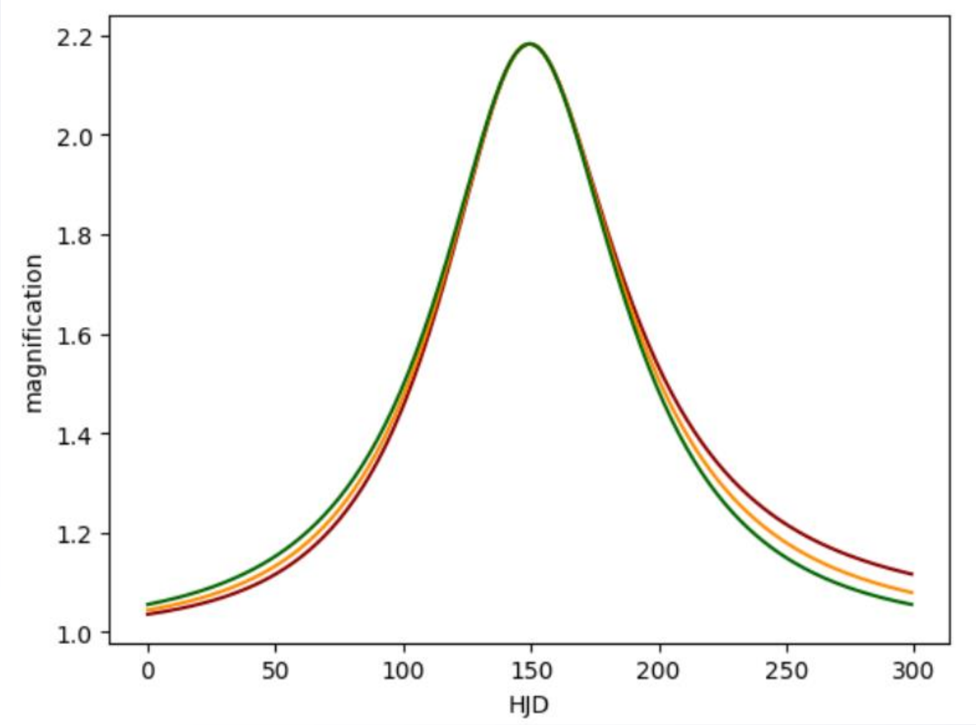
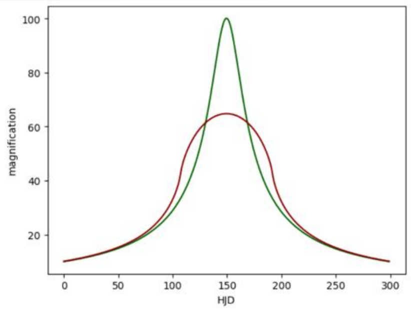
Higher order effects in microlensing



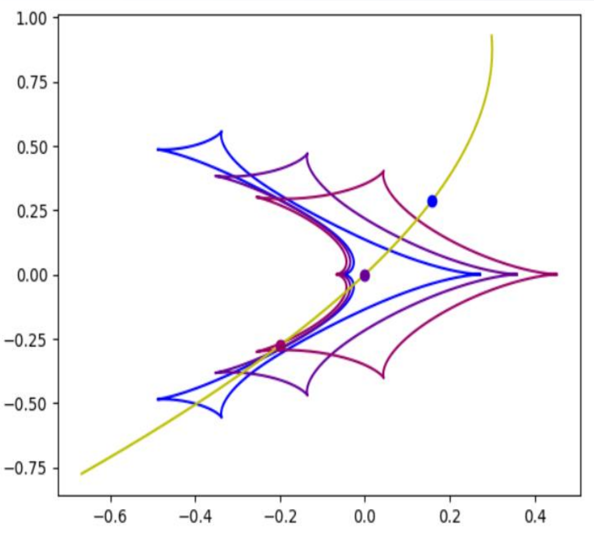
Valerio Bozza
University of Salerno, Italy

Higher order effects in microlensing

- Additional physical effects → Standard rulers → Break degeneracies!



- Sources are not point-like.
- The **finite-size** of the source matters



- The relative lens-source proper motion is never straight and uniform.
- The «static» approximation is valid only for short enough events.
- Observer, lens and source may all move in a non-uniform way.

- The Earth orbits the Sun → **parallax**
- A binary or planetary lens orbits around the c.o.m. → **lens orbital motion**
- The source may orbit a companion → source orbital motion (**xallarap**)

Finite-size of the source

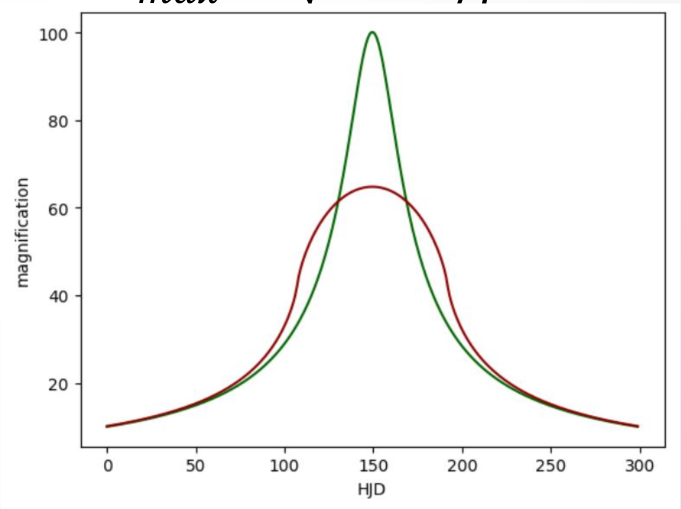
- Paczynski formula $A = \frac{u^2+2}{u\sqrt{u^2+4}}$ assumes a point-source.
- Stars have a finite **angular size**

$$\theta_* = 0.58\mu as \left(\frac{R_*}{R_\odot}\right)\left(\frac{D_S}{8kpc}\right)^{-1}$$

- This is typically negligible compared to the Einstein angle $\theta_E \sim 0.3mas$

- Finite size matters for giant sources and very small impact parameters u_0
- The control parameter is $\rho_* \equiv \theta_*/\theta_E$.
- Introduces a cut-off in magnification

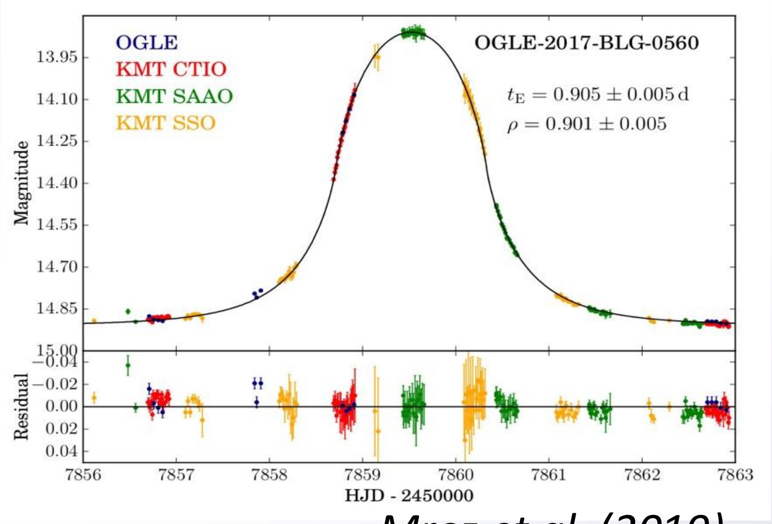
$$A_{max} = \sqrt{1 + 4/\rho_*^2}$$



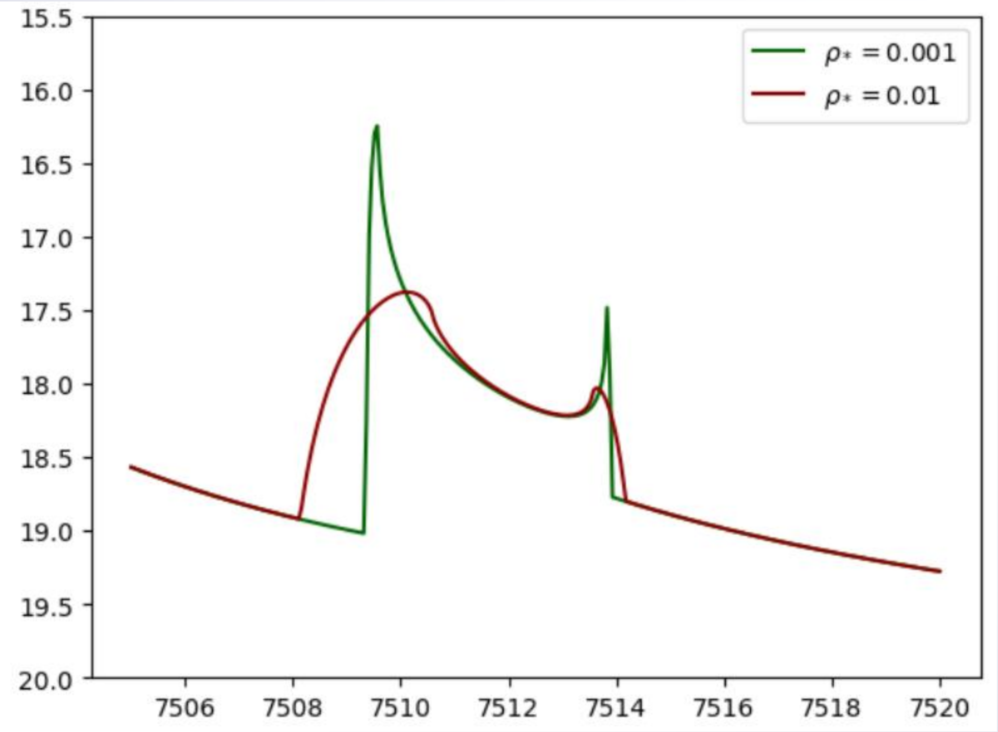
Finite-size of the source

- Rarely measured in single-lens events (high-magnification events)
- Very common for Free-Floating Planets, since $\theta_E \approx \mu as$.

- In binary lensing, every caustic crossing is smoothed by the finite-size of the source



Mroz et al. (2019)



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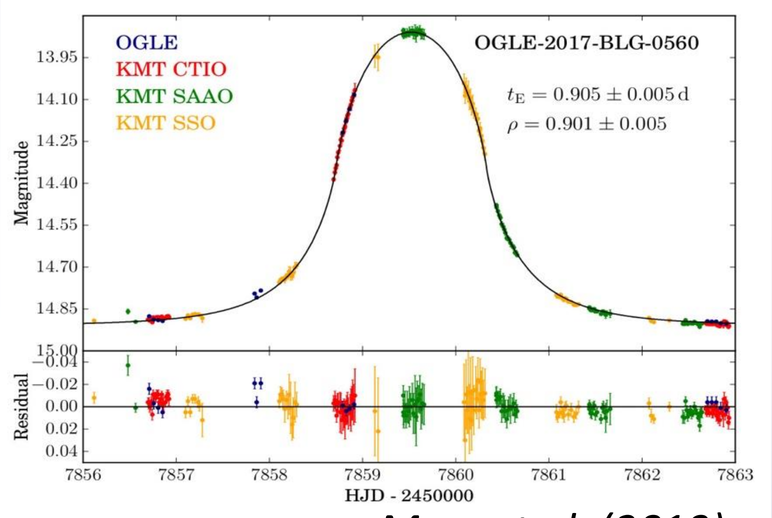

