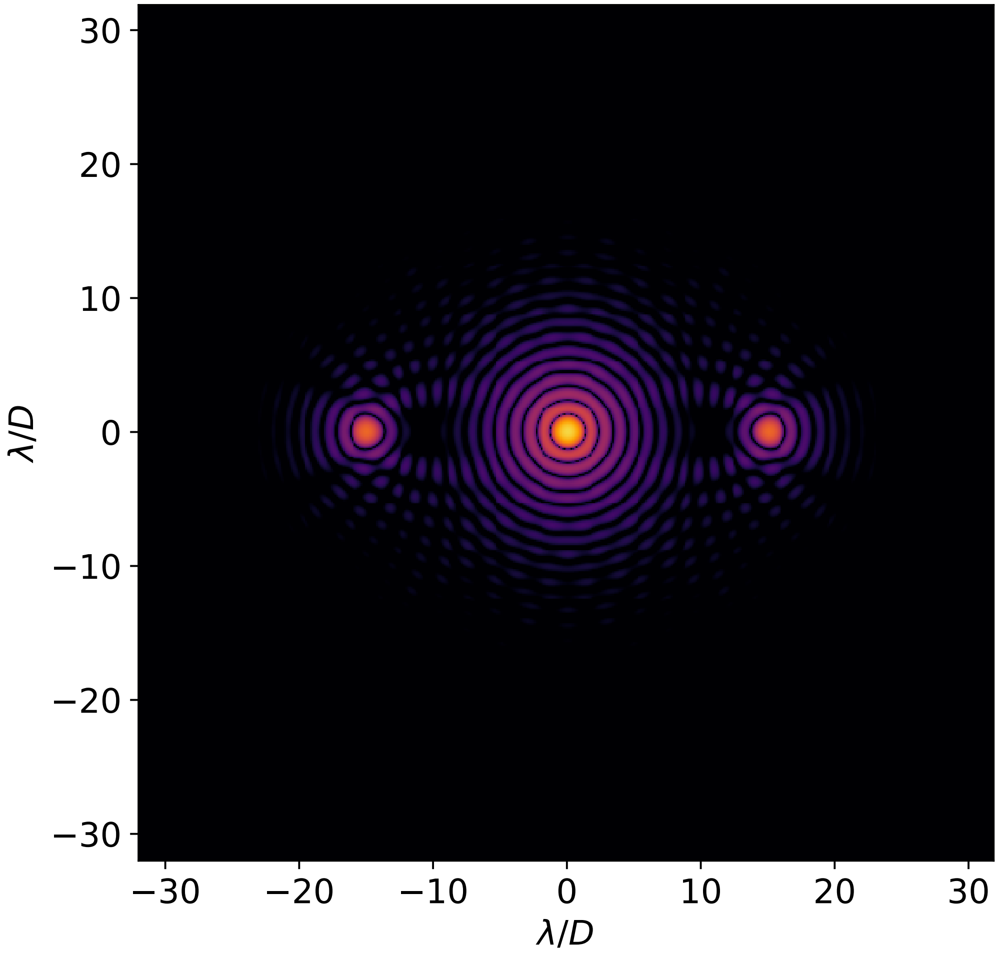


Phase ripples across pupil

15 cycles per pupil

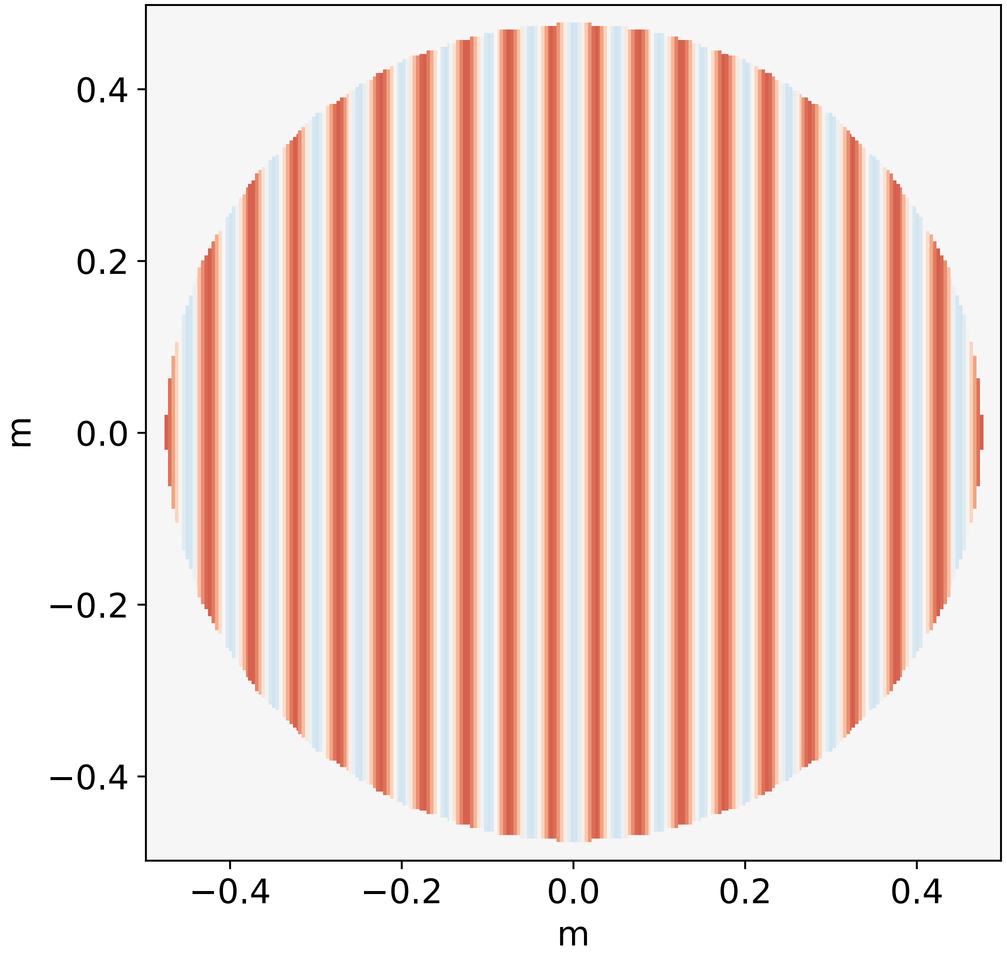


Pair of speckles at 15 λ/D

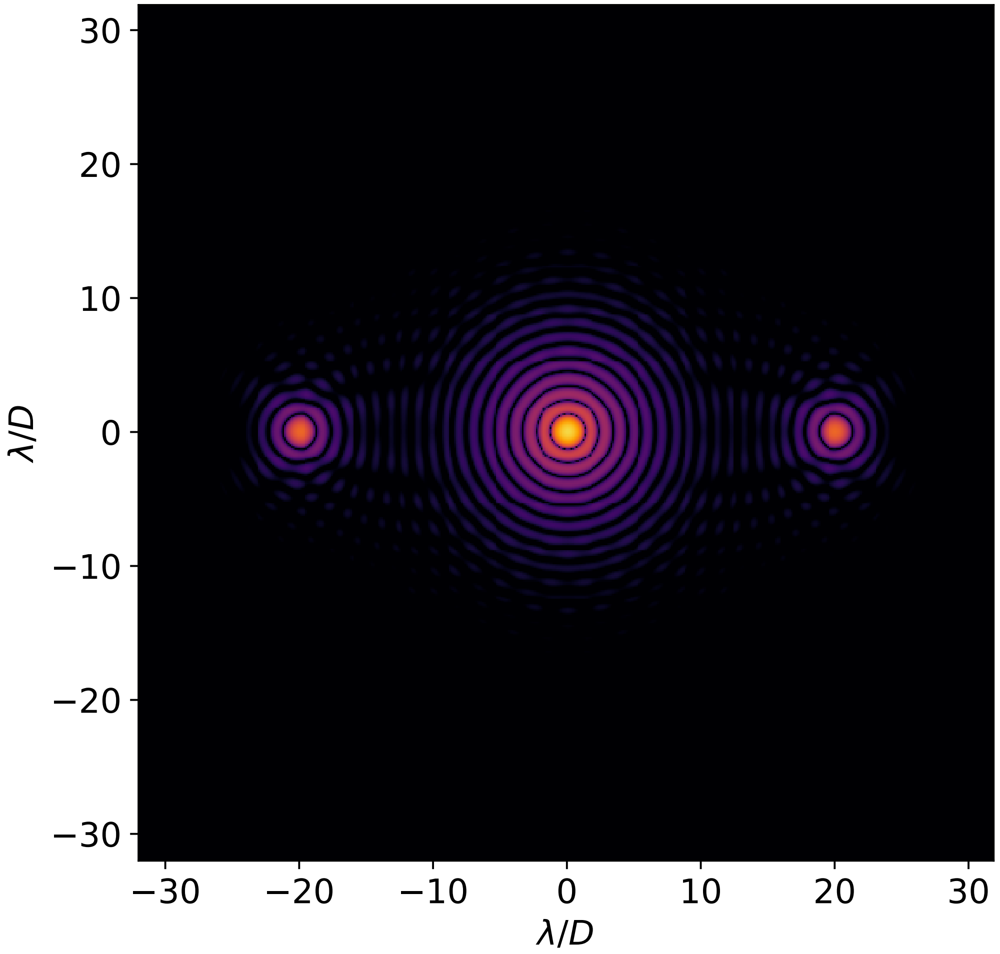


Phase ripples across pupil

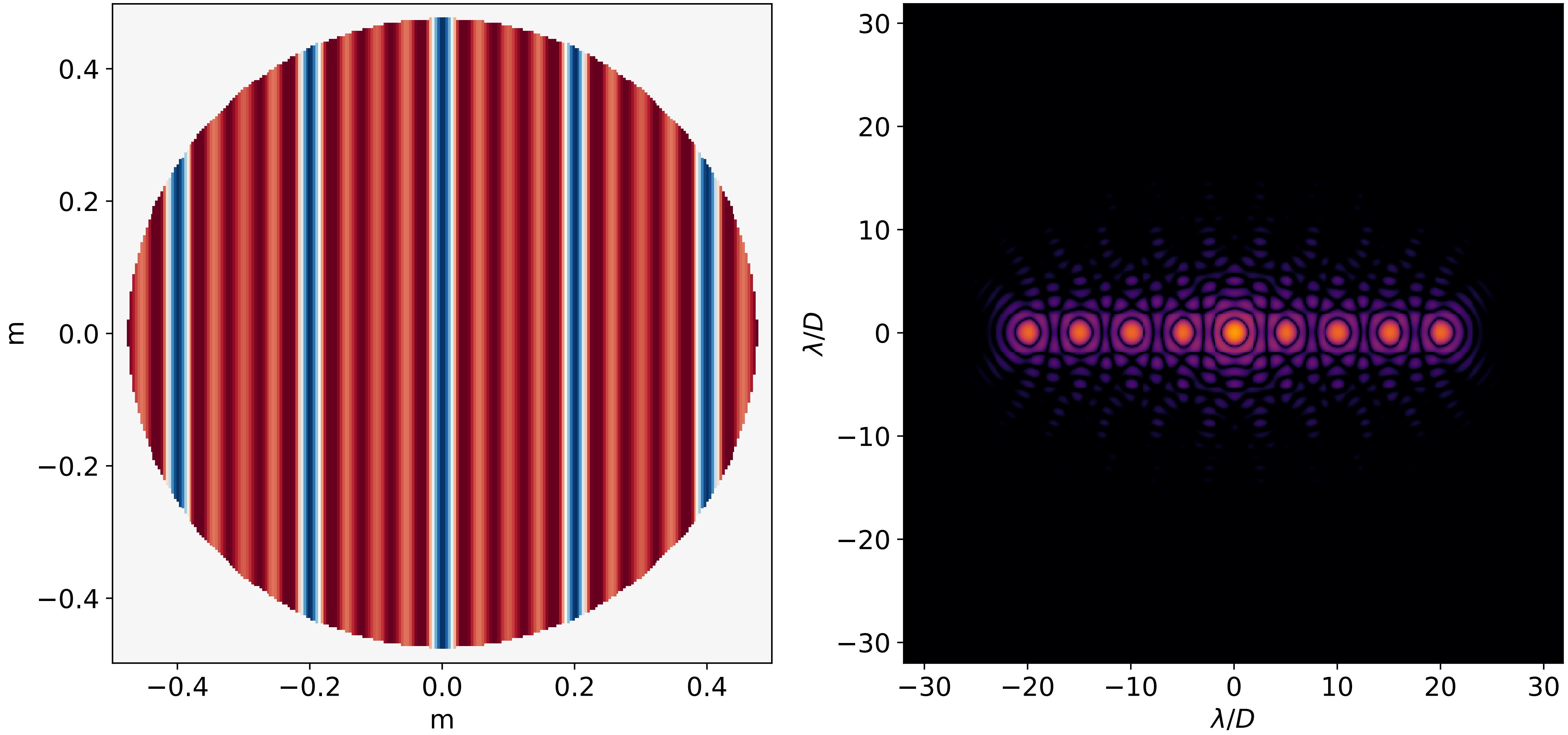
20 cycles per pupil



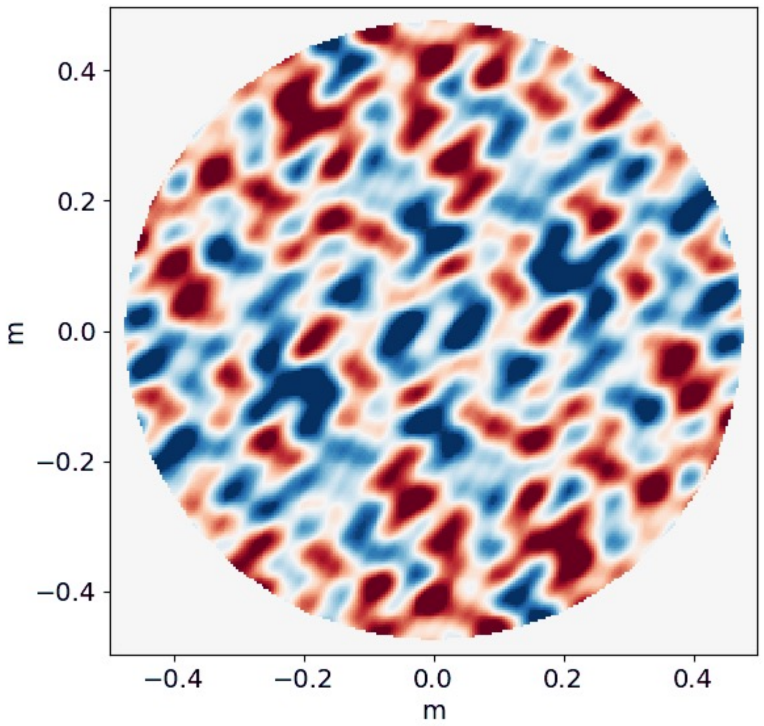