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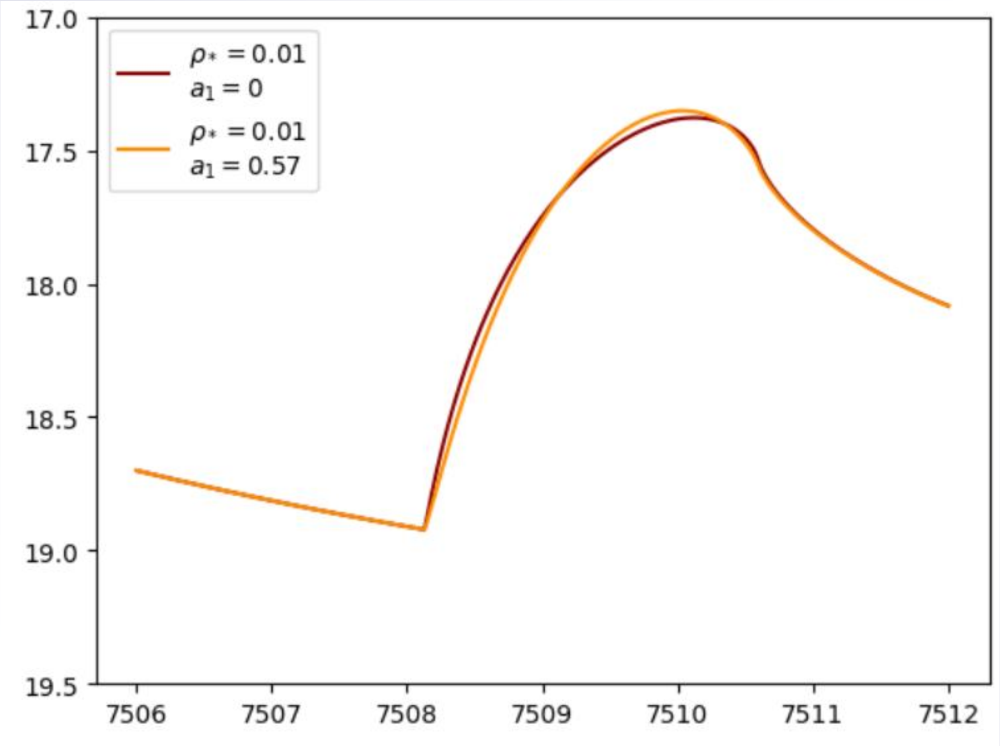


- Microlensing is also sensitive the source **limb darkening** profile

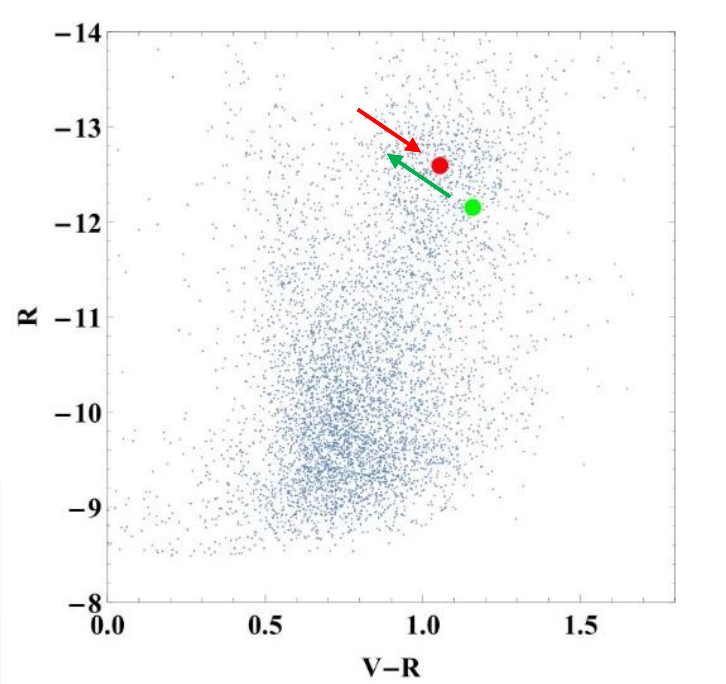
$$I(r) = I(0)\left[1 - a_1\sqrt{1 - \frac{r^2}{\rho_*^2}}\right]$$

Constraints from finite-size

- If we know the true angular size of the source θ_*,
- We can measure the **Einstein angle** $\theta_E = \theta_*/\rho_*$
- A very good characterization of the source is necessary
- Also useful to correctly address limb darkening.

- Find the red clump in your CMD
- Compare with the dereddened position from Bensby et al. (2011) and Nataf et al. (2013)
- Find the extinction vector
- Apply the correction to the source to find its dereddened magnitude and color

- Use isochrones or empirical relations to get θ_* and limb darkening (e.g. Boyajian et al. 2017 …)



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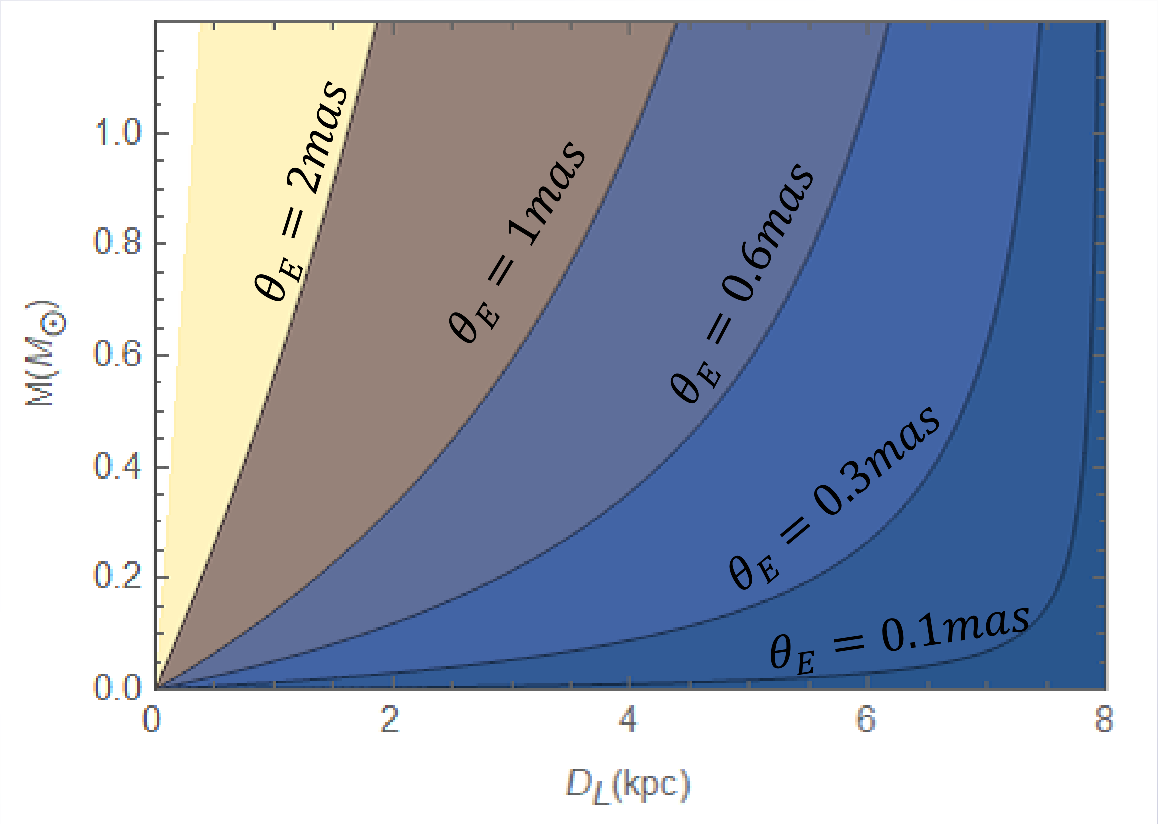
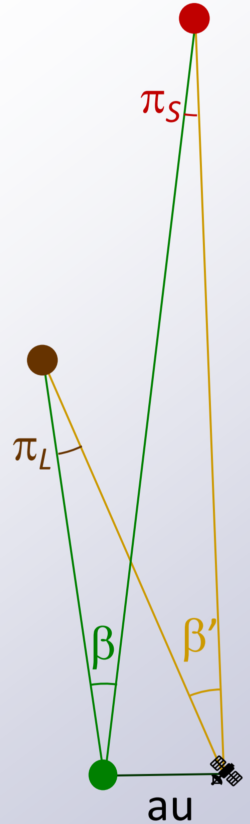


Microlensing parallax

- Microlensing depends on the angular distance β between the source and the lens relative to the Einstein angle θ_E.
- Seen from a different observer spaced by 1 au, the angular distance changes as $\beta' = \beta + (\underbrace{\pi_L - \pi_S}_{\pi_{rel}})$
- $\pi_{rel} \equiv \pi_L - \pi_S = \frac{\text{au}}{D_L} - \frac{\text{au}}{D_S}$ is the relative parallax.
- The change in the microlensing magnification will depend on $\frac{\Delta\beta}{\theta_E} = \frac{\pi_{rel}}{\theta_E}$.
- Therefore, we define the microlensing parallax parameter as

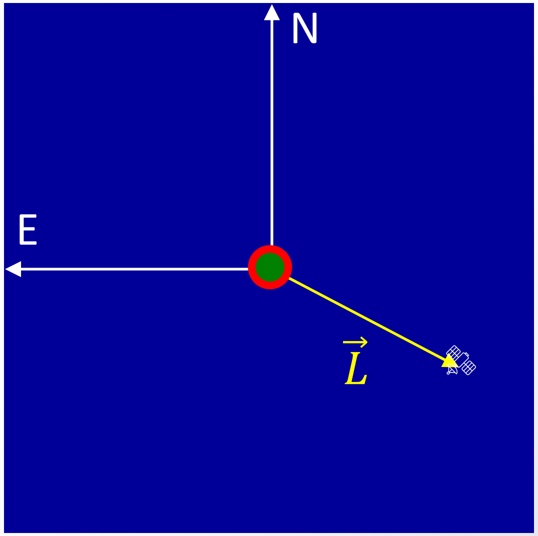
$$\pi_E \equiv \frac{\pi_{rel}}{\theta_E}$$

- In 3-dimensions we also need to specify the orientation of the displacement.



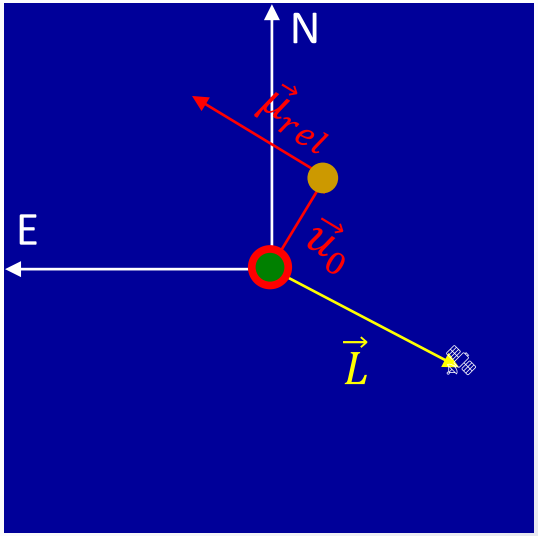
Microlensing parallax

- In the North-East celestial frame, we may project the baseline $\vec{L}$ between the two observers.

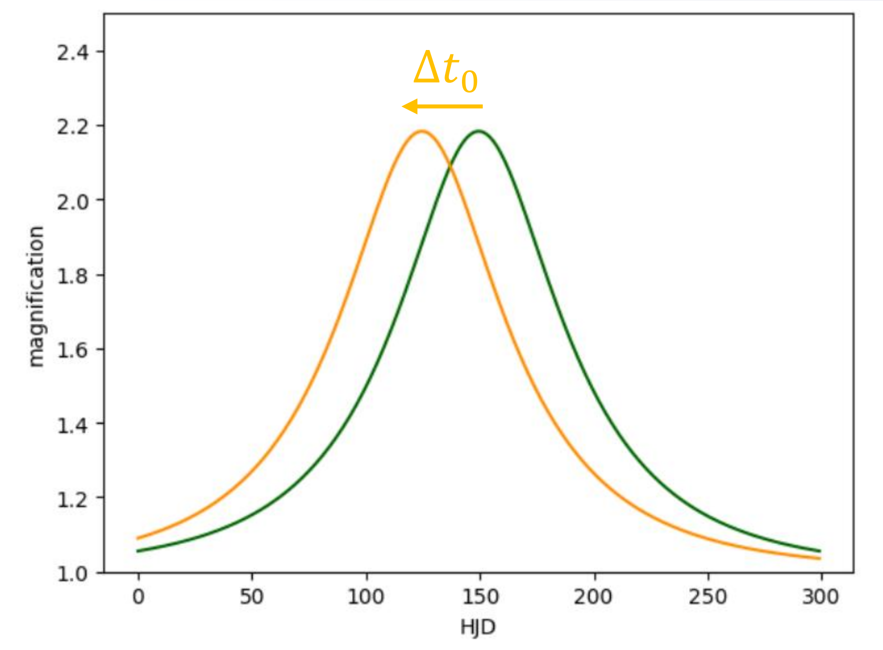


Microlensing parallax

- In the North-East celestial frame, we may project the baseline $\vec{L}$ between the two observers.
- Depending on the direction of the relative lens-source proper motion, the second observer will see a different light curve.



$$\Delta t_0 = -\frac{L}{\mathrm{au}}\pi_E t_E$$



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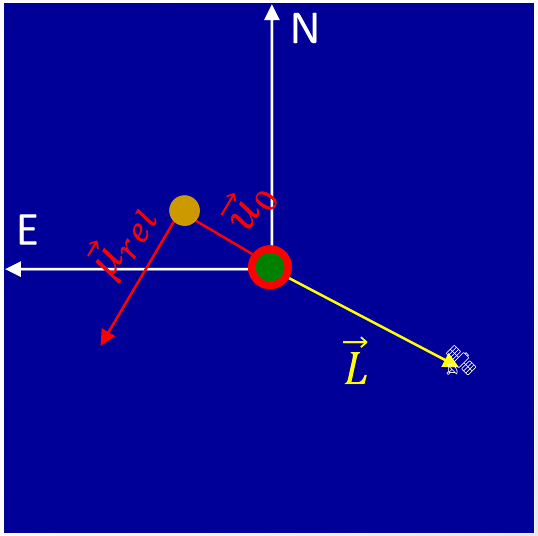
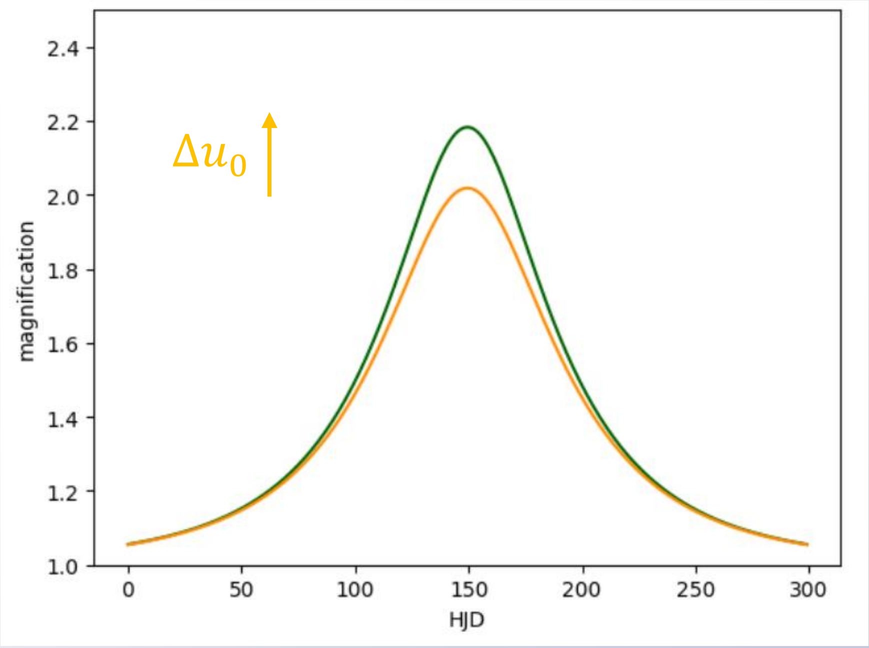


$$\Delta u_0 = +\frac{L}{\mathrm{au}}\pi_E$$



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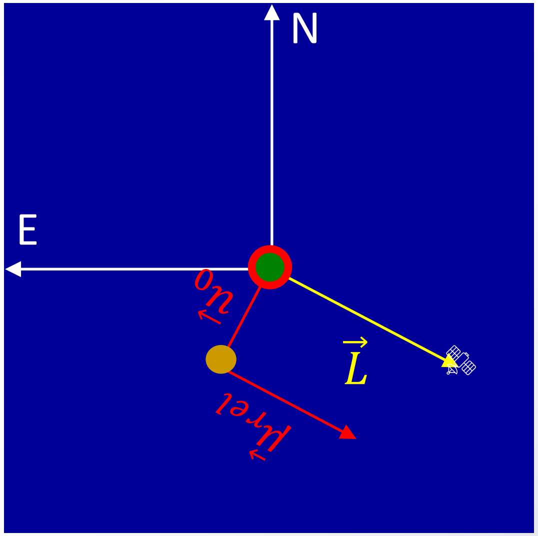

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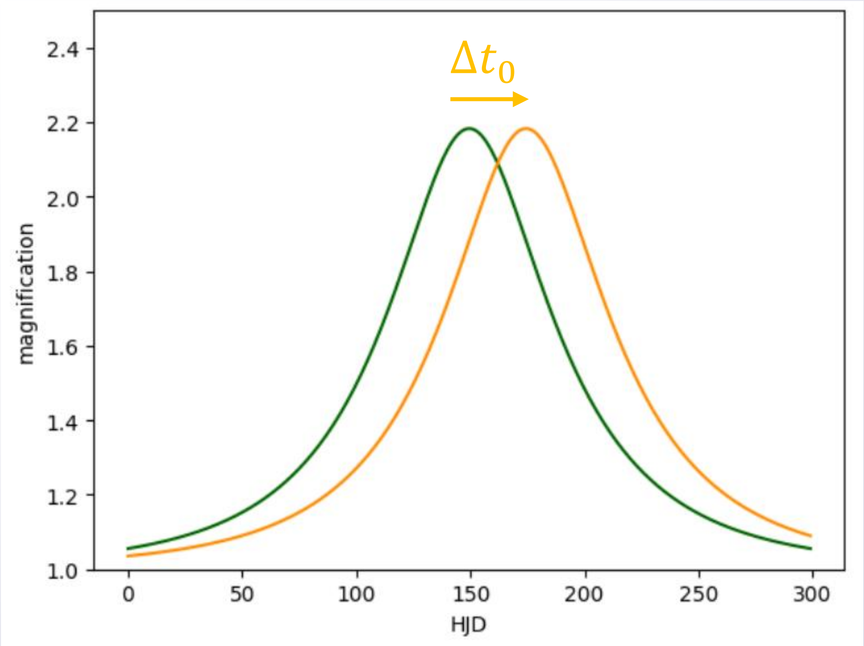

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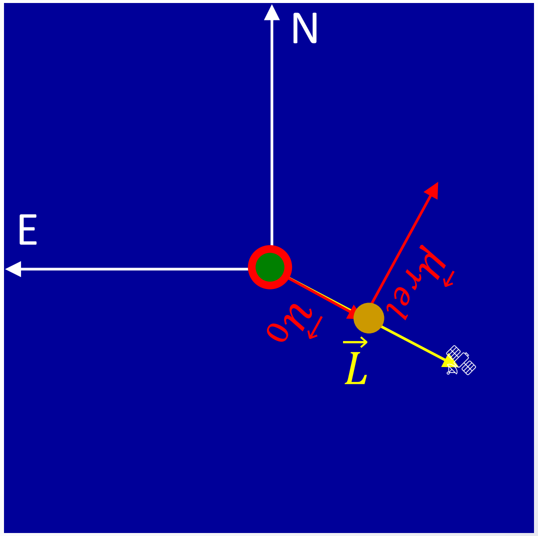
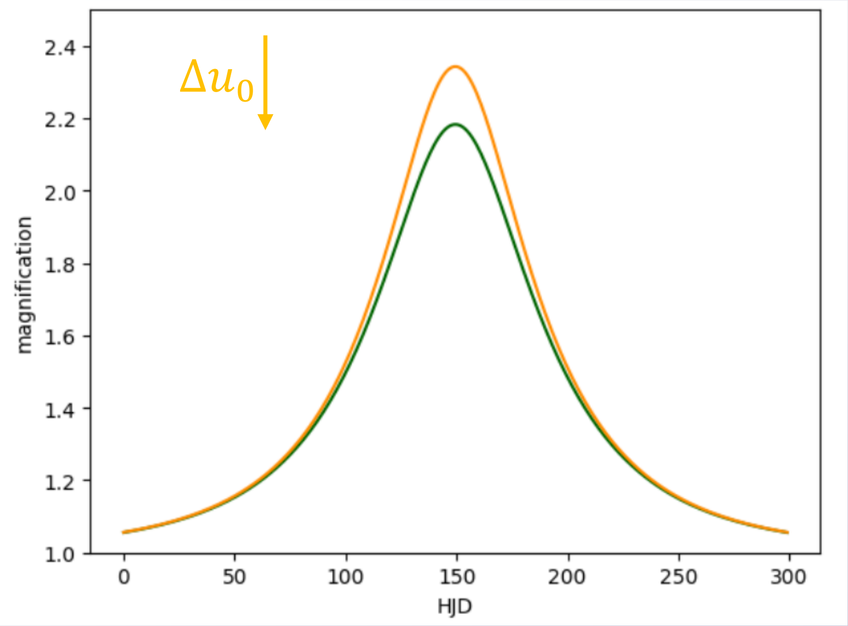
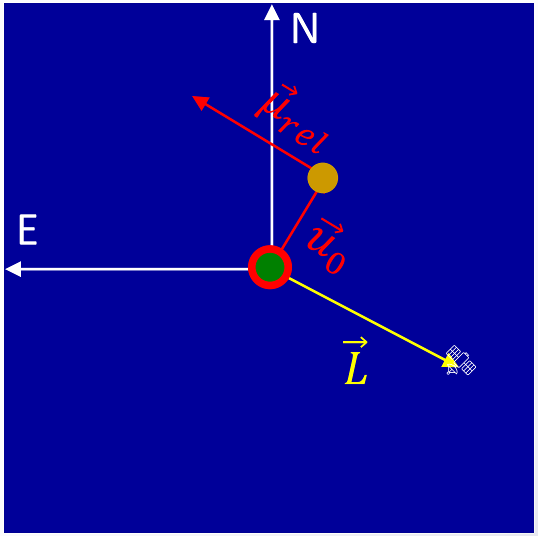




$$\Delta u_0 = -\frac{L}{\mathrm{au}}\pi_E$$

Microlensing parallax

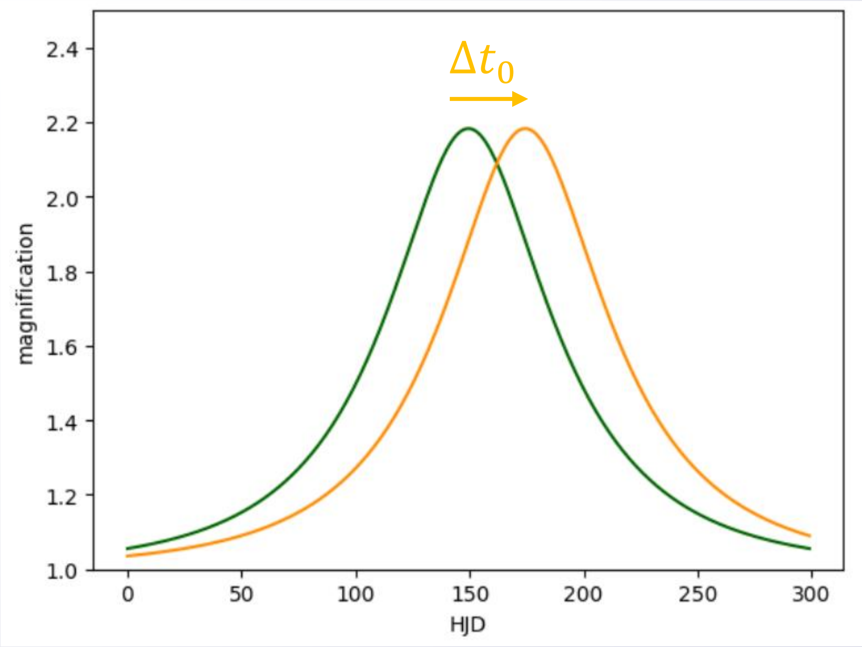
- In the North-East celestial frame, we may project the baseline $\vec{L}$ between the two observers.
- Depending on the direction of the relative lens-source proper motion, the second observer will see a different light curve.
- We can combine the direction of the proper motion and the microlensing parallax into a **parallax vector**