Pair of speckles at 20 λ/D

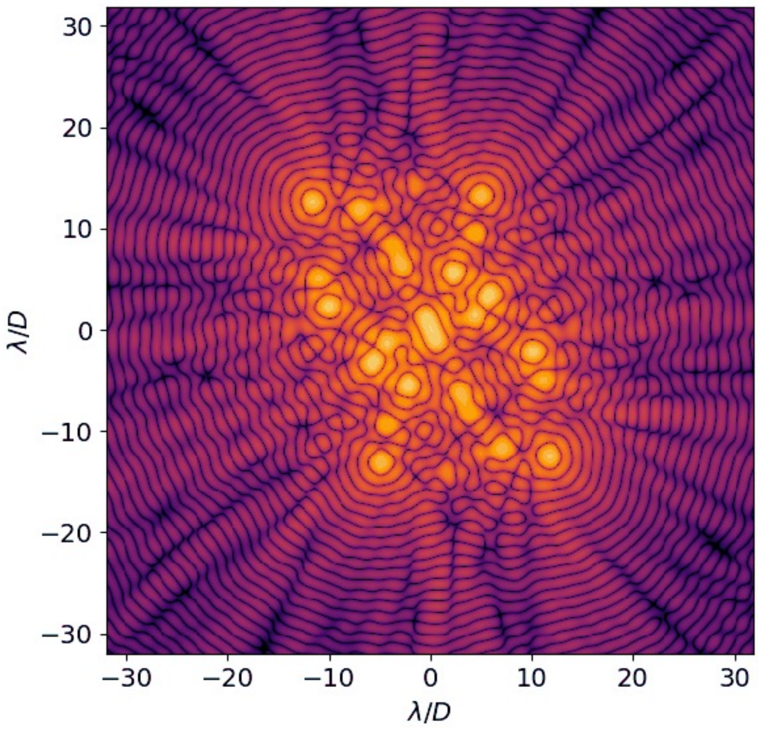


Linear combinations of sine waves in pupil



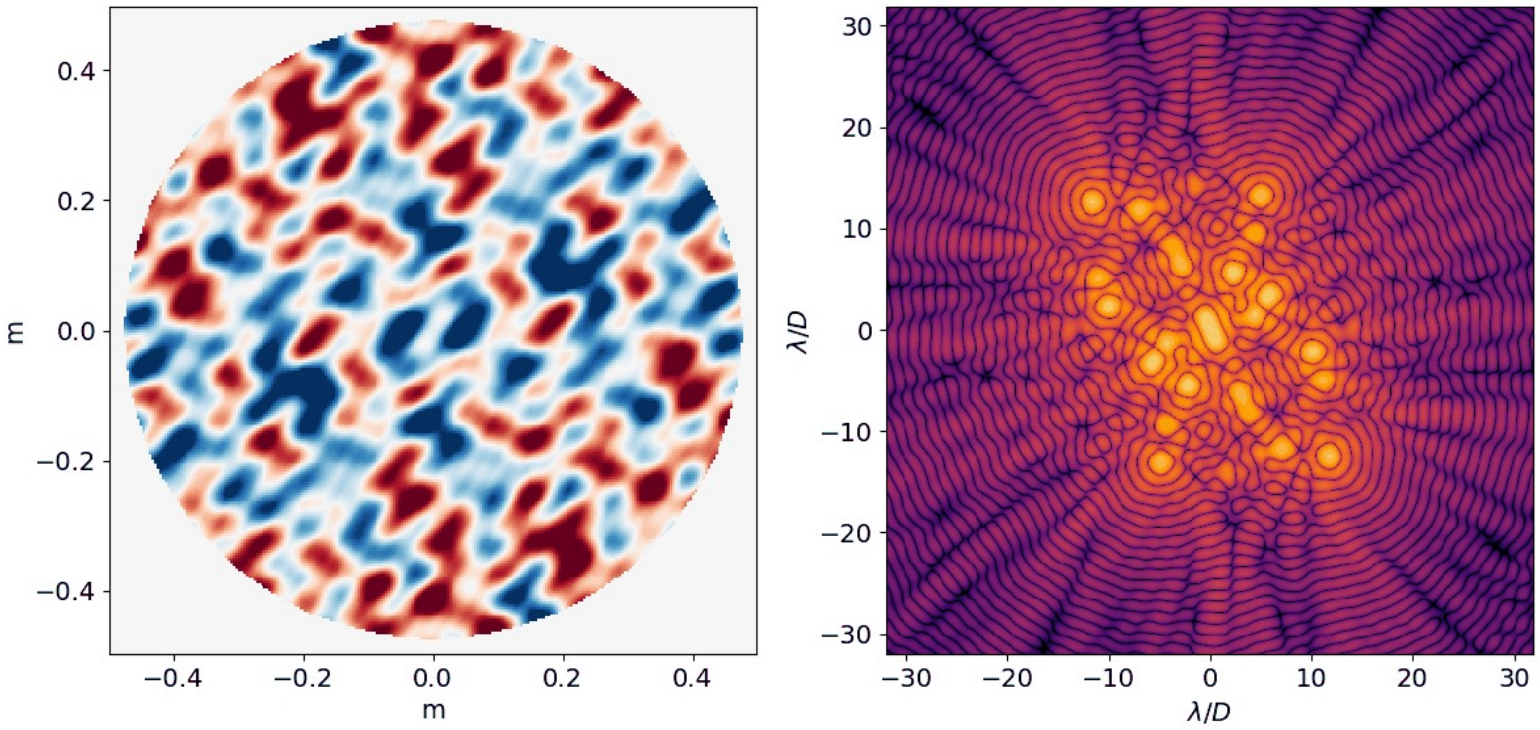
Any aberration can be expressed as linear combination of sine waves





- Break down aberrations by their **spatial frequency**
- General division due to occurrence of aberrations in **low, mid and high spatial frequencies**