$$\vec{\pi}_E \equiv \pi_E \frac{\vec{\mu}_{rel}}{|\vec{\mu}_{rel}|}$$

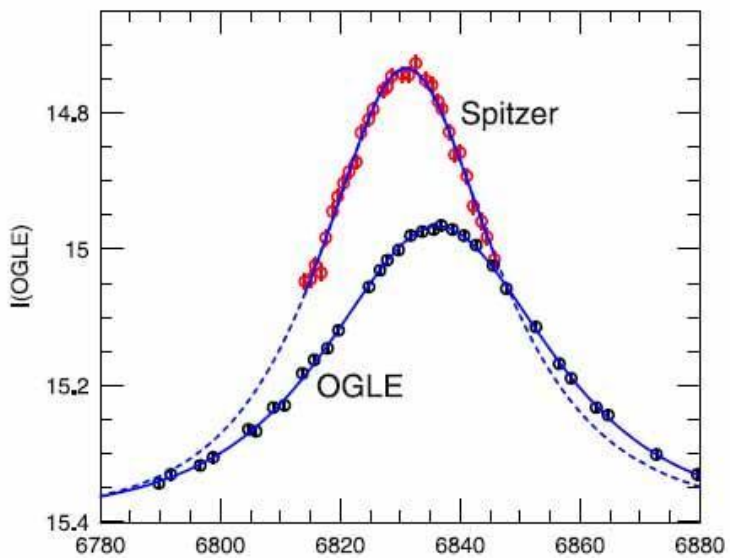
- The two components of the parallax vector are denoted by

$$\vec{\pi}_E \equiv (\pi_{E,N}, \pi_{E,E})$$

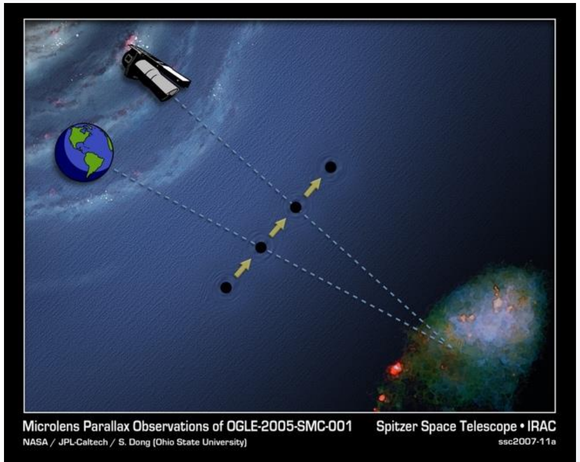


Satellite parallax

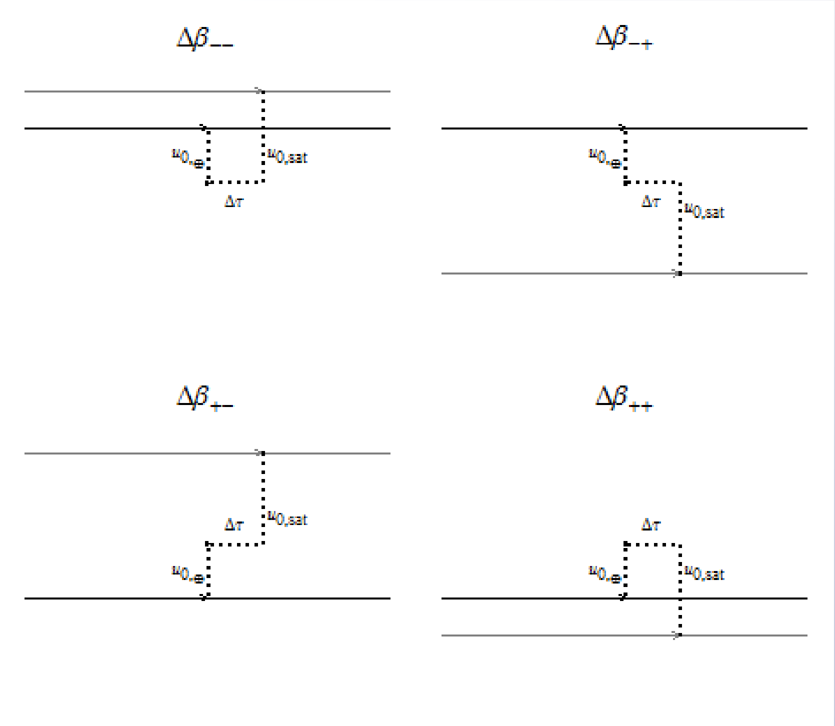
- The satellite microlensing parallax *(Refsdal 1966, Gould 1994)* has been measured in hundred events by Spitzer ($L \approx 1.2$au)



OGLE-2014-BLG-0939 *(Yee et al. 2015)*

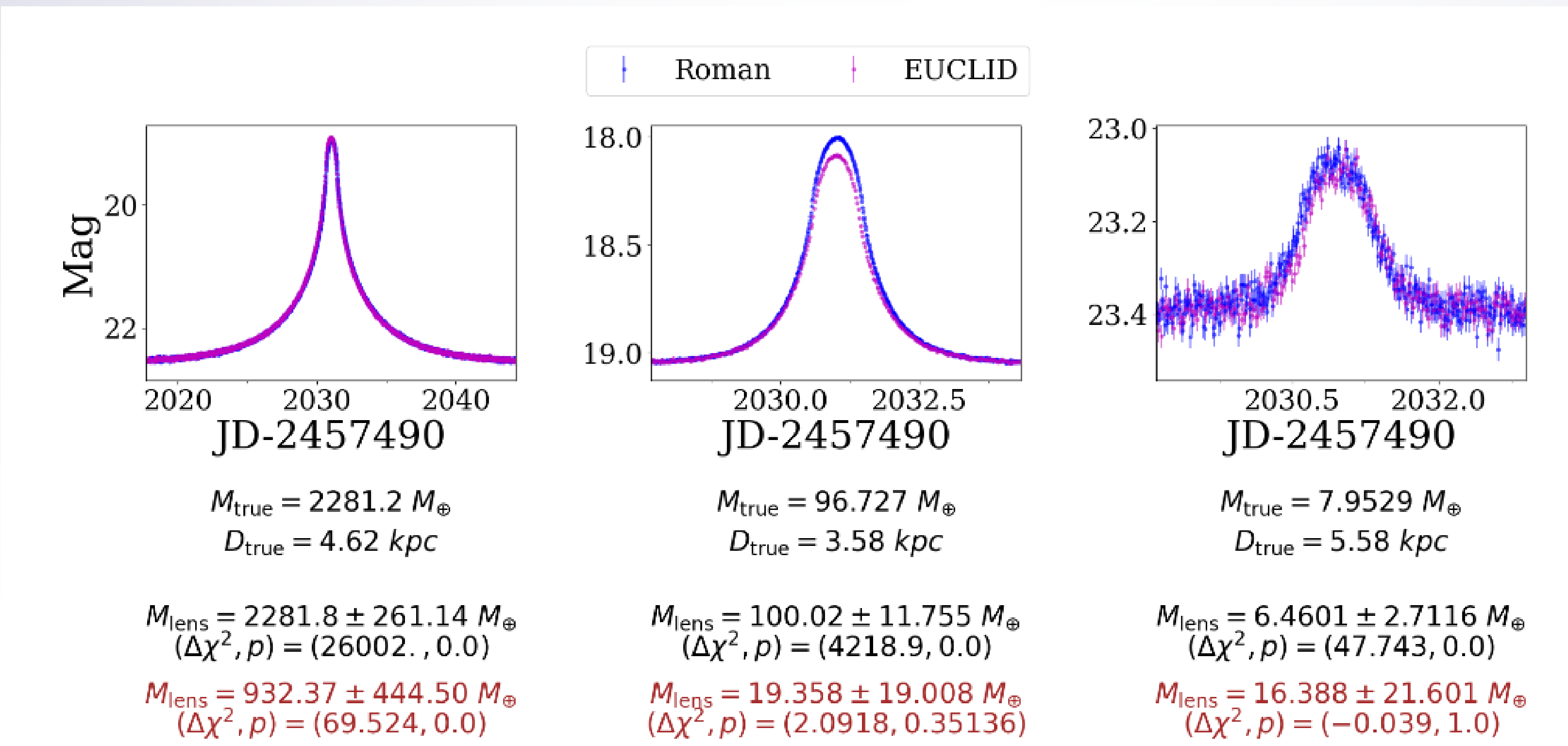


- Unfortunately, since we do not know the sign of u_0 for either observer, there is a four-fold degeneracy *(Gould 1994,1995)*



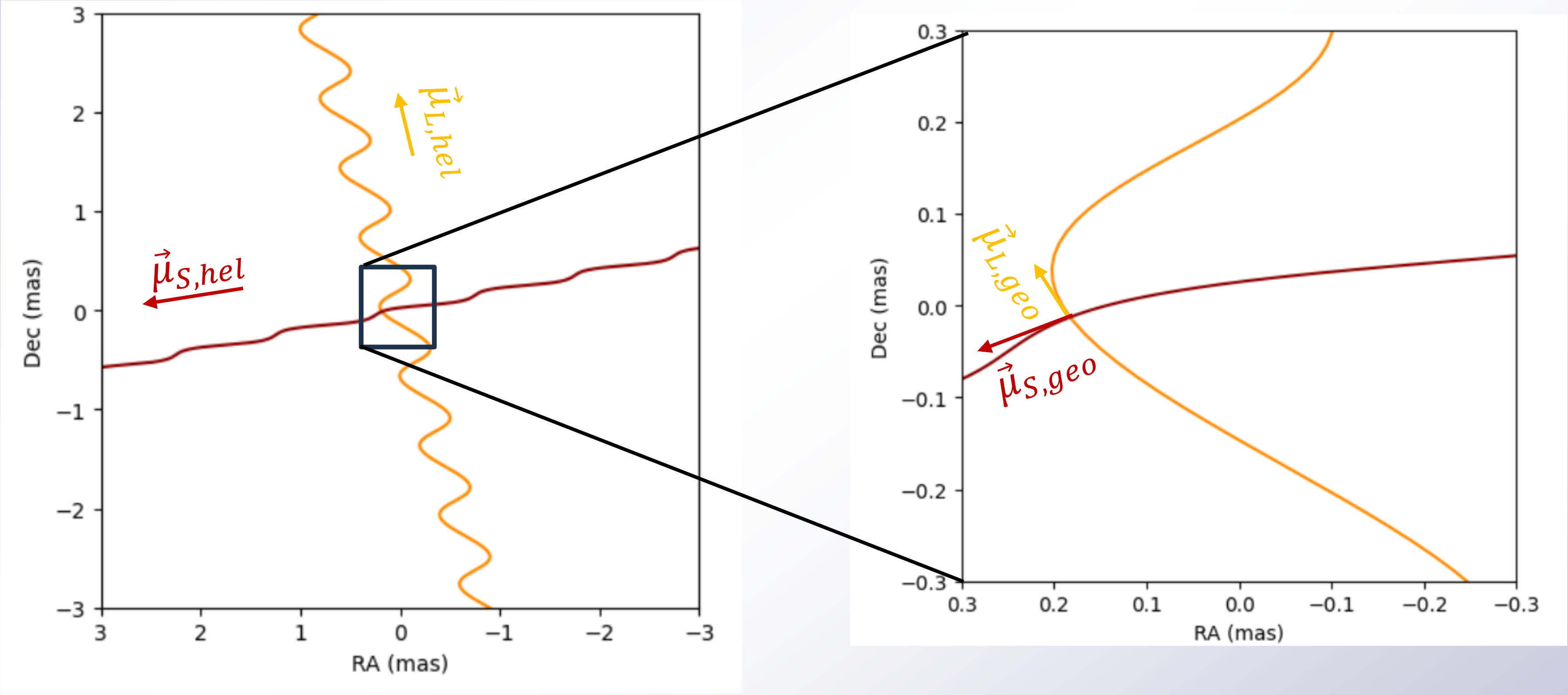
Roman Free-Floating planets

- Free-floating planets generate very short microlensing events $t_E < 1\mathrm{d}$
- Simultaneous observations from *Roman* and the ground or from *Roman* and *Euclid* ($L \approx 0.01\mathrm{au}$) would solve the microlensing degeneracy and lead to accurate mass measurements *(Bachelet et al. 2022)*



Annual parallax

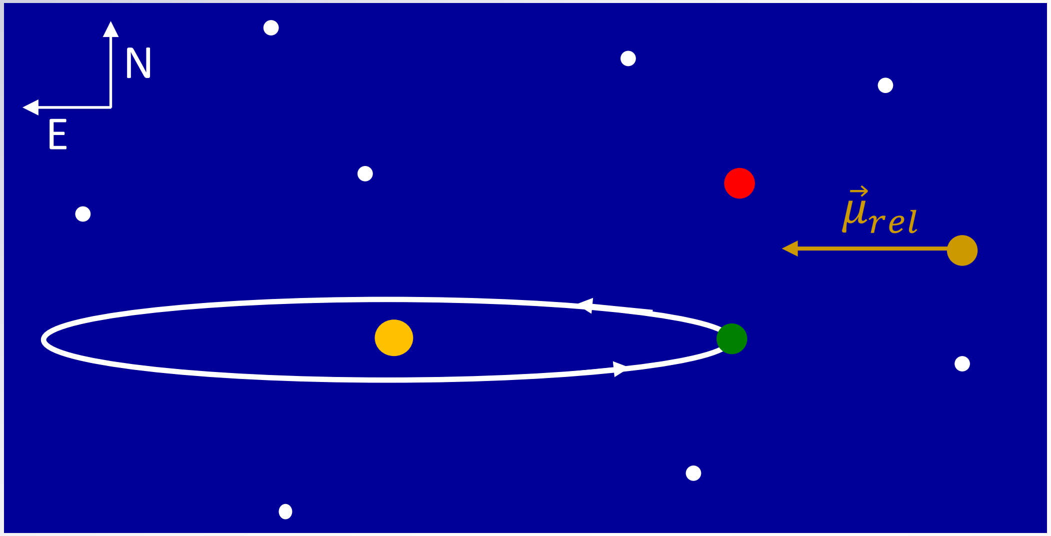
- As the Earth orbits the Sun, both the lens and the source move on a parallactic ellipse.
- The uniform motion approximation breaks down for long enough events! *(Gould 1992)*



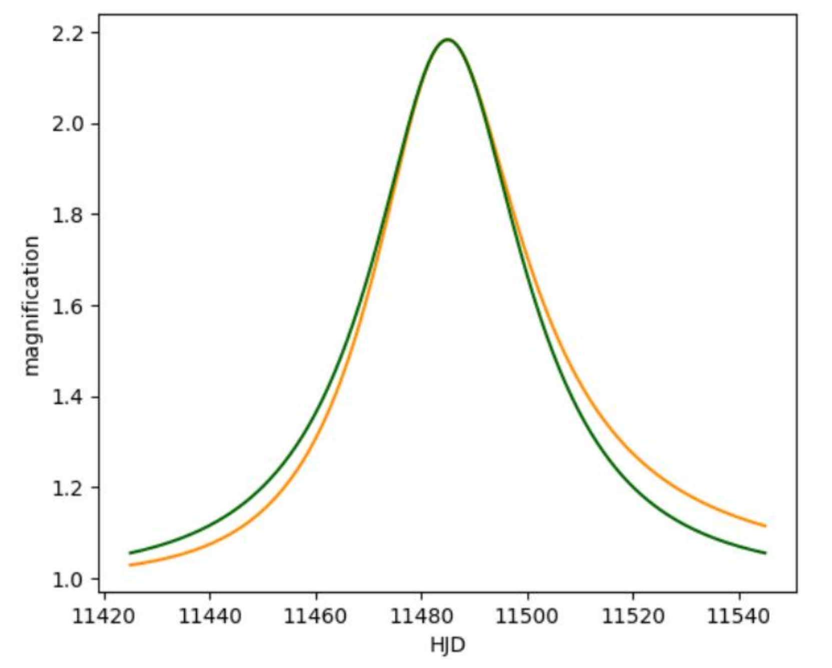
- Note that the instantaneous (geocentric) proper motions are different from the average (heliocentric) proper motions: $\vec{\mu}_{hel} = \vec{\mu}_{geo} + \vec{v}_{\oplus}/D$

Annual parallax

- The Earth motion is known.
- The distortion on the light curve depends on the direction of the proper motion and the time of the year.



- Example: t_0 at the vernal equinox.
- $\pi_{E,N} = 0 \quad \pi_{E,E} > 0$
- The rise is steeper than the descent



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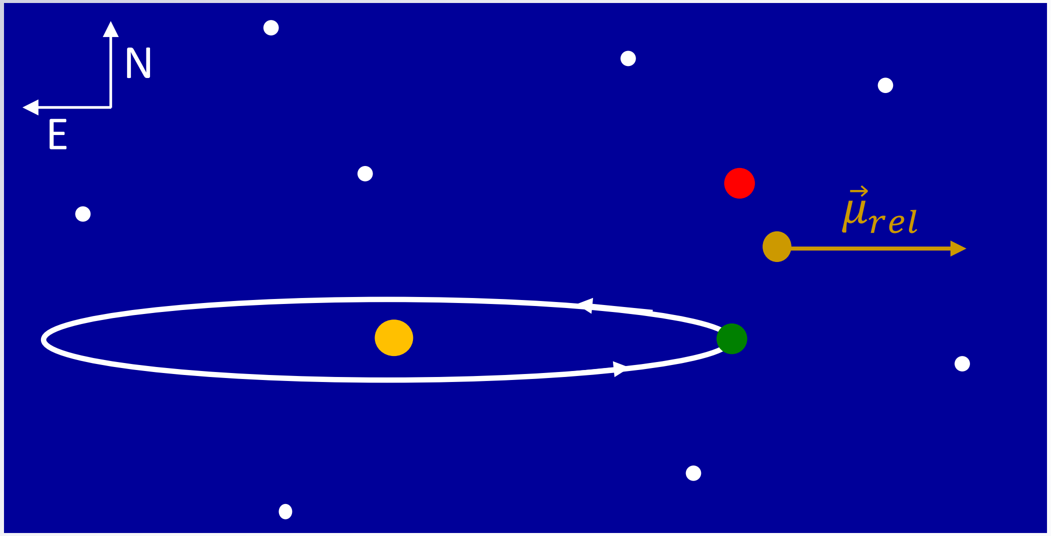
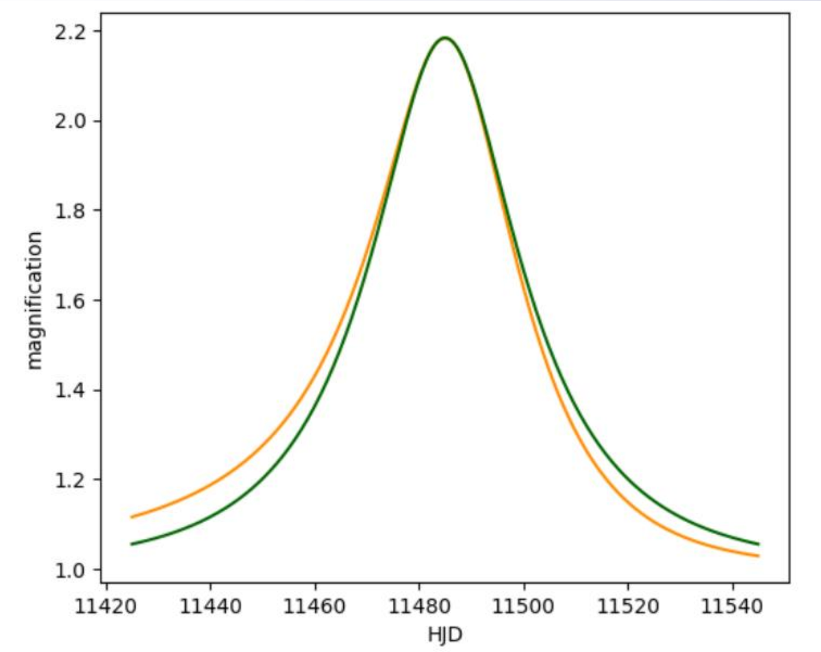


- Example: t_0 at the vernal equinox.
- $\pi_{E,N} = 0 \quad \pi_{E,E} < 0$
- The rise is slower than the descent



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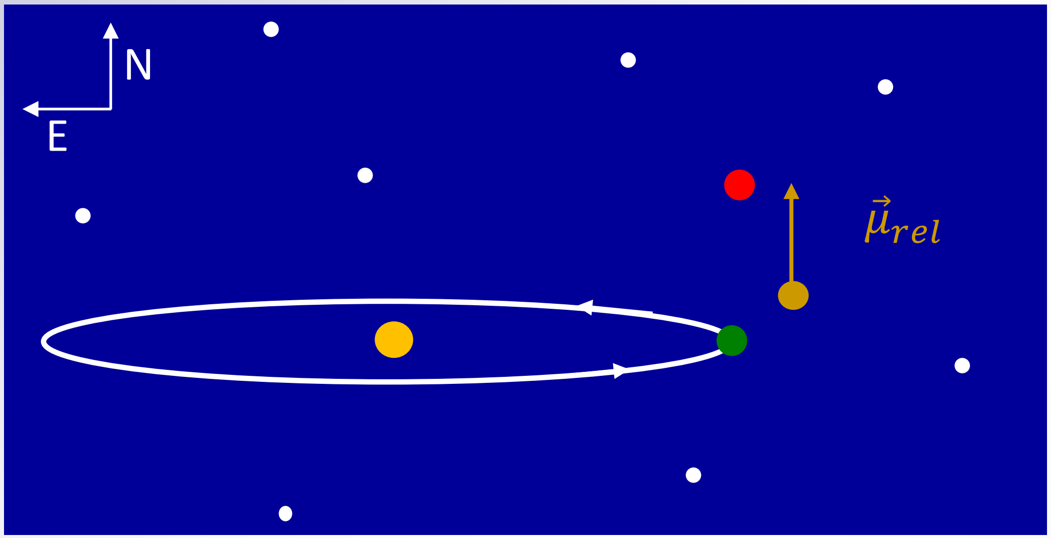
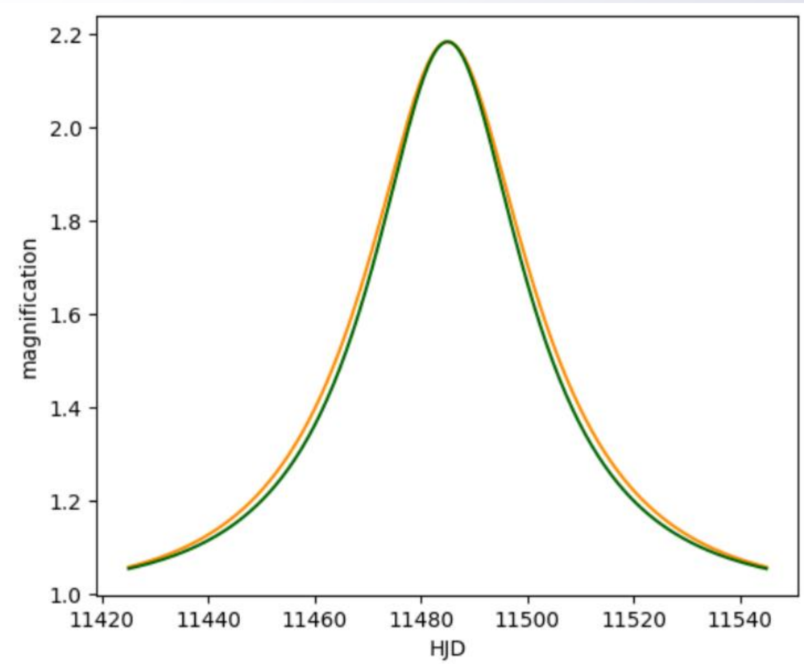


- Example: t_0 at the vernal equinox.
- $\pi_{E,N} > 0 \quad \pi_{E,E} = 0$
- Event slightly longer (for $u_0 > 0$)