Any aberration can be expressed as linear combination of sine waves

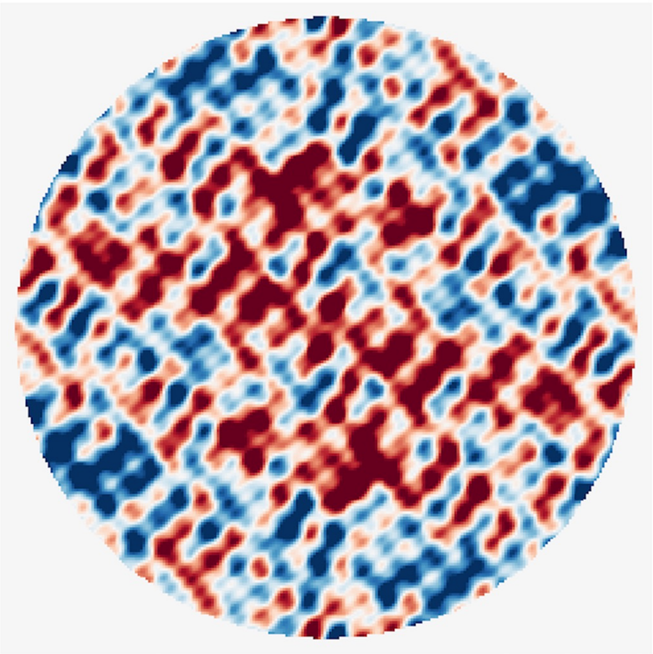


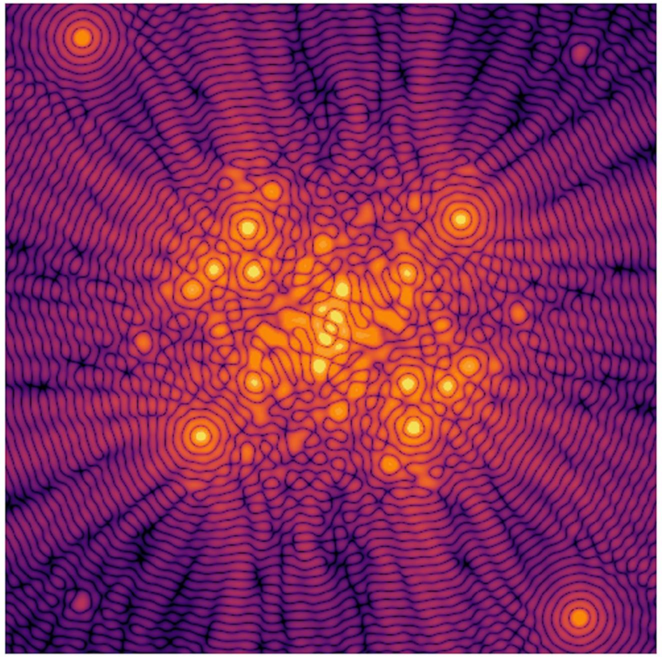
- Break down aberrations by their **spatial frequency**
- General division due to occurrence of aberrations in **low, mid and high spatial frequencies**