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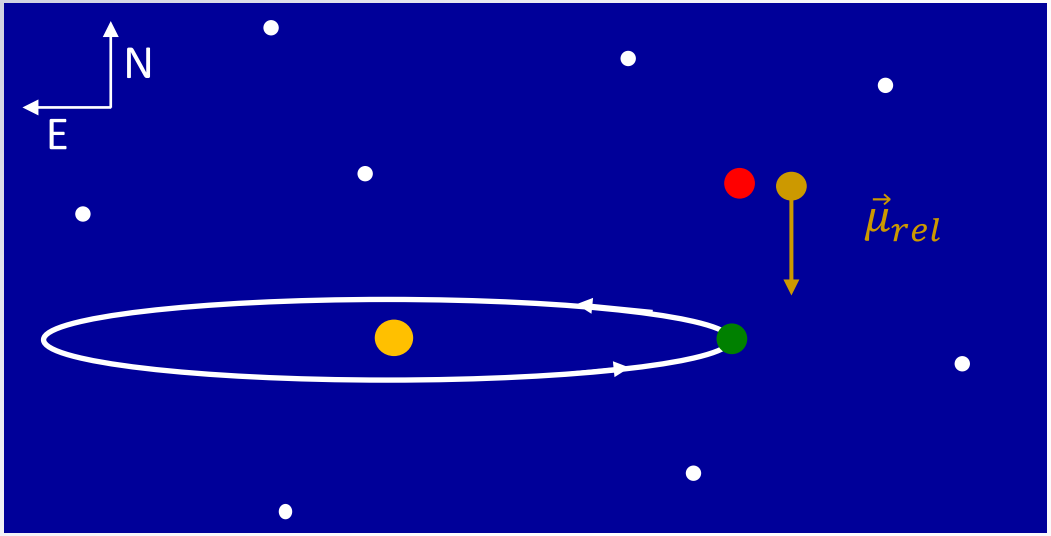
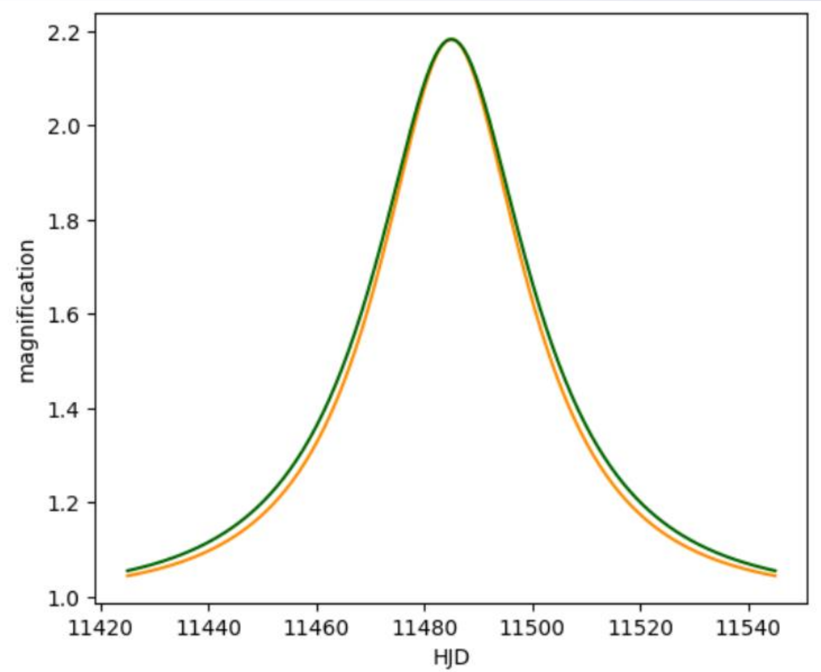
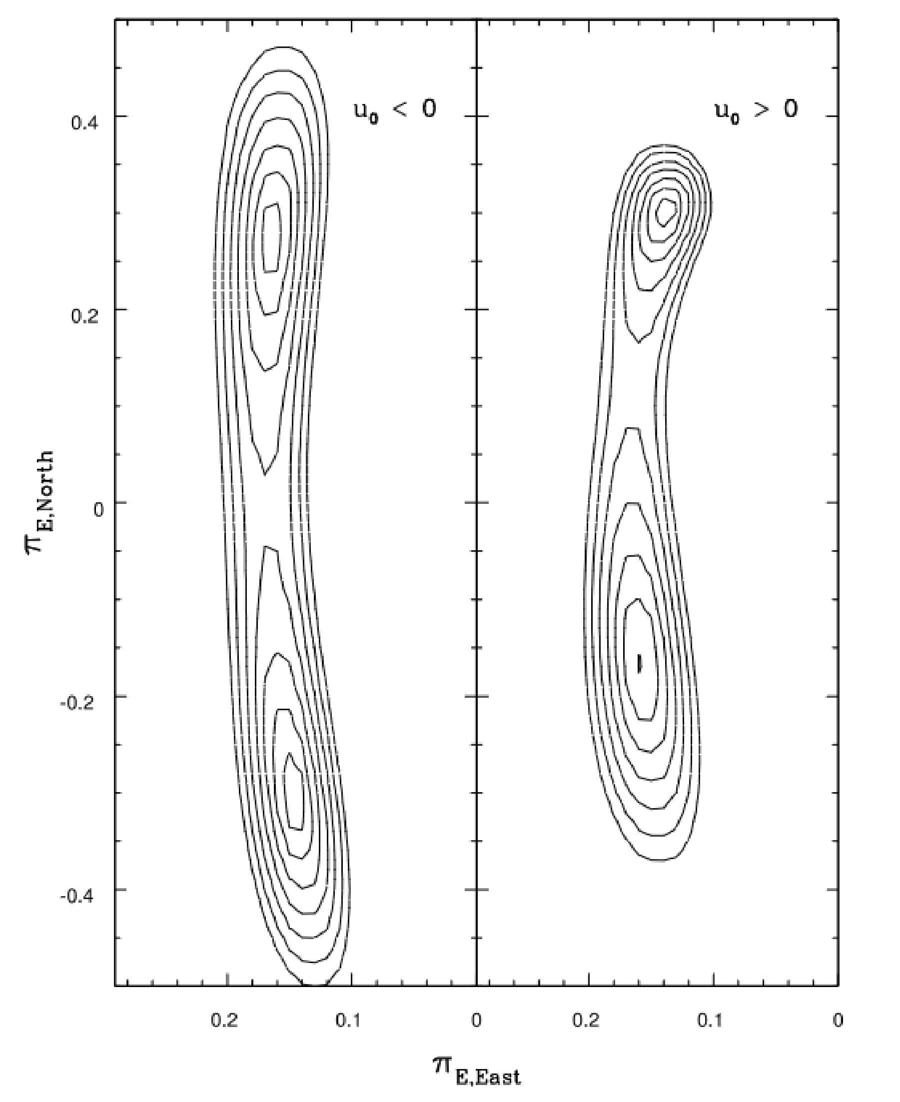


- Example: t_0 at the vernal equinox.
- $\pi_{E,N} < 0 \quad \pi_{E,E} = 0$
- Event slightly shorter (for $u_0 > 0$)



- The effects for $\pi_{E,N}$ are reversed if $u_0 < 0$.
- Degeneracy with t_E and u_0.

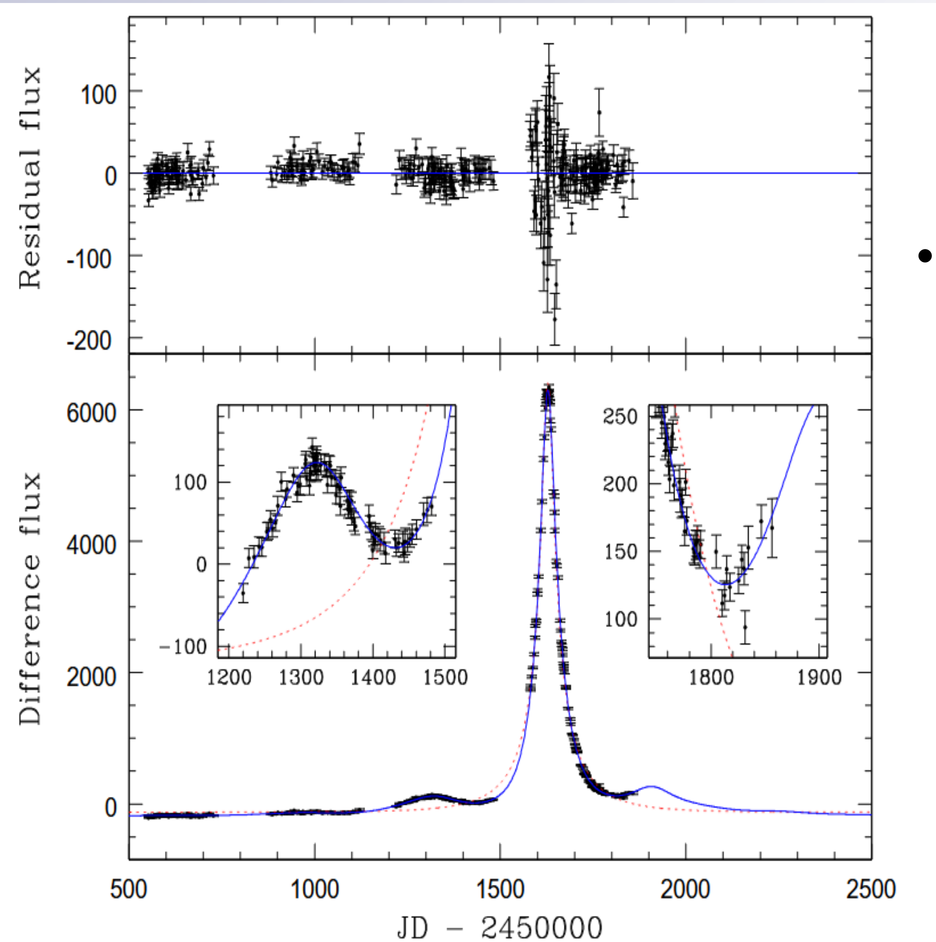
1-dimensional parallax



(Poindexter et al. 2005)

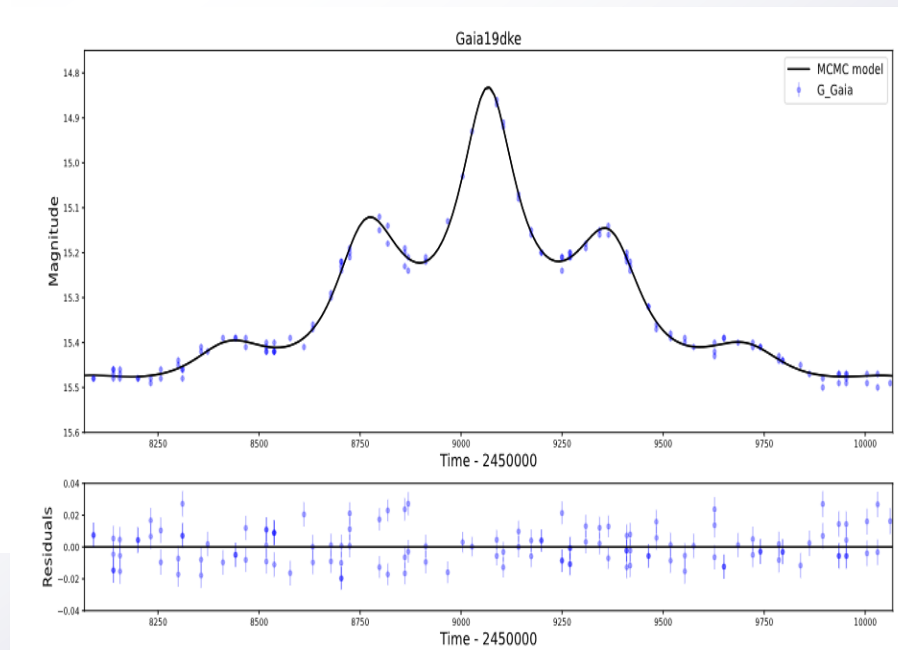
- Degeneracies in the annual parallax may result in a
  good measurement for $\pi_{E,E}$
  and a bad measurement for $\pi_{E,N}$
  (1-d parallax).

Multi-peak events



Smith et al. (2002)

- For very slow proper-motion, the lens may align with the source several times.

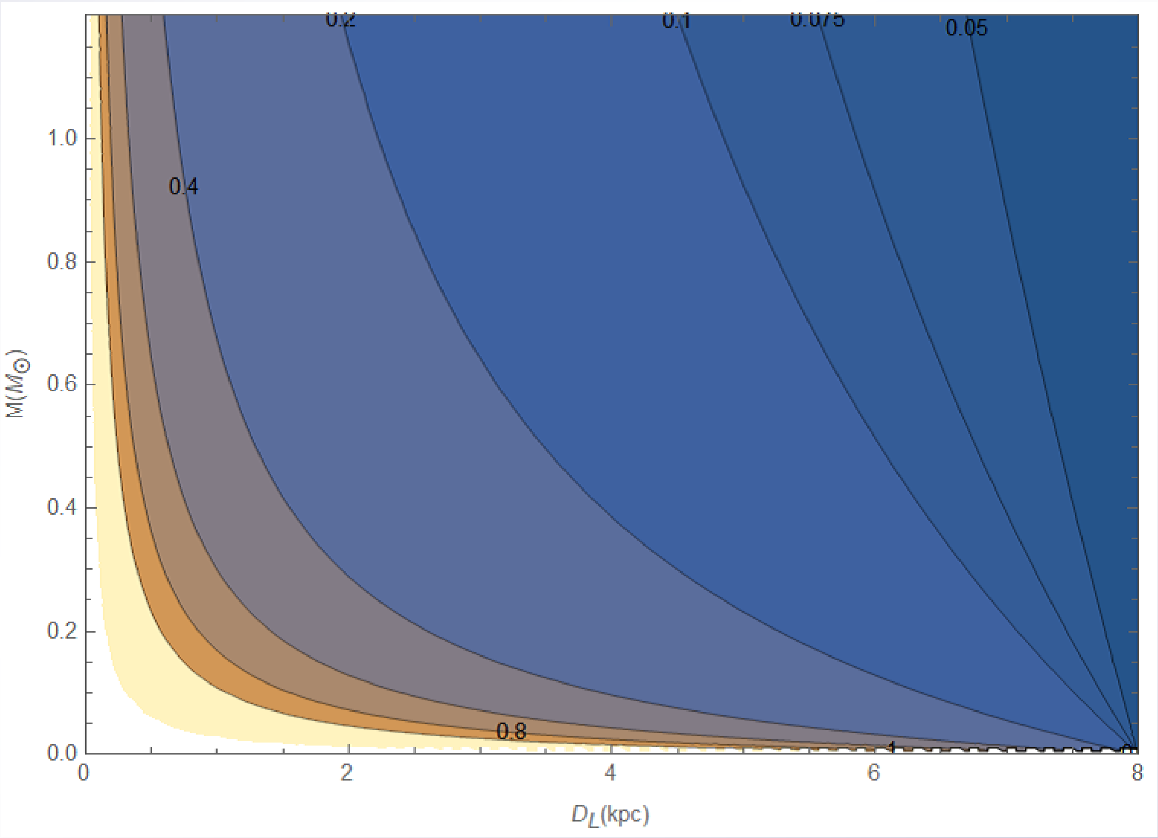


Maskoliunas et al. (2023)

Parallax constraint

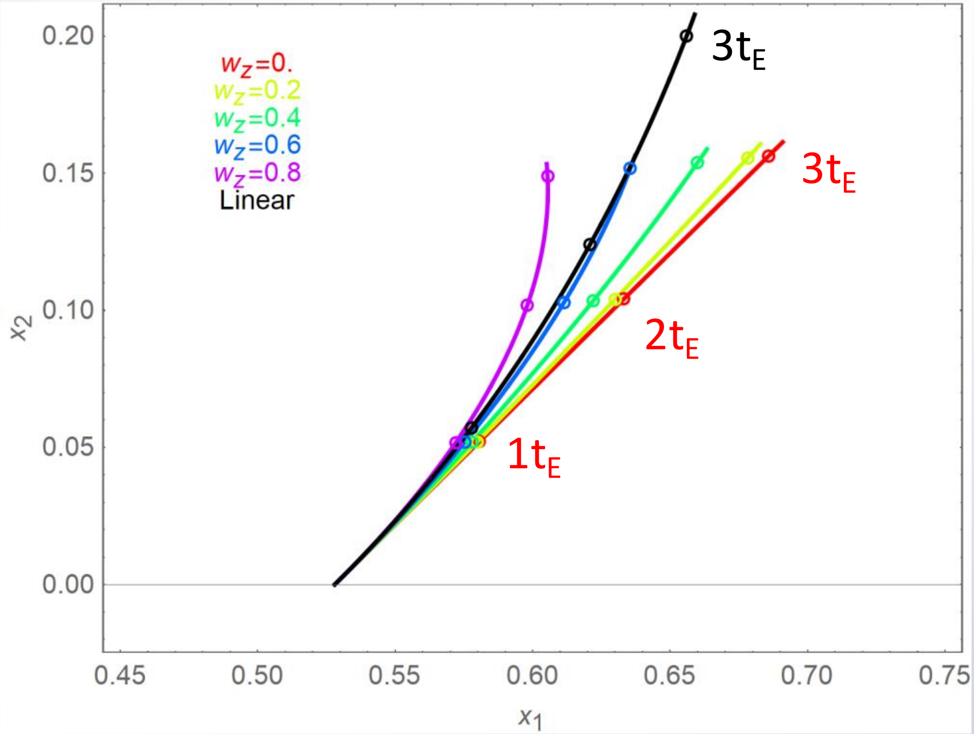
- Given the source distance, the measurement of the microlensing parallax provides a constraint in the mass-distance plane

$$\pi_E \equiv \frac{\pi_{rel}}{\theta_E} \Leftrightarrow M = \frac{c^2}{4G}\left(\frac{1}{D_L} - \frac{1}{D_S}\right)\left(\frac{\text{au}}{\pi_E}\right)^2$$



Lens orbital motion

- Binary lenses orbit around the common center of mass.
- The separation changes by $\gamma_{\parallel} \equiv \frac{1}{s}\frac{ds}{dt}$
- The binary axis rotates as $\gamma_{\perp} \equiv \frac{d\alpha}{dt}$
- We may approximate these change rates as constant throughout the duration of the event (linear approximation) *(Albrow et al. 2000)*.
- However, on long times, this approximation is unphysical.

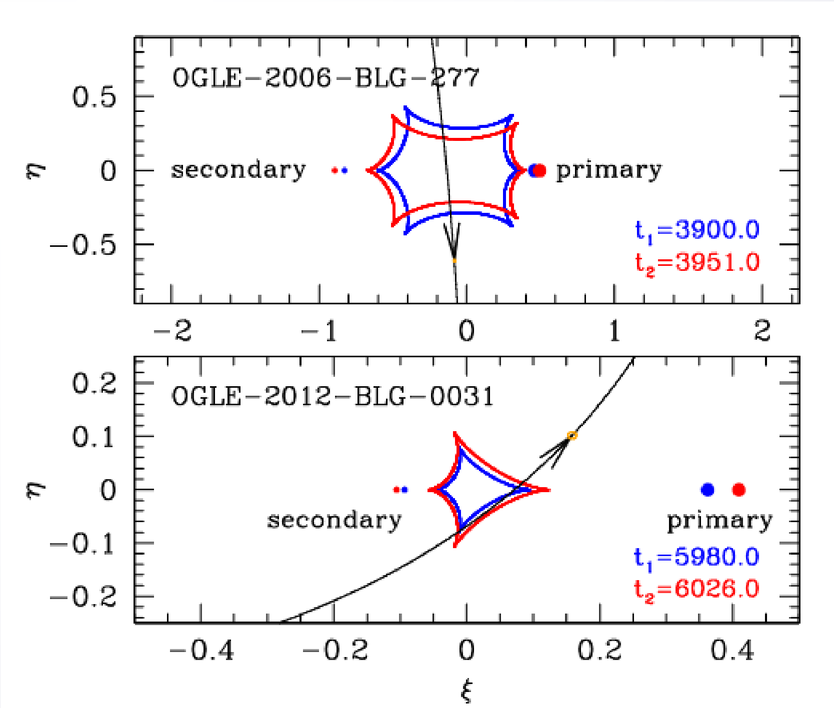


Circular orbital motion

- The minimal physical assumption is the circular orbital motion
- Besides $\gamma_{\parallel} \equiv \frac{1}{s}\frac{ds}{dt}$, $\gamma_{\perp} \equiv \frac{d\alpha}{dt}$, we also introduce the longitudinal velocity $\gamma_z \equiv \frac{1}{s}\frac{ds_z}{dt}$ *(Skowron et al. 2011)*.
- Then $s_z = -s\frac{\gamma_{\parallel}}{\gamma_z}$ and we can track the circular motion.

- For real microlensing events, γ_z will be poorly constrained.
- Overall, only long enough events will be sensitive to orbital motion.
- Resonant caustics are very sensitive to small changes in *s*.

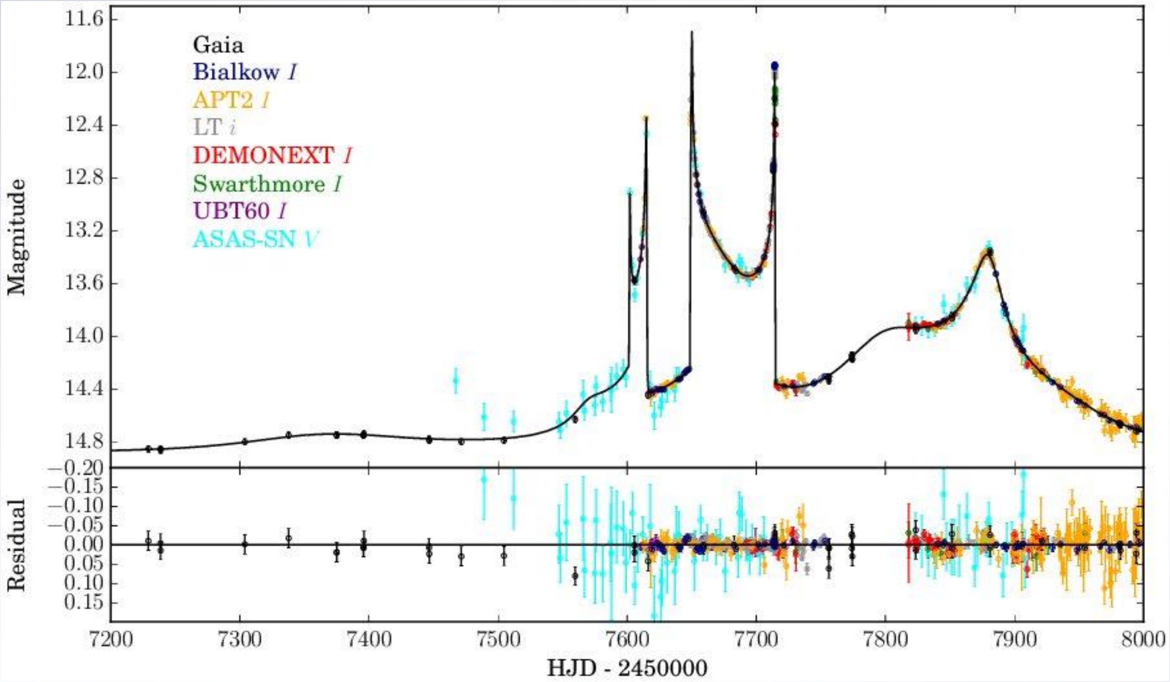
- Note that the microlensing parallax is often degenerate with weak orbital motion.

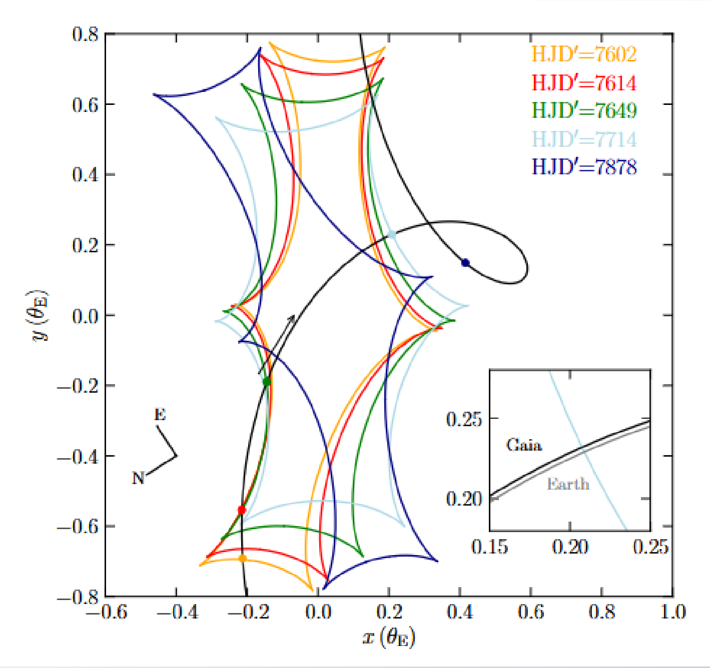


Park et al. (2018)

Keplerian orbital motion

- Binary systems may have highly eccentric orbits.
- Two more parameters are needed to fully characterize an elliptic orbit, e.g. $r_s \equiv \frac{s_z}{s}$, $a_s = \frac{a}{s_{tot}}$.