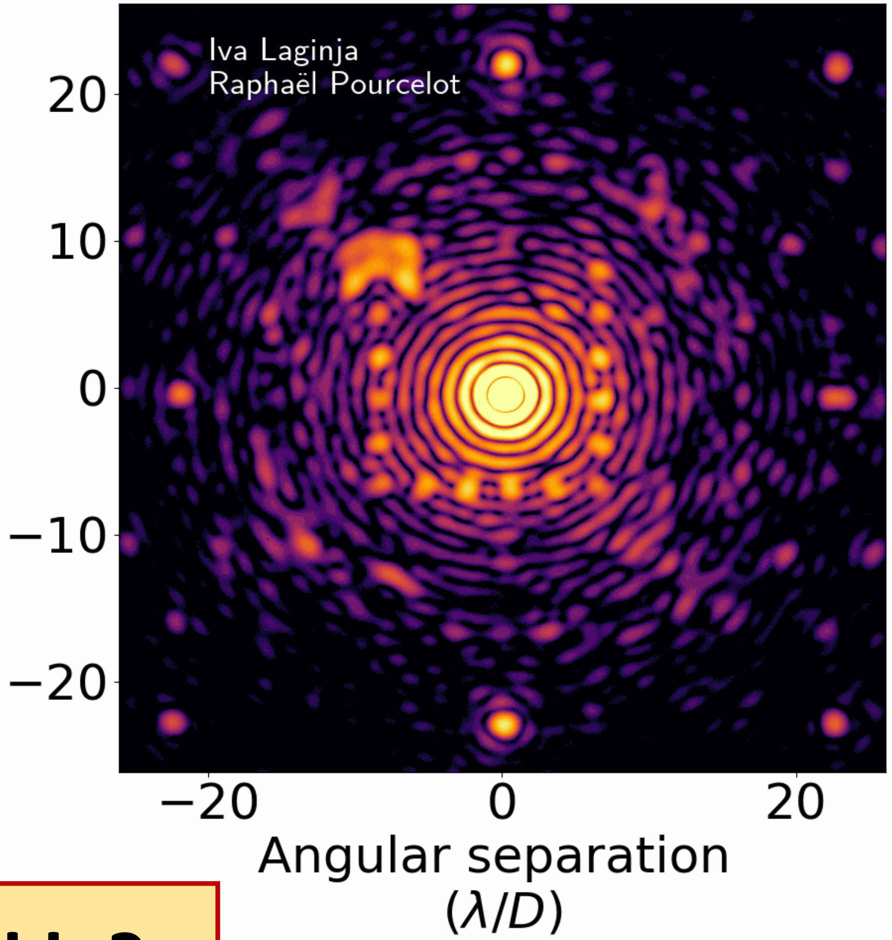
Is it a planet or is it a speckle?

Any aberration can be expressed as linear combination of sine waves

Laginja & Pourcelot 2023

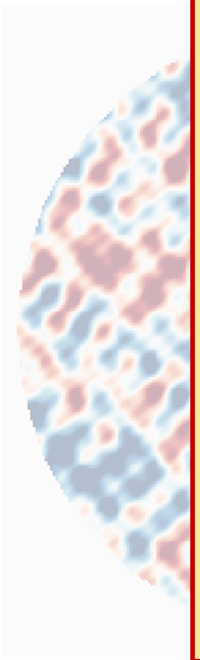






Is it a planet or is it a speckle?

Any aberration can be expressed as linear combination of sine waves



It's not as easy as letting Pacman clean up…

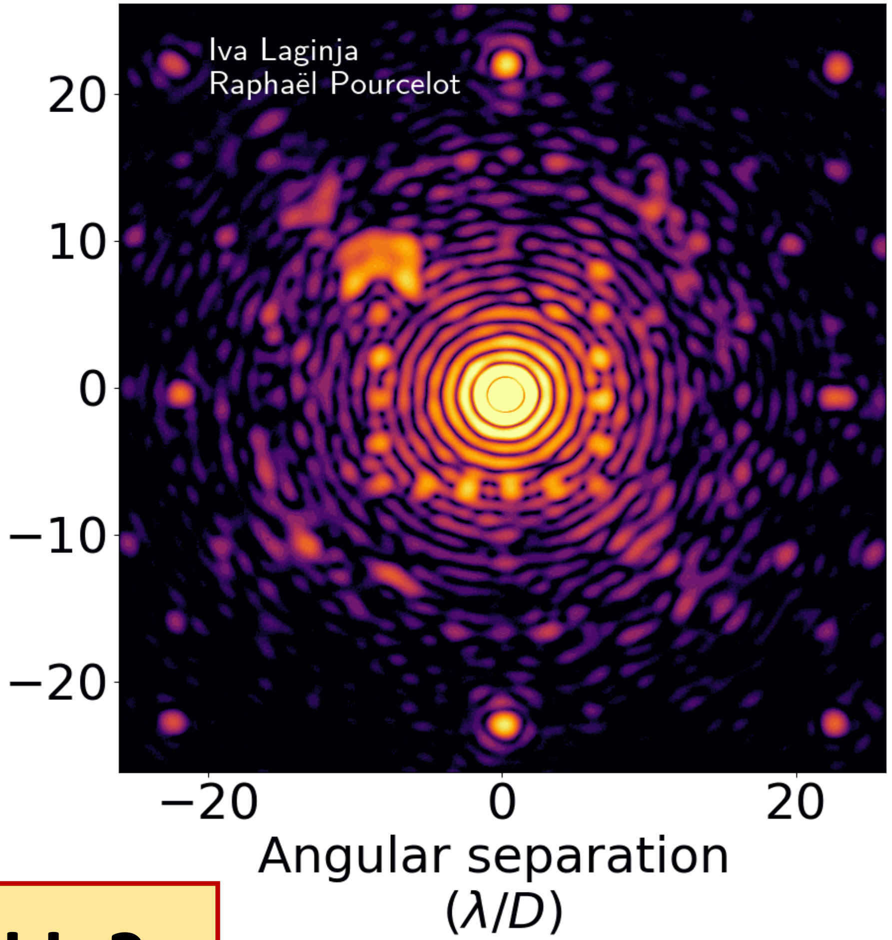
→ need wavefront sensing and control (WFS&C)

(see talk by Becky Jensen-Clem)

→ need post processing

(see talk by Faustine Cantalloube)

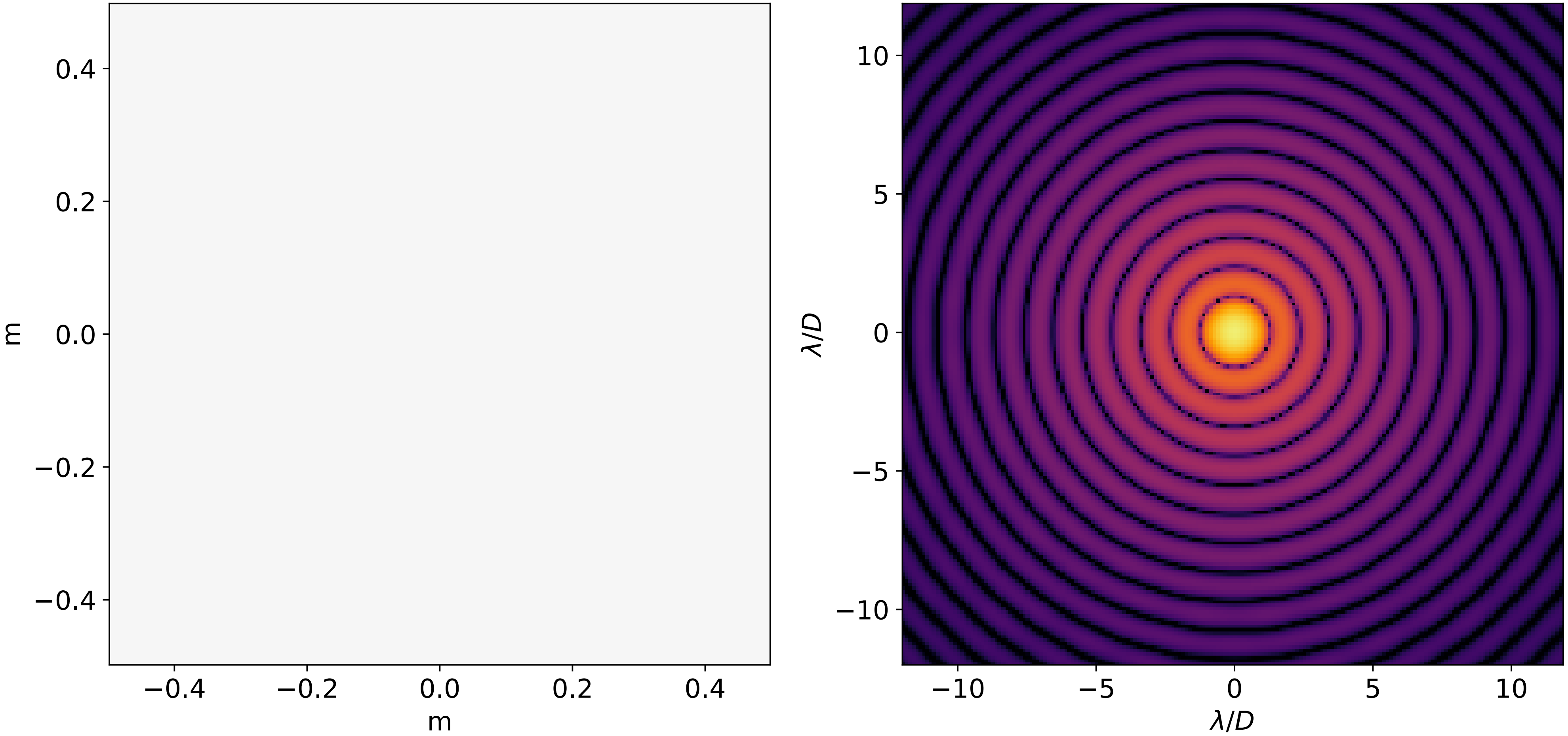
Laginja & Pourcelot 2023



Is it a planet or is it a speckle?

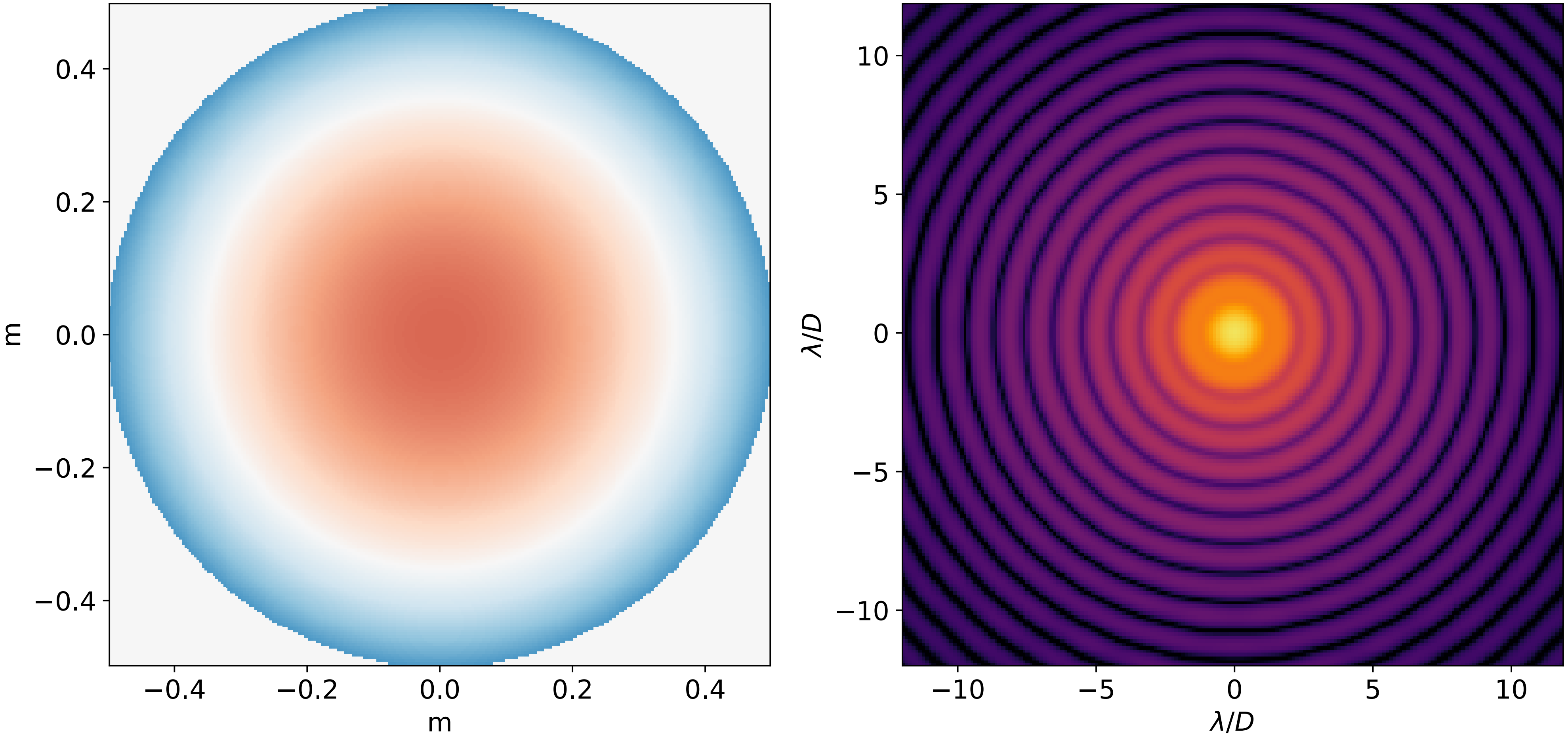
Low spatial frequency aberrations

No aberration



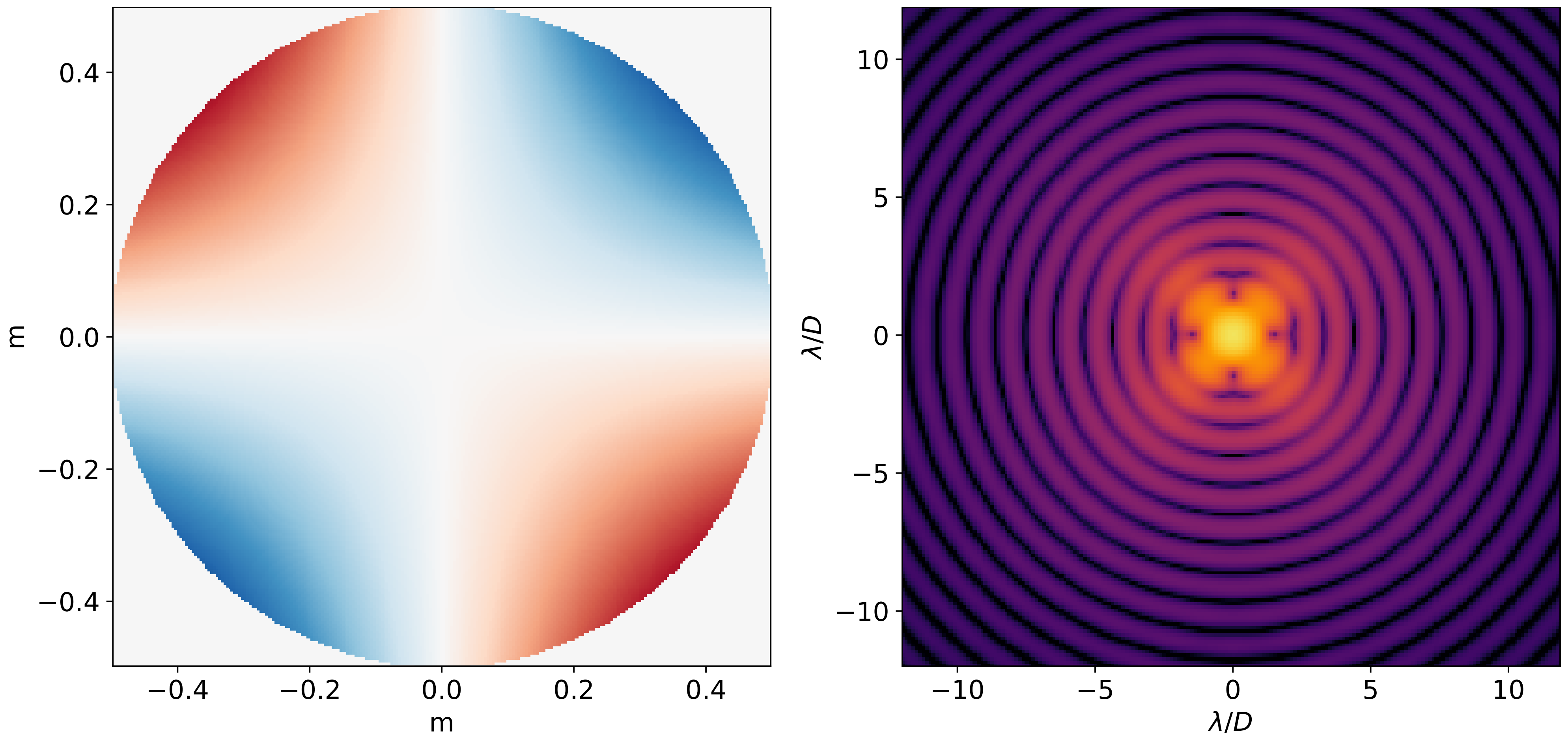
Low spatial frequency aberrations

Defocus



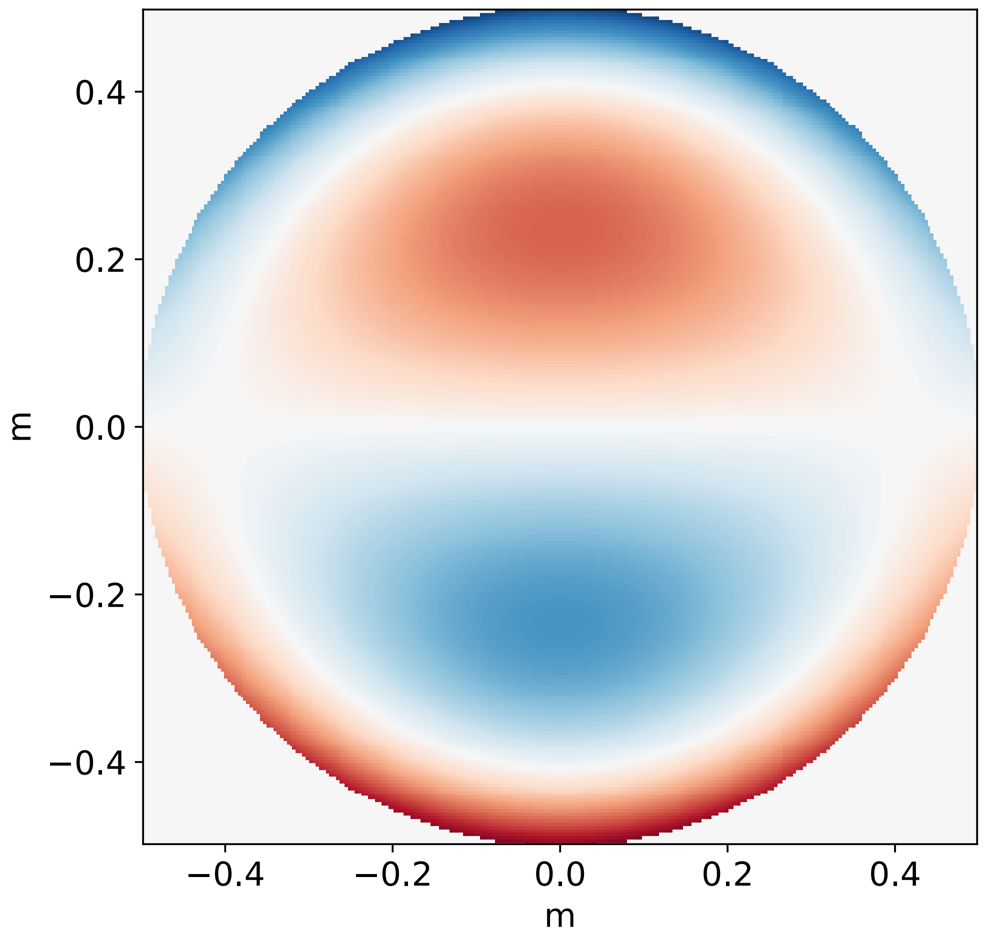
Low spatial frequency aberrations

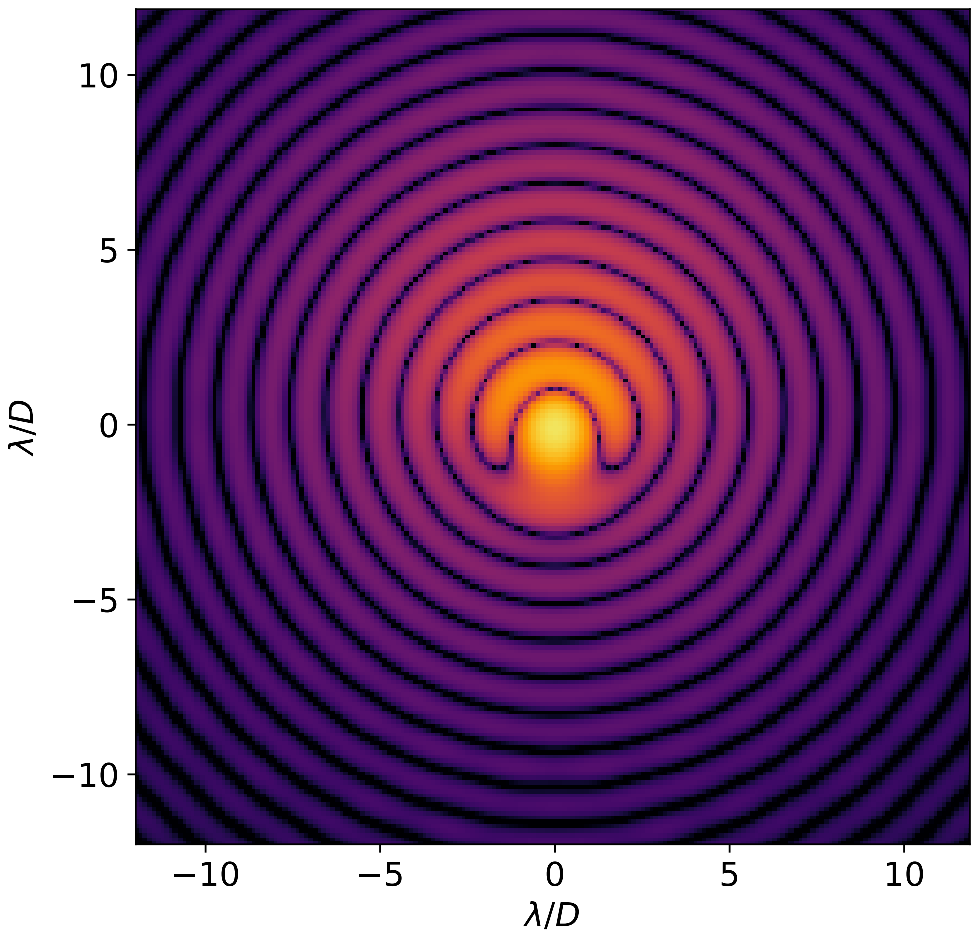
Astigmatism



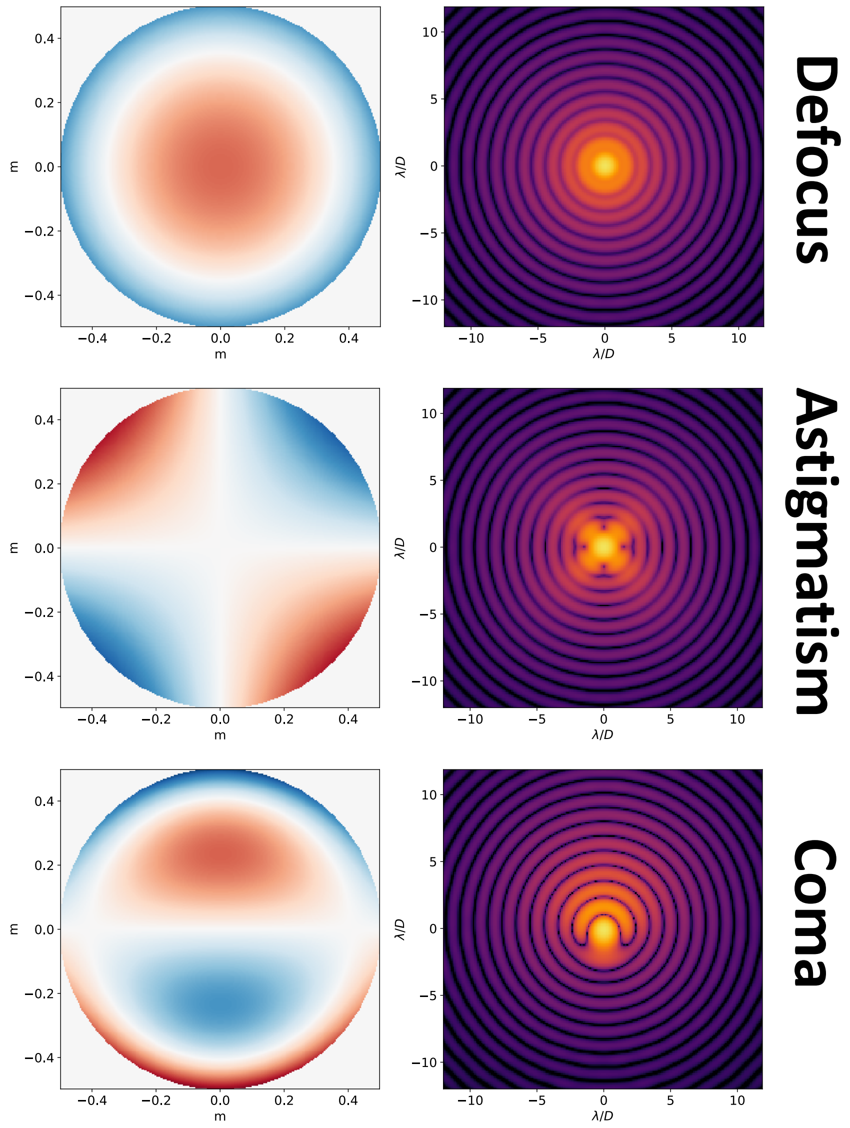
Low spatial frequency aberrations

Coma



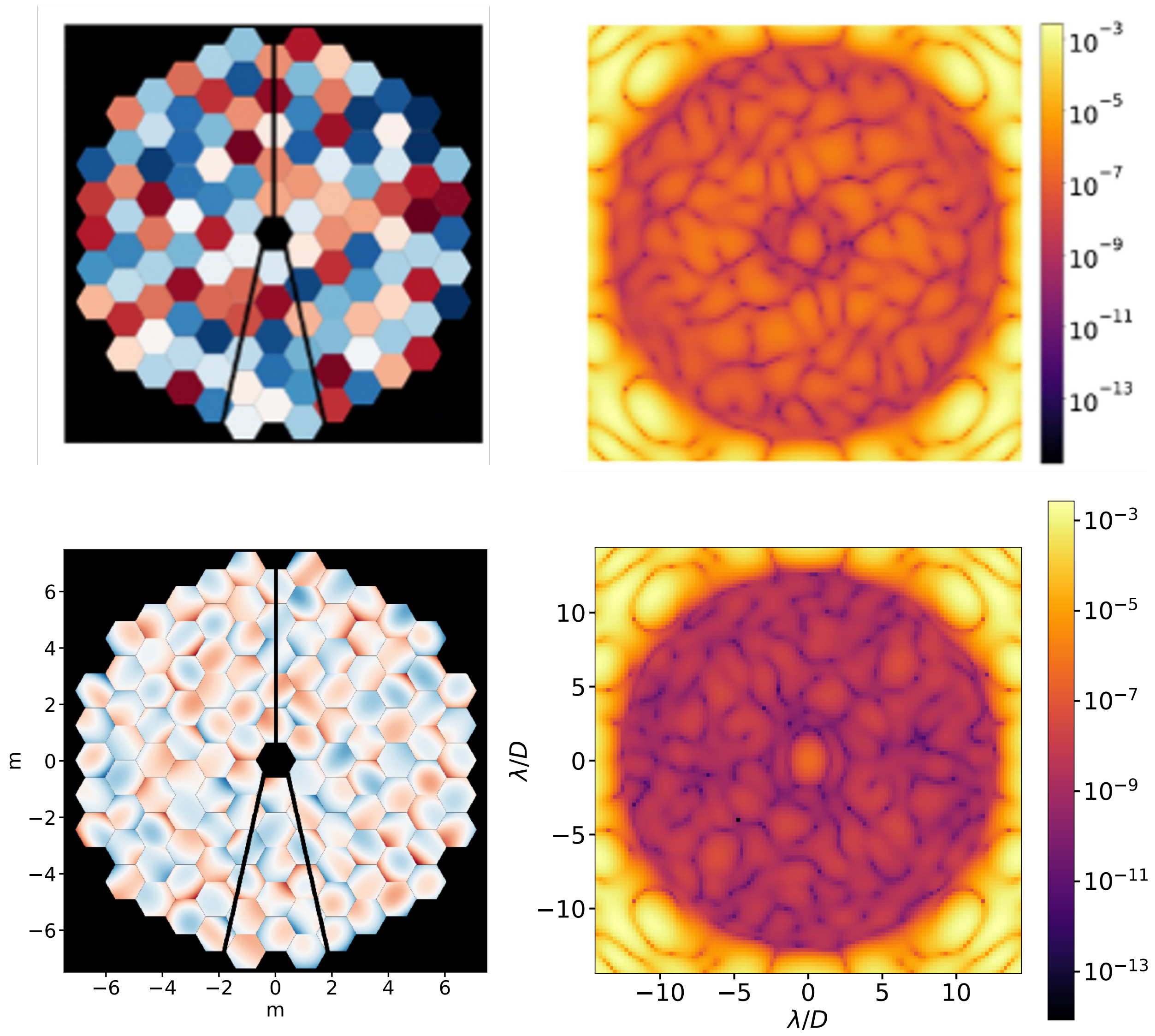


Low spatial frequency aberrations



- Aberration **sources**:
  - Thermal settling of telescope
  - Misalignment of optics
  - Fast tip-tilt jitter
- Results in focal-plane contamination **close to optical axis → close to star**
- Low-order aberrations contaminate the prime area of interest for detection of close-in exoplanets
  → motivation for **low-order WFS&C**

Mid spatial frequency aberrations

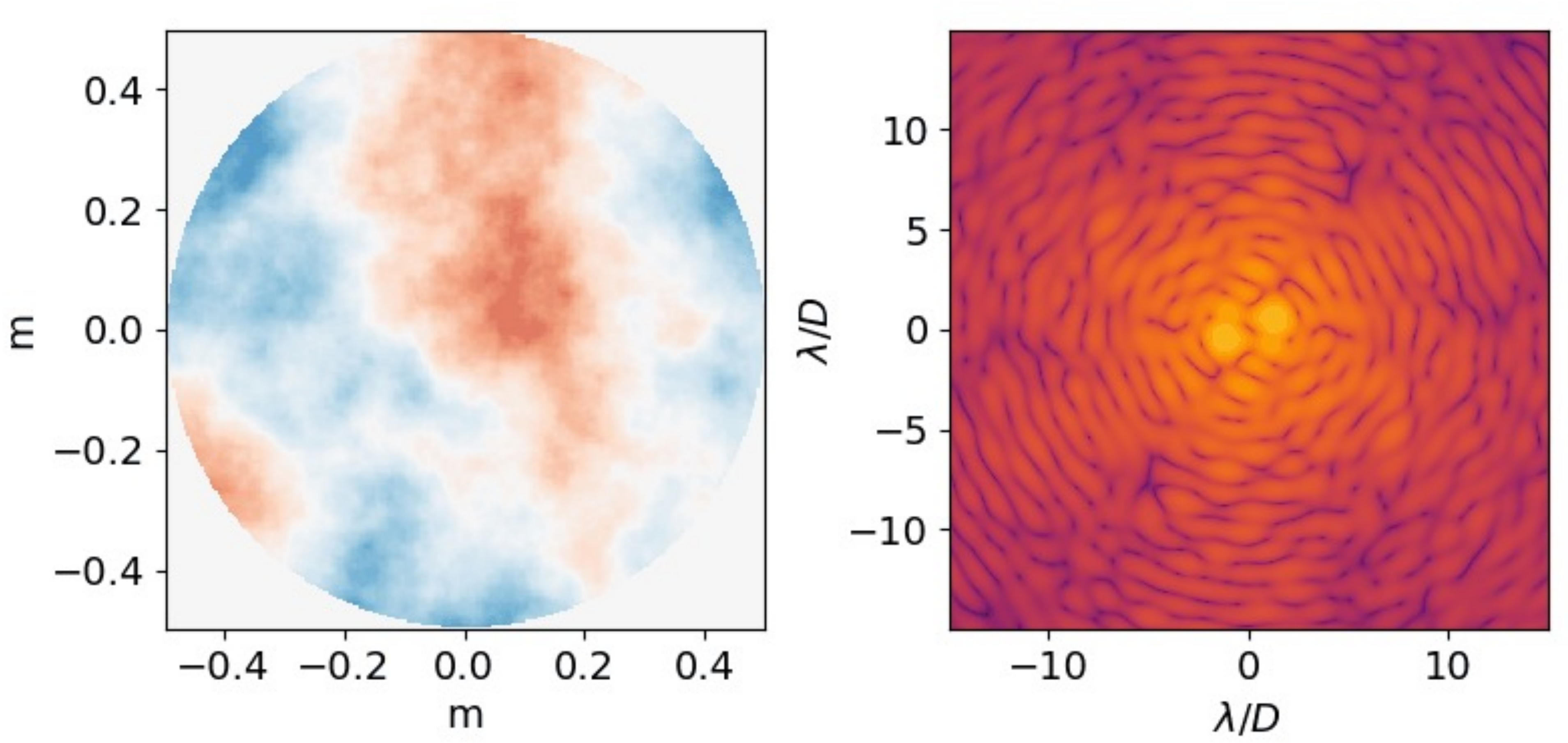


Aberration sources:

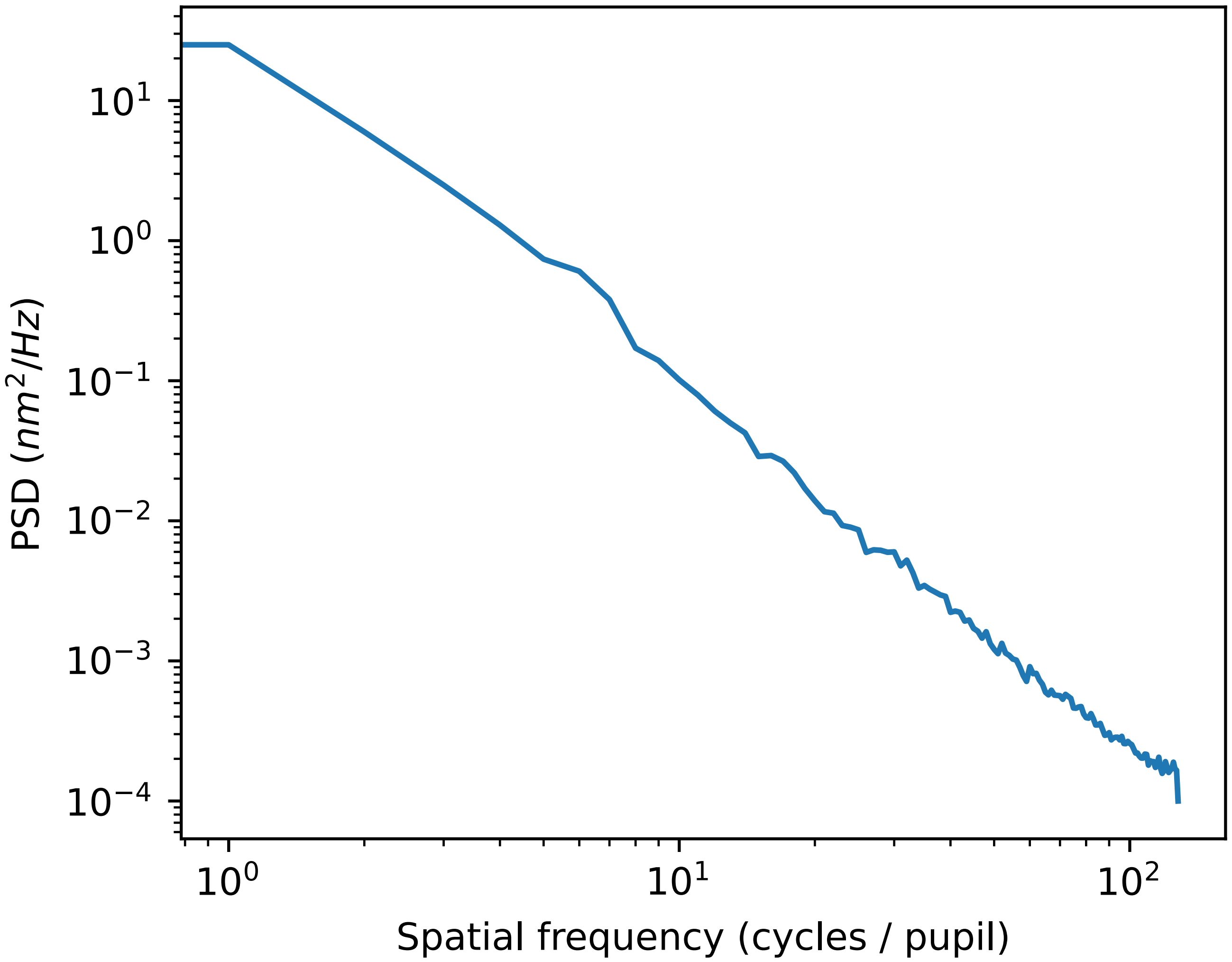
- Bad cophasing of segments
- Misalignment of segments
- Radius of curvature error on segments

* Telescope shown here contains coronagraph.

Phase screens have energy in a range of spatial frequencies



Phase screens have energy in a range of spatial frequencies



Summary

- We need to use **wave optics** to describe **diffraction** in a telescope
- We can model light as a **scalar field** and describe its propagation between **pupil and focal planes** with **Fourier transforms**
- The **telescope pupil** defines the ideal diffraction pattern at the **diffraction limit**
- Faint planets “drown” in the **wings of the PSF**, especially **at small angular separations**
- A planet at a certain **angular separation** manifests as a **shifted PSF**
- **Aberrations** contaminate the focal-plane images and make planets even harder to detect