Gaia16aye (Wyrzykowski et al. 2022)

Constraints from orbital motion

- The 3rd Kepler's law can be imposed if the orbital period is well constrained: $\frac{4\pi^2}{T^2} = \frac{GM}{a^3}$

- If only two components of the orbital velocity are known, we can still check that the system is bound *(Dong et al. 2009)*:

$$\frac{1}{2}\left(v_x^2 + v_y^2 + v_z^2\right) = K_3 < -U_3 = G\frac{M}{\sqrt{x^2 + y^2 + z^2}}$$

- Removing the z components, this constraint becomes

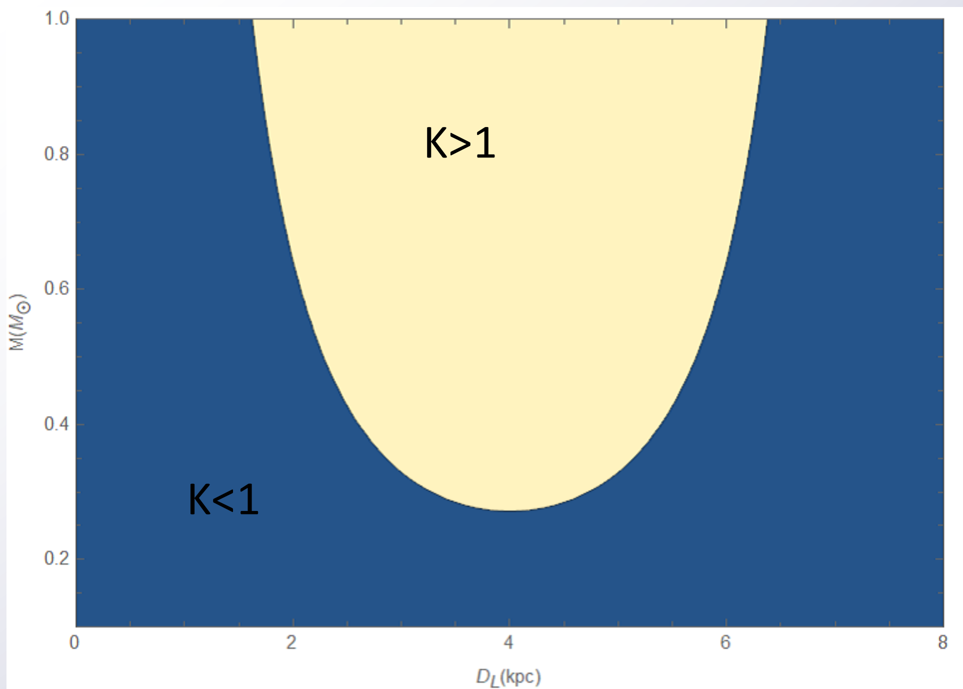
$$\frac{1}{2}\left(v_x^2 + v_y^2\right) < K_3 < -U_3 < G\frac{M}{\sqrt{x^2 + y^2}}$$

- In terms of microlensing parameters, we have

$$K = \frac{\left(\gamma_\parallel^2 + \gamma_\perp^2\right)s^3\theta_E^3 D_L^3}{2GM} < 1$$

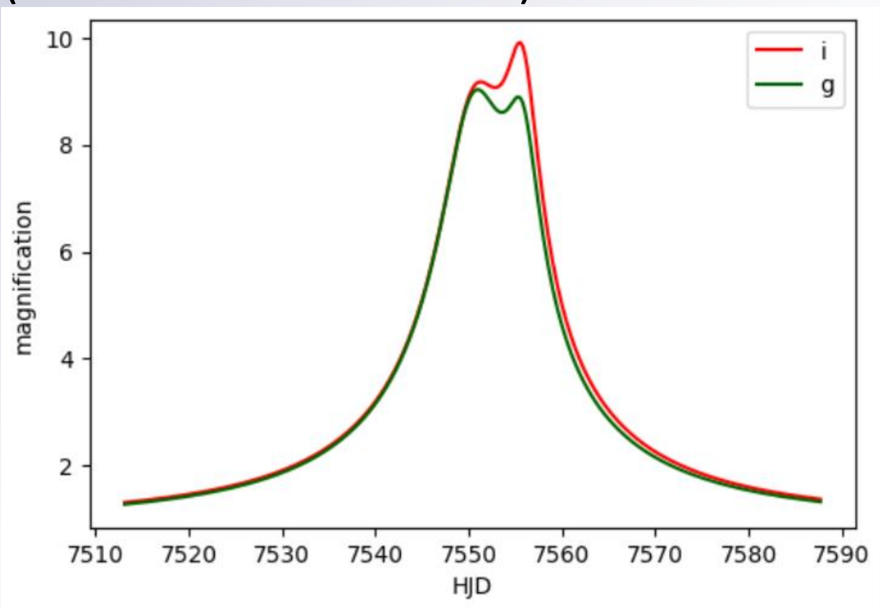
which becomes an upper limit for the mass of the system.

Remember! $\theta_E = \sqrt{\kappa M \pi_{rel}}$

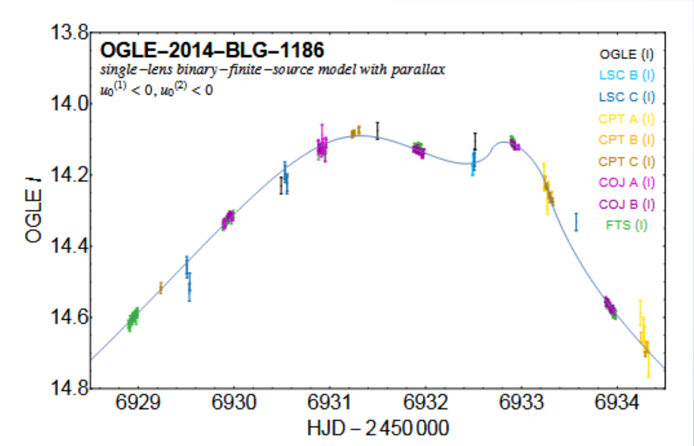


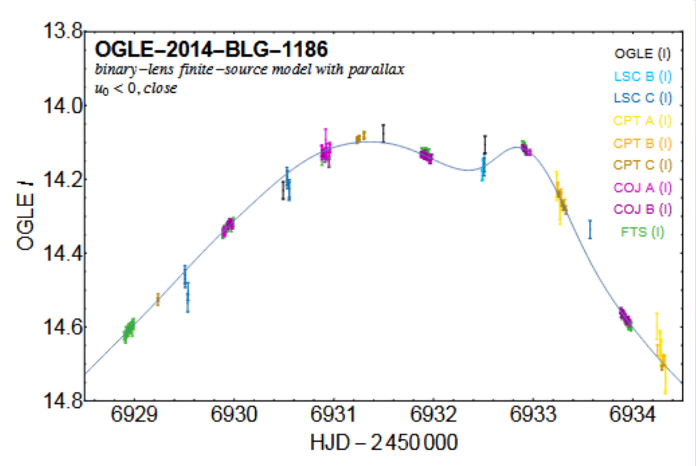
Binary sources

- Binary sources generate the sum of two Paczynski light curves *(Griest & Hu 1992)*.
- Additional parameters are $t_{0,2}$, $u_{0,2}$ and the flux ratio q_F (different for each band).



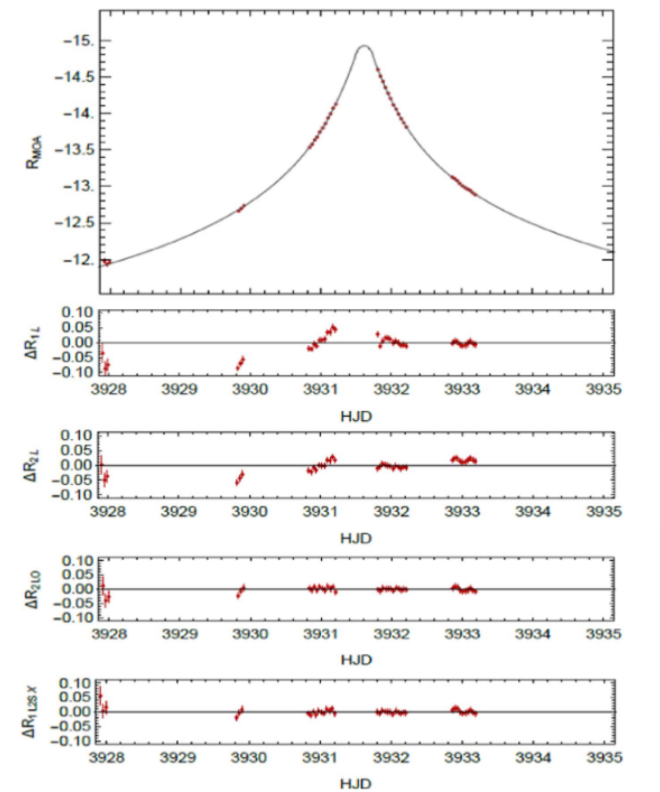
- With poor sampling, a planetary signal may be mimicked by a binary source. *(Dominik et al. 2019)*



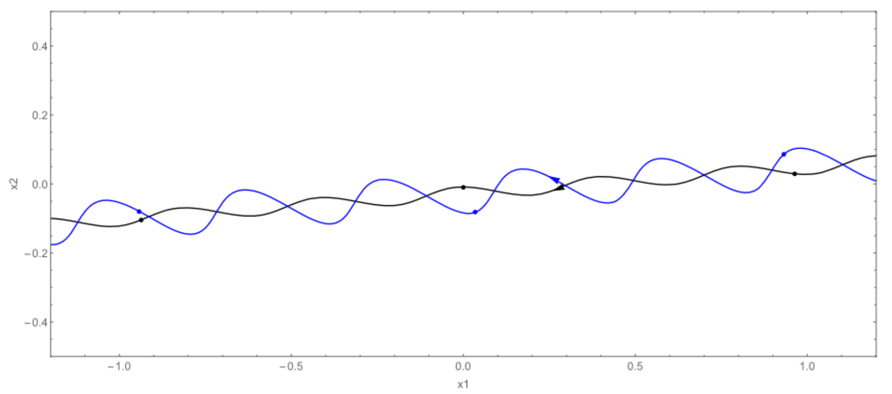


Xallarap

- A companion to the source may manifest through the orbital motion around the common center of mass.
- As the annual parallax is due to the orbital motion of the observer,
- The orbital motion of the source is called «Xallarap» *(Han & Gould 1997)*



Similarly to lens orbital motion, we may start from a circular motion with 3 parameters and expand to full Keplerian motion.

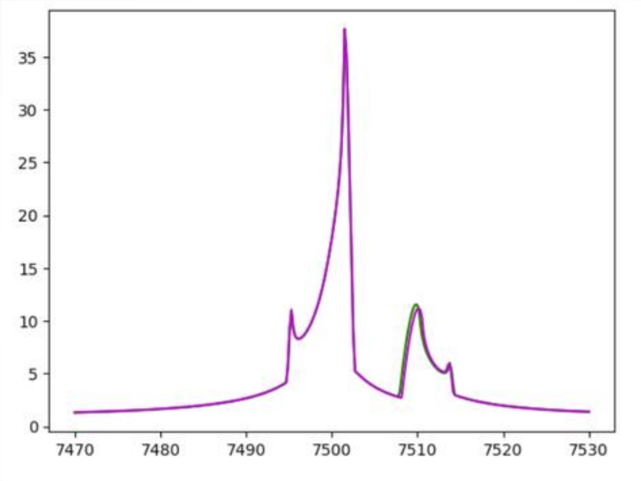


MOA-2006-BLG-074 (Rota et al. 2021)

- It is always difficult to exclude xallarap as a possible alternative.
- After all, most stars are in binary systems.
- If the orbital period is measured, the 3rd Kepler law can be implemented.

Libraries including Higher-order effects

- VBMicrolensing *(VB et al. 2010, 2018, 2021, 2025)* is the reference package for binary and multiple lens calculations *(github.com/valboz/VBMicrolensing)*
- All higher order effects can be included.
- RTModel *(VB 2024)* provides a framework for fitting microlensing events with all higher orders included *(github.com/valboz/RTModel)*.



- MulensModel *(github.com/rpoleski/MulensModel)*
- pyLIMA *(github.com/ebachelet/pyLIMA)*
- Provide independent implementations of higher orders effects and flexible solutions for microlensing fitting.
- Rely on VBMicrolensing for binary lens calculations.

- BAGLE *(github.com/MovingUniverseLab/BAGLE_Microlensing)*
- Accurate and rich framework for parallax and astrometry.
- In rapid development